Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.4-f-x-^m-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [156]. This is test number [48].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (156)	% 0.00 (0)
Mathematica	% 94.23 (147)	% 5.77 (9)
Maple	% 87.82 (137)	% 12.18 (19)
Maxima	% 44.23 (69)	% 55.77 (87)
Fricas	% 69.23 (108)	% 30.77 (48)
Sympy	% 48.08 (75)	% 51.92 (81)
Giac	% 70.51 (110)	% 29.49 (46)
Mupad	% 78.21 (122)	% 21.79 (34)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

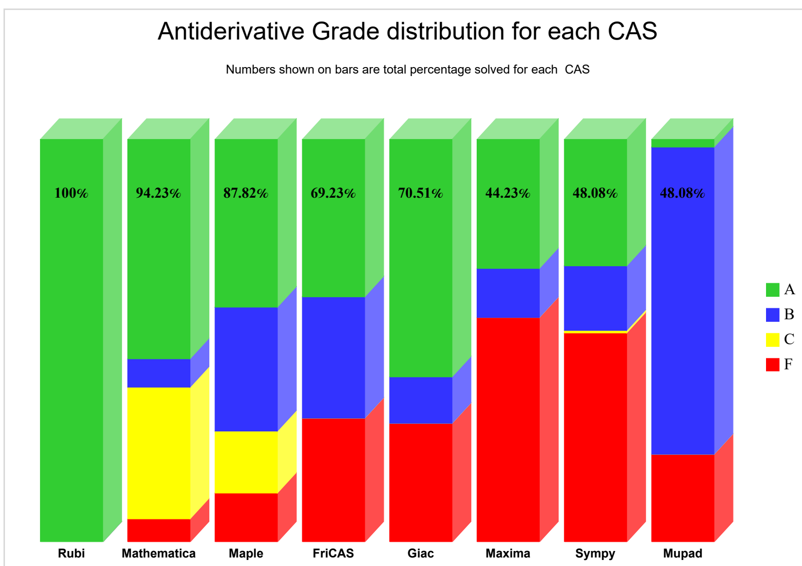
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

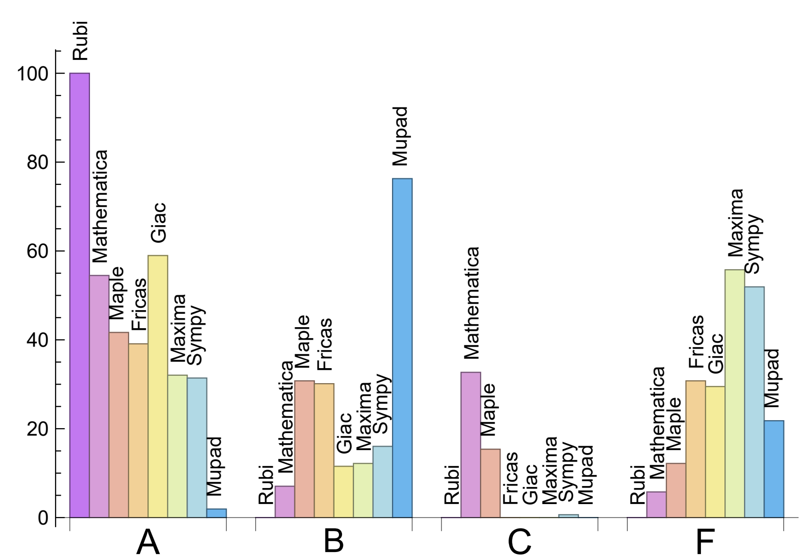
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	54.49	7.05	32.69	5.77
Maple	41.67	30.77	15.38	12.18
Maxima	32.05	12.18	0.00	55.77
Fricas	39.10	30.13	0.00	30.77
Sympy	31.41	16.03	0.64	51.92
Giac	58.97	11.54	0.00	29.49
Mupad	1.92	76.28	0.00	21.79

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	100.00 %	0.00 %	0.00 %
Maple	19	100.00 %	0.00 %	0.00 %
Maxima	87	66.67 %	4.60 %	28.74 %
Fricas	48	64.58 %	35.42 %	0.00 %
Sympy	81	3.70 %	90.12 %	6.17 %
Giac	46	78.26 %	10.87 %	10.87 %
Mupad	34	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

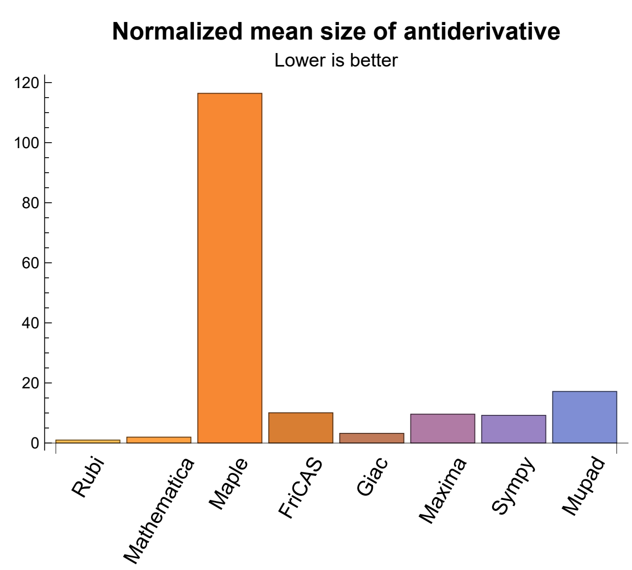
1.3 Performance

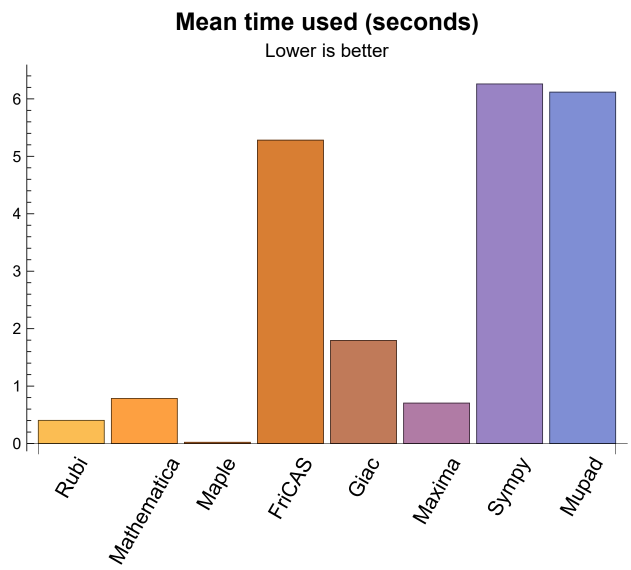
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.40	208.71	0.98	153.50	1.00
Mathematica	0.79	457.80	1.96	80.00	0.96
Maple	0.02	2589.11	116.42	70.00	1.24
Maxima	0.70	214.09	9.61	37.00	1.00
Fricas	5.28	940.94	10.07	216.00	2.49
Sympy	6.26	225.01	9.21	87.00	1.02
Giac	1.80	220.13	3.21	70.00	1.00
Mupad	6.12	3286.17	17.16	295.00	3.15

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{86, 155, 156}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {79, 143, 152, 153, 154}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

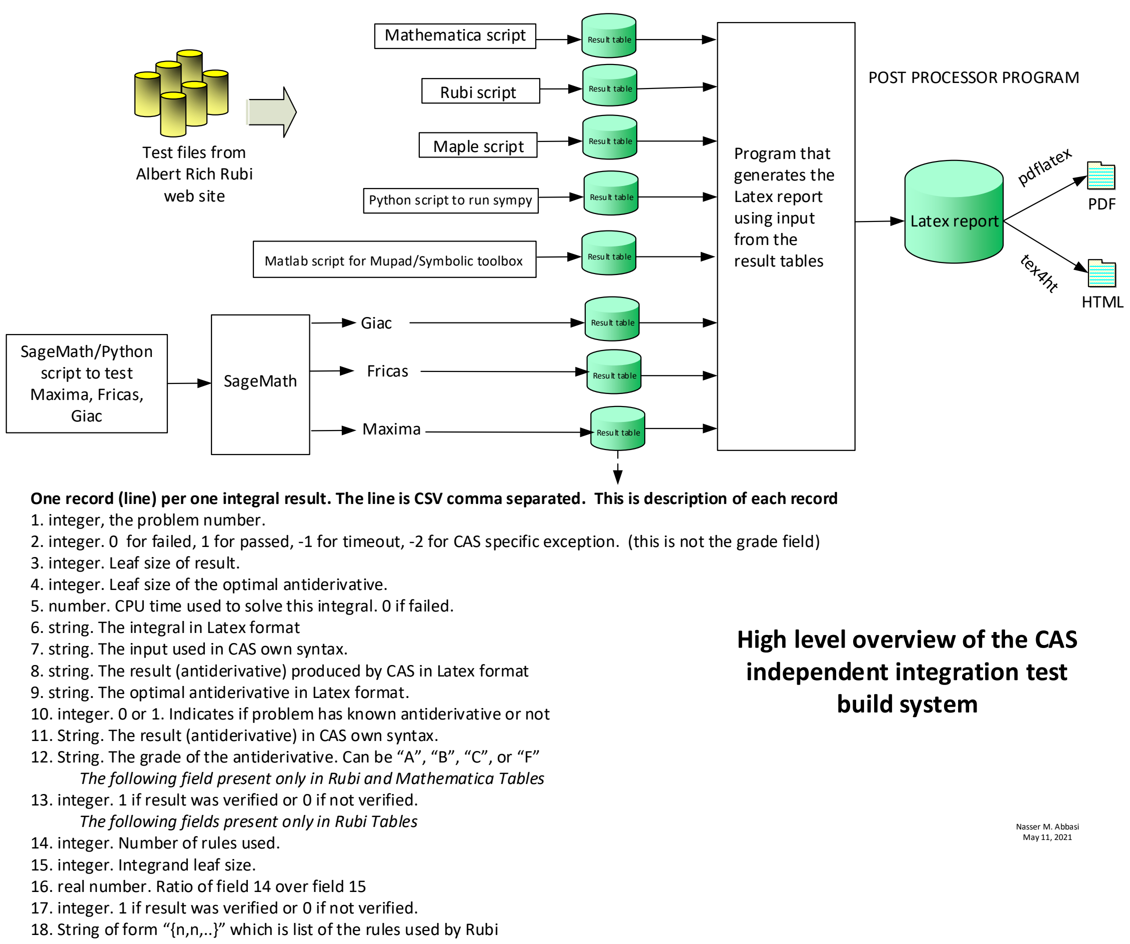
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 32, 44, 46, 53, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 96, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 144, 149, 152, 153, 154, 155, 156 }

B grade: { 93, 94, 95, 97, 98, 99, 101, 102, 103, 142, 143 }

C grade: { 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85, 138, 139, 140 }

F grade: { 90, 91, 92, 145, 146, 147, 148, 150, 151 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 53, 55, 57, 59, 64, 65, 66, 67, 74, 86, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 155, 156 }

B grade: { 6, 9, 35, 36, 37, 38, 39, 40, 41, 42, 46, 50, 61, 62, 63, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 125, 126, 127, 128 }

C grade: { 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 43, 45, 47, 49, 51, 52, 54, 56, 58, 60, 140 }

F grade: { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 32, 33, 53, 57, 86, 93, 97, 101, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 121, 122, 123, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 94, 95, 96, 98, 99, 100, 102, 103, 104, 110, 111, 112, 118, 119, 120, 124, 126, 127, 128 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 55, 57, 58, 61, 62, 63, 64, 65, 66, 86, 105, 106, 107, 108, 113, 114, 115, 116, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 8, 16, 17, 25, 26, 27, 28, 29, 30, 31, 45, 46, 47, 50, 54, 56, 59, 60, 71, 72, 73, 74, 75, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 112, 117, 118, 119, 120, 125, 126, 127, 128 }

C grade: { }

F grade: { 14, 15, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 49, 51, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 52, 53, 54, 55, 56, 57, 58, 59, 60, 105, 106, 107, 113, 114, 115, 121, 122, 123, 124, 137 }

B grade: { 9, 10, 11, 44, 93, 94, 95, 97, 98, 99, 101, 102, 103, 109, 110, 111, 117, 118, 119, 125, 126, 127, 129, 133, 138 }

C grade: { 144 }

F grade: { 12, 13, 14, 15, 16, 17, 18, 19, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 100, 104, 108, 112, 116, 120, 128, 130, 131, 132, 134, 135, 136, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 155, 156 }

B grade: { 25, 26, 27, 28, 29, 30, 31, 46, 50, 93, 94, 95, 96, 97, 98, 99, 100, 104 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 49, 51, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.8 Mupad

A grade: { 86, 155, 156 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

C grade: { }

F grade: { 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	164	169	166	182	187	173	158
normalized size	1	1.00	1.01	1.04	1.02	1.12	1.15	1.06	0.97
time (sec)	N/A	0.185	0.047	0.002	0.813	0.687	0.101	0.340	1.599

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	135	147	151	141	130
normalized size	1	1.00	1.00	1.01	1.00	1.09	1.12	1.04	0.96
time (sec)	N/A	0.125	0.037	0.000	0.720	0.903	0.093	0.341	0.056

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	102	112	117	109	102
normalized size	1	1.00	1.01	1.00	0.99	1.09	1.14	1.06	0.99
time (sec)	N/A	0.097	0.029	0.000	0.653	0.803	0.085	0.304	0.043

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	76	75	76	70
normalized size	1	1.00	1.00	0.96	0.95	1.04	1.03	1.04	0.96
time (sec)	N/A	0.062	0.022	0.000	0.747	0.826	0.077	0.332	0.035

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	40	39	43	38
normalized size	1	1.00	1.00	0.88	0.86	0.95	0.93	1.02	0.90
time (sec)	N/A	0.028	0.009	0.001	0.552	0.907	0.066	0.325	0.043

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	176	313	169	465	175	173	165
normalized size	1	1.00	0.94	1.66	0.90	2.47	0.93	0.92	0.88
time (sec)	N/A	0.211	0.155	0.007	1.523	0.896	0.901	0.372	0.269

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	199	345	204	697	206	199	187
normalized size	1	1.00	0.93	1.62	0.96	3.27	0.97	0.93	0.88
time (sec)	N/A	0.226	0.192	0.008	1.600	0.972	1.676	0.380	1.801

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	209	362	240	941	246	224	221
normalized size	1	1.00	0.86	1.50	0.99	3.89	1.02	0.93	0.91
time (sec)	N/A	0.262	0.267	0.010	1.694	0.556	5.234	0.401	0.288

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	126	260	0	430	620	131	3586
normalized size	1	1.00	0.95	1.97	0.00	3.26	4.70	0.99	27.17
time (sec)	N/A	0.218	0.068	0.006	0.000	1.839	55.468	0.999	2.404

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	175	0	305	434	95	2624
normalized size	1	1.00	0.96	1.80	0.00	3.14	4.47	0.98	27.05
time (sec)	N/A	0.120	0.070	0.003	0.000	1.360	17.489	1.065	2.950

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	216	287	70	1632
normalized size	1	1.00	0.99	1.38	0.00	3.00	3.99	0.97	22.67
time (sec)	N/A	0.073	0.051	0.004	0.000	1.095	6.548	1.206	2.627

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	240	0	76	4149
normalized size	1	1.00	1.03	1.36	0.00	3.08	0.00	0.97	53.19
time (sec)	N/A	0.128	0.035	0.006	0.000	1.427	0.000	1.046	6.765

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	130	191	0	385	0	128	7282
normalized size	1	1.00	1.16	1.71	0.00	3.44	0.00	1.14	65.02
time (sec)	N/A	0.197	0.050	0.008	0.000	2.321	0.000	1.071	9.569

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	723	88	70	0	0	0	0	13112
normalized size	1	1.00	0.12	0.10	0.00	0.00	0.00	0.00	18.14
time (sec)	N/A	1.813	0.048	0.013	0.000	0.000	0.000	0.000	42.007

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	88	67	0	0	0	0	11453
normalized size	1	1.00	0.12	0.09	0.00	0.00	0.00	0.00	15.95
time (sec)	N/A	1.457	0.050	0.004	0.000	0.000	0.000	0.000	30.152

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	59	49	0	13607	0	0	7457
normalized size	1	1.00	0.09	0.08	0.00	21.46	0.00	0.00	11.76
time (sec)	N/A	0.728	0.030	0.005	0.000	93.486	0.000	0.000	24.559

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	61	47	0	14094	0	0	7469
normalized size	1	1.00	0.10	0.07	0.00	22.23	0.00	0.00	11.78
time (sec)	N/A	0.654	0.030	0.007	0.000	27.687	0.000	0.000	18.962

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	85	70	0	0	0	0	11174
normalized size	1	1.00	0.13	0.11	0.00	0.00	0.00	0.00	17.11
time (sec)	N/A	1.175	0.047	0.013	0.000	0.000	0.000	0.000	38.020

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	89	68	0	0	0	0	13466
normalized size	1	1.00	0.14	0.10	0.00	0.00	0.00	0.00	20.56
time (sec)	N/A	1.110	0.047	0.010	0.000	0.000	0.000	0.000	37.903

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	37	37	42	37	39
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.058	0.016	0.003	0.988	0.719	0.137	0.418	0.056

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	32	24	26
normalized size	1	1.00	1.00	0.81	0.77	0.77	1.03	0.77	0.84
time (sec)	N/A	0.035	0.008	0.003	0.960	0.499	0.121	0.577	0.038

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
normalized size	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.040	0.009	0.003	0.951	0.615	0.135	0.568	0.046

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	38	34	41	35	36
normalized size	1	1.00	1.07	0.85	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.055	0.013	0.006	0.973	0.861	0.149	0.592	1.859

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	24	28	36	24	26
normalized size	1	1.00	1.45	0.81	0.77	0.90	1.16	0.77	0.84
time (sec)	N/A	0.045	0.013	0.006	0.960	0.804	0.144	0.449	0.045

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	1036	31	642	332
normalized size	1	1.00	0.11	0.11	0.00	2.48	0.07	1.54	0.79
time (sec)	N/A	0.538	0.011	0.007	0.000	1.018	0.182	0.576	0.652

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	48	44	0	1588	32	817	309
normalized size	1	1.00	0.13	0.12	0.00	4.16	0.08	2.14	0.81
time (sec)	N/A	0.328	0.013	0.004	0.000	1.143	0.185	0.692	2.279

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	46	41	0	1030	24	632	330
normalized size	1	1.00	0.12	0.11	0.00	2.72	0.06	1.67	0.87
time (sec)	N/A	0.259	0.013	0.006	0.000	0.992	0.180	0.626	2.376

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	55	44	0	1583	22	821	281
normalized size	1	1.00	0.13	0.11	0.00	3.85	0.05	2.00	0.68
time (sec)	N/A	0.276	0.013	0.005	0.000	1.220	0.181	0.582	2.264

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	1031	26	637	319
normalized size	1	1.00	0.14	0.11	0.00	2.51	0.06	1.55	0.78
time (sec)	N/A	0.277	0.012	0.004	0.000	1.121	0.179	0.722	2.299

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	47	46	0	1598	31	829	313
normalized size	1	1.00	0.11	0.11	0.00	3.84	0.07	1.99	0.75
time (sec)	N/A	0.275	0.014	0.007	0.000	1.317	0.195	0.708	0.398

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	1062	32	642	332
normalized size	1	1.00	0.11	0.11	0.00	2.54	0.08	1.54	0.79
time (sec)	N/A	0.359	0.012	0.007	0.000	1.167	0.200	0.642	2.399

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	33	32	32	37	32	34
normalized size	1	1.00	1.03	0.92	0.89	0.89	1.03	0.89	0.94
time (sec)	N/A	0.039	0.010	0.003	0.977	0.962	0.134	0.462	1.845

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	38	34	41	35	36
normalized size	1	1.00	1.41	0.90	0.97	0.87	1.05	0.90	0.92
time (sec)	N/A	0.056	0.014	0.006	0.989	1.050	0.150	0.540	1.847

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	0	34	41	35	36
normalized size	1	1.00	1.41	0.90	0.00	0.87	1.05	0.90	0.92
time (sec)	N/A	0.063	0.010	0.006	0.000	1.050	0.144	0.410	0.039

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	103	1070	0	0	400	0	-1
normalized size	1	1.00	0.26	2.70	0.00	0.00	1.01	0.00	-0.00
time (sec)	N/A	0.416	0.176	0.255	0.000	1.079	9.014	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	101	1010	0	0	257	0	-1
normalized size	1	1.00	0.28	2.84	0.00	0.00	0.72	0.00	-0.00
time (sec)	N/A	0.311	0.150	0.043	0.000	1.193	5.918	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	98	956	0	0	124	0	-1
normalized size	1	1.00	0.31	3.03	0.00	0.00	0.39	0.00	-0.00
time (sec)	N/A	0.246	0.128	0.036	0.000	1.010	3.449	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	98	907	0	0	119	0	-1
normalized size	1	1.00	0.35	3.26	0.00	0.00	0.43	0.00	-0.00
time (sec)	N/A	0.182	0.090	0.033	0.000	1.042	2.904	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	102	934	0	0	119	0	-1
normalized size	1	1.00	0.35	3.23	0.00	0.00	0.41	0.00	-0.00
time (sec)	N/A	0.189	0.105	0.046	0.000	0.999	15.073	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	129	1005	0	0	119	0	-1
normalized size	1	1.00	0.42	3.25	0.00	0.00	0.39	0.00	-0.00
time (sec)	N/A	0.211	0.138	0.051	0.000	1.026	86.879	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	166	1095	0	0	0	0	-1
normalized size	1	1.00	0.48	3.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.188	0.054	0.000	1.021	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	200	1182	0	0	0	0	-1
normalized size	1	1.00	0.51	3.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	0.238	0.056	0.000	0.805	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	0	0	0	50213
normalized size	1	1.00	0.20	0.15	0.00	0.00	0.00	0.00	115.97
time (sec)	N/A	1.132	0.075	0.006	0.000	0.000	0.000	0.000	9.632

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	216	287	70	3704
normalized size	1	1.00	0.99	1.38	0.00	3.00	3.99	0.97	51.44
time (sec)	N/A	0.072	0.057	0.004	0.000	1.015	18.295	20.738	4.206

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	59	51	0	13521	0	0	29445
normalized size	1	1.00	0.16	0.14	0.00	36.06	0.00	0.00	78.52
time (sec)	N/A	0.456	0.046	0.003	0.000	48.370	0.000	0.000	9.569

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	179	340	0	1535	0	1406	4501
normalized size	1	1.00	0.97	1.85	0.00	8.34	0.00	7.64	24.46
time (sec)	N/A	0.213	0.152	0.020	0.000	1.470	0.000	20.309	7.053

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	61	47	0	13304	0	0	36707
normalized size	1	1.00	0.16	0.13	0.00	35.48	0.00	0.00	97.89
time (sec)	N/A	0.351	0.047	0.002	0.000	9.666	0.000	0.000	8.746

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	240	0	78	8454
normalized size	1	1.00	1.03	1.36	0.00	3.08	0.00	1.00	108.38
time (sec)	N/A	0.126	0.033	0.007	0.000	2.397	0.000	20.628	5.271

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	85	72	0	0	0	0	39028
normalized size	1	1.00	0.22	0.18	0.00	0.00	0.00	0.00	99.56
time (sec)	N/A	0.683	0.064	0.007	0.000	0.000	0.000	0.000	9.459

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	89	365	0	2772	0	3006	15013
normalized size	1	1.00	0.45	1.83	0.00	13.93	0.00	15.11	75.44
time (sec)	N/A	0.311	0.045	0.023	0.000	2.630	0.000	22.519	7.620

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	86	68	0	0	0	0	65350
normalized size	1	1.00	0.22	0.17	0.00	0.00	0.00	0.00	165.86
time (sec)	N/A	0.625	0.071	0.007	0.000	0.000	0.000	0.000	10.224

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	46	34	0	218	170	208	56
normalized size	1	1.00	0.17	0.12	0.00	0.78	0.61	0.75	0.20
time (sec)	N/A	0.300	0.016	0.007	0.000	0.963	0.226	0.452	1.920

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
normalized size	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.042	0.014	0.006	1.507	0.887	0.146	0.628	0.049

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	55	46	0	715	27	253	248
normalized size	1	1.00	0.15	0.13	0.00	2.01	0.08	0.71	0.70
time (sec)	N/A	0.289	0.016	0.008	0.000	1.195	3.164	0.481	1.985

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	39	0	41	42	31	20
normalized size	1	1.00	0.88	0.78	0.00	0.82	0.84	0.62	0.40
time (sec)	N/A	0.040	0.016	0.017	0.000	1.055	0.129	0.440	1.889

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	715	26	253	208
normalized size	1	1.00	0.16	0.12	0.00	2.01	0.07	0.71	0.59
time (sec)	N/A	0.216	0.014	0.000	0.000	0.923	3.226	0.536	0.002

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	38	34	41	38	36
normalized size	1	1.00	1.07	0.85	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.053	0.013	0.007	0.965	0.792	0.160	0.453	1.886

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	47	38	0	224	168	210	58
normalized size	1	1.00	0.17	0.14	0.00	0.80	0.60	0.75	0.21
time (sec)	N/A	0.208	0.015	0.010	0.000	0.971	0.234	0.582	1.860

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	70	0	188	76	81	56
normalized size	1	1.00	0.55	0.79	0.00	2.11	0.85	0.91	0.63
time (sec)	N/A	0.090	0.016	0.011	0.000	0.841	0.232	0.528	0.099

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	47	46	0	608	32	258	479
normalized size	1	1.00	0.13	0.12	0.00	1.64	0.09	0.70	1.29
time (sec)	N/A	0.269	0.015	0.009	0.000	1.130	3.175	0.444	0.068

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	283	662	0	1027	0	295	2490
normalized size	1	1.00	1.01	2.36	0.00	3.67	0.00	1.05	8.89
time (sec)	N/A	0.597	0.229	0.014	0.000	80.215	0.000	0.376	6.206

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	218	512	0	798	0	224	2051
normalized size	1	1.00	1.00	2.35	0.00	3.66	0.00	1.03	9.41
time (sec)	N/A	0.395	0.172	0.010	0.000	35.524	0.000	0.368	5.242

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	178	388	0	596	0	185	1367
normalized size	1	1.00	1.01	2.20	0.00	3.39	0.00	1.05	7.77
time (sec)	N/A	0.285	0.185	0.007	0.000	10.870	0.000	0.401	4.339

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	275	0	405	0	149	966
normalized size	1	1.00	0.89	1.85	0.00	2.72	0.00	1.00	6.48
time (sec)	N/A	0.210	0.119	0.006	0.000	3.568	0.000	0.366	3.668

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	169	0	305	0	127	801
normalized size	1	1.00	0.86	1.36	0.00	2.46	0.00	1.02	6.46
time (sec)	N/A	0.145	0.074	0.006	0.000	1.490	0.000	0.392	3.407

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	105	168	0	305	0	126	521
normalized size	1	1.00	0.85	1.37	0.00	2.48	0.00	1.02	4.24
time (sec)	N/A	0.107	0.073	0.006	0.000	1.453	0.000	0.341	3.817

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	152	285	0	0	0	164	2399
normalized size	1	1.01	0.96	1.80	0.00	0.00	0.00	1.04	15.18
time (sec)	N/A	0.271	0.186	0.007	0.000	0.000	0.000	0.351	5.400

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	194	412	0	0	0	210	2388
normalized size	1	1.00	1.01	2.13	0.00	0.00	0.00	1.09	12.37
time (sec)	N/A	0.343	0.174	0.011	0.000	0.000	0.000	0.342	20.389

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	252	562	0	0	0	279	3530
normalized size	1	1.00	1.00	2.23	0.00	0.00	0.00	1.11	14.01
time (sec)	N/A	0.428	0.220	0.013	0.000	0.000	0.000	0.348	26.162

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	338	943	0	0	0	565	3503
normalized size	1	1.00	0.99	2.75	0.00	0.00	0.00	1.65	10.21
time (sec)	N/A	0.907	0.356	0.014	0.000	0.000	0.000	0.420	8.039

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	269	765	0	2139	0	476	2495
normalized size	1	1.00	0.98	2.79	0.00	7.81	0.00	1.74	9.11
time (sec)	N/A	0.563	0.287	0.011	0.000	103.931	0.000	0.401	6.003

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	207	580	0	1465	0	412	2037
normalized size	1	1.00	0.84	2.36	0.00	5.96	0.00	1.67	8.28
time (sec)	N/A	0.395	0.227	0.008	0.000	36.698	0.000	0.422	5.112

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	159	389	0	1120	0	331	1585
normalized size	1	1.00	0.82	2.01	0.00	5.77	0.00	1.71	8.17
time (sec)	N/A	0.306	0.225	0.010	0.000	13.249	0.000	0.347	6.091

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	148	328	0	1059	0	323	1768
normalized size	1	1.00	0.81	1.79	0.00	5.79	0.00	1.77	9.66
time (sec)	N/A	0.236	0.244	0.008	0.000	14.162	0.000	0.366	8.070

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	151	386	0	1079	0	331	1782
normalized size	1	1.00	0.80	2.04	0.00	5.71	0.00	1.75	9.43
time (sec)	N/A	0.305	0.211	0.009	0.000	7.015	0.000	0.354	8.109

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	249	246	589	0	0	0	391	3510
normalized size	1	1.00	0.99	2.38	0.00	0.00	0.00	1.58	14.15
time (sec)	N/A	0.409	0.253	0.013	0.000	0.000	0.000	0.408	25.284

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	287	791	0	0	0	487	4948
normalized size	1	1.00	0.99	2.72	0.00	0.00	0.00	1.67	17.00
time (sec)	N/A	0.563	0.340	0.014	0.000	0.000	0.000	0.361	31.159

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	370	993	0	0	0	587	7144
normalized size	1	1.00	0.99	2.67	0.00	0.00	0.00	1.58	19.20
time (sec)	N/A	0.851	0.425	0.016	0.000	0.000	0.000	0.488	45.611

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	981	981	10904	11938	0	0	0	0	-1
normalized size	1	1.00	11.12	12.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.172	14.199	0.161	0.000	1.256	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	778	778	7531	9182	0	0	0	0	-1
normalized size	1	1.00	9.68	11.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.349	13.735	0.066	0.000	0.864	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	5350	6302	0	0	0	0	-1
normalized size	1	1.00	8.41	9.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.993	13.051	0.052	0.000	1.305	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	693	4361	0	0	0	0	-1
normalized size	1	1.00	1.26	7.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	11.771	0.047	0.000	1.194	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	955	955	1258	3023	0	0	0	0	-1
normalized size	1	1.00	1.32	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.313	10.527	0.047	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	929	929	1372	3553	0	0	0	0	-1
normalized size	1	1.00	1.48	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.725	11.413	0.049	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1287	1287	811	4957	0	0	0	0	-1
normalized size	1	1.00	0.63	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.302	12.532	0.057	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.195	0.432	0.000	0.912	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	249	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.310	0.089	0.000	0.946	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	189	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.159	0.093	0.000	0.806	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	136	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.077	0.087	0.000	0.850	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.110	0.109	0.000	0.627	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.168	0.112	0.000	0.648	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	0.706	0.111	0.000	0.481	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	201	46548	14	1446	1326	216	1203
normalized size	1	1.00	12.56	2909.25	0.88	90.38	82.88	13.50	75.19
time (sec)	N/A	0.060	0.176	0.003	0.430	0.463	0.350	0.434	3.341

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1240	1454	1384	246	1210
normalized size	1	1.00	12.94	2586.22	68.89	80.78	76.89	13.67	67.22
time (sec)	N/A	0.329	0.177	0.003	0.490	0.511	0.338	0.661	3.234

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1240	1454	1394	246	1210
normalized size	1	1.00	12.94	2586.22	68.89	80.78	77.44	13.67	67.22
time (sec)	N/A	0.302	0.182	0.003	0.546	0.643	0.342	0.609	3.183

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	2042	2041	1297	0	1693	1395
normalized size	1	1.00	0.96	88.78	88.74	56.39	0.00	73.61	60.65
time (sec)	N/A	0.056	0.068	0.062	0.857	0.736	0.000	1.002	5.778

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	201	47685	16	1450	1326	218	1208
normalized size	1	1.00	11.17	2649.17	0.89	80.56	73.67	12.11	67.11
time (sec)	N/A	0.069	0.175	0.004	0.444	0.502	0.360	0.473	1.376

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1242	1454	1384	246	1214
normalized size	1	1.00	11.65	2384.40	62.10	72.70	69.20	12.30	60.70
time (sec)	N/A	0.322	0.169	0.002	0.525	0.496	0.359	0.572	3.251

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1242	1454	1394	246	1214
normalized size	1	1.00	11.65	2384.40	62.10	72.70	69.70	12.30	60.70
time (sec)	N/A	0.310	0.167	0.001	0.505	0.543	0.360	0.683	1.282

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	2046	2045	1299	0	1693	1401
normalized size	1	1.00	0.96	81.84	81.80	51.96	0.00	67.72	56.04
time (sec)	N/A	0.060	0.054	0.059	0.827	0.763	0.000	1.089	5.776

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	13	154	175	13	154
normalized size	1	1.00	11.47	10.33	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.014	0.005	0.001	0.435	0.478	0.129	0.402	2.088

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	15	156
normalized size	1	1.00	11.38	9.81	9.75	9.75	11.38	0.94	9.75
time (sec)	N/A	0.054	0.006	0.001	0.433	0.770	0.127	0.383	2.080

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	15	156
normalized size	1	1.00	11.62	9.81	9.75	9.75	11.56	0.94	9.75
time (sec)	N/A	0.056	0.006	0.001	0.439	0.701	0.127	0.508	2.079

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	0	189	229
normalized size	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90
time (sec)	N/A	0.033	0.117	0.035	0.478	0.683	0.000	0.433	2.628

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	12	11	11	10	12	11
normalized size	1	1.00	0.91	1.09	1.00	1.00	0.91	1.09	1.00
time (sec)	N/A	0.004	0.003	0.002	0.446	0.752	0.155	0.347	1.962

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.019	0.006	0.002	0.428	0.714	0.280	1.734	1.956

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.024	0.006	0.001	0.439	0.632	0.413	1.092	0.052

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	23	19	0	19	121
normalized size	1	1.00	1.00	1.26	1.21	1.00	0.00	1.00	6.37
time (sec)	N/A	0.027	0.105	0.025	0.598	0.704	0.000	0.464	2.317

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	14	350	359	14	358
normalized size	1	1.00	0.94	0.94	0.88	21.88	22.44	0.88	22.38
time (sec)	N/A	0.005	0.012	0.000	0.438	0.644	4.791	0.395	3.616

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
normalized size	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.020	0.012	0.000	0.948	0.734	7.662	6.776	12.162

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
normalized size	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.023	0.013	0.002	0.954	0.647	11.757	22.371	18.211

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	416	394	0	21	496
normalized size	1	1.00	0.96	0.96	18.09	17.13	0.00	0.91	21.57
time (sec)	N/A	0.027	0.060	0.064	2.393	0.748	0.000	0.633	23.011

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	13	13	10	14	13
normalized size	1	1.00	0.92	1.08	1.00	1.00	0.77	1.08	1.00
time (sec)	N/A	0.005	0.005	0.001	0.435	0.495	0.151	0.390	0.049

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.019	0.007	0.000	0.437	0.555	0.281	1.624	0.049

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.024	0.007	0.001	0.431	0.615	0.392	1.040	0.059

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	25	21	0	21	199
normalized size	1	1.00	1.00	1.24	1.19	1.00	0.00	1.00	9.48
time (sec)	N/A	0.029	0.117	0.025	0.599	0.676	0.000	0.374	2.676

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	17	16	354	359	16	358
normalized size	1	1.00	0.89	0.94	0.89	19.67	19.94	0.89	19.89
time (sec)	N/A	0.004	0.013	0.002	0.428	0.674	5.060	0.427	5.221

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
normalized size	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.020	0.017	0.000	0.967	0.763	8.089	7.187	11.044

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
normalized size	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.024	0.017	0.001	0.938	0.535	11.792	22.351	16.597

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	419	397	0	23	496
normalized size	1	1.00	0.92	0.96	16.76	15.88	0.00	0.92	19.84
time (sec)	N/A	0.029	0.066	0.067	2.423	0.705	0.000	0.790	22.399

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
normalized size	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.004	0.004	0.002	0.440	0.807	0.122	0.456	0.051

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
normalized size	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.024	0.006	0.006	0.426	0.839	0.191	0.486	0.064

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
normalized size	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.030	0.007	0.006	0.427	0.841	0.202	0.340	1.991

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	47	17	48	17	28
normalized size	1	1.00	1.00	1.20	3.13	1.13	3.20	1.13	1.87
time (sec)	N/A	0.035	0.012	0.022	0.442	0.873	31.234	0.374	2.225

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	13	81	87	13	12
normalized size	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.004	0.020	0.016	0.422	0.819	0.879	0.318	4.296

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.021	0.029	0.019	0.520	0.883	1.384	0.450	2.332

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.025	0.037	0.013	0.514	0.811	1.897	0.606	5.065

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	105	0	20	107
normalized size	1	1.00	1.00	9.67	29.14	5.00	0.00	0.95	5.10
time (sec)	N/A	0.032	0.181	0.052	0.663	0.741	0.000	0.440	2.357

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	28	104	20	39
normalized size	1	1.00	0.95	1.05	1.00	1.40	5.20	1.00	1.95
time (sec)	N/A	0.005	0.009	0.003	0.425	0.968	57.114	0.414	2.037

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	0	23	49
normalized size	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96
time (sec)	N/A	0.019	0.011	0.004	0.595	0.757	0.000	0.460	2.092

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	0	23	49
normalized size	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96
time (sec)	N/A	0.024	0.012	0.006	0.578	0.786	0.000	0.319	2.115

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	40	39	38	0	27	56
normalized size	1	1.00	0.96	1.48	1.44	1.41	0.00	1.00	2.07
time (sec)	N/A	0.028	0.029	0.059	0.722	0.878	0.000	0.845	2.569

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	104	22	42
normalized size	1	1.00	0.95	1.05	1.00	1.45	4.73	1.00	1.91
time (sec)	N/A	0.005	0.011	0.001	0.422	0.860	56.660	0.397	2.049

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	0	25	52
normalized size	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93
time (sec)	N/A	0.020	0.015	0.003	0.597	0.907	0.000	0.454	2.047

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	0	25	52
normalized size	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93
time (sec)	N/A	0.025	0.016	0.006	0.588	0.597	0.000	0.385	2.079

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	45	43	42	0	29	59
normalized size	1	1.00	0.97	1.55	1.48	1.45	0.00	1.00	2.03
time (sec)	N/A	0.028	0.034	0.060	0.708	0.928	0.000	0.820	2.536

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	19	26	46	19	23
normalized size	1	1.00	0.89	1.26	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.004	0.010	0.005	0.428	0.811	0.662	0.410	2.034

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	85	22	31
normalized size	1	1.00	4.04	1.29	1.46	1.29	3.54	0.92	1.29
time (sec)	N/A	0.014	0.073	0.003	0.588	0.763	17.117	0.400	2.074

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	0	22	31
normalized size	1	1.00	4.04	1.29	1.46	1.29	0.00	0.92	1.29
time (sec)	N/A	0.020	0.074	0.003	0.590	0.731	0.000	0.671	2.069

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	111	155	40	36	0	26	34
normalized size	1	1.00	4.27	5.96	1.54	1.38	0.00	1.00	1.31
time (sec)	N/A	0.079	0.130	0.106	0.766	0.644	0.000	0.834	2.126

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	158	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.305	0.034	0.000	0.759	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	5363	0	0	0	0	0	-1
normalized size	1	1.00	14.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.380	6.519	0.041	0.000	0.597	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	816	816	20515	0	0	0	0	0	-1
normalized size	1	1.00	25.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.552	7.537	0.061	0.000	1.011	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	34	33	126	0	31
normalized size	1	1.00	1.00	0.77	0.72	0.70	2.68	0.00	0.66
time (sec)	N/A	0.062	0.023	0.002	0.445	0.885	6.321	0.000	2.463

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	0.161	0.092	0.000	0.978	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.501	0.161	0.092	0.000	0.653	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.132	0.090	0.000	0.618	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.083	0.089	0.000	0.654	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	218	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	0.675	0.095	0.000	0.689	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.489	0.132	0.091	0.000	0.634	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.125	0.089	0.000	0.715	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	391	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	1.136	0.069	0.000	0.733	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	273	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	0.600	0.052	0.000	0.665	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.302	0.050	0.000	0.556	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.190	0.088	0.000	0.821	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.269	0.092	0.000	0.850	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [.4231]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	22	0.045
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	22	0.045
5	A	2	1	1.00	20	0.050
6	A	8	8	1.00	22	0.364
7	A	8	8	1.00	22	0.364
8	A	8	8	1.00	22	0.364
9	A	7	6	1.00	25	0.240
10	A	6	6	1.00	25	0.240
11	A	5	5	1.00	25	0.200
12	A	7	6	1.00	25	0.240
13	A	7	6	1.00	25	0.240
14	A	14	8	1.00	25	0.320
15	A	14	8	1.00	25	0.320
16	A	13	7	1.00	23	0.304
17	A	13	7	1.00	22	0.318
18	A	14	8	1.00	25	0.320
19	A	14	8	1.00	25	0.320
20	A	7	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	4	4	1.00	23	0.174
22	A	5	5	1.00	23	0.217
23	A	7	6	1.00	23	0.261
24	A	5	4	1.00	23	0.174
25	A	15	9	1.00	23	0.391
26	A	15	9	1.00	23	0.391
27	A	14	8	1.00	23	0.348
28	A	13	7	1.00	21	0.333
29	A	13	7	1.00	20	0.350
30	A	14	8	1.00	23	0.348
31	A	15	9	1.00	23	0.391
32	A	5	5	1.00	21	0.238
33	A	7	6	1.00	21	0.286
34	A	8	7	1.00	18	0.389
35	A	6	4	1.00	24	0.167
36	A	5	4	1.00	24	0.167
37	A	4	4	1.00	24	0.167
38	A	3	3	1.00	24	0.125
39	A	3	3	1.00	24	0.125
40	A	3	3	1.00	24	0.125
41	A	4	4	1.00	24	0.167
42	A	5	4	1.00	24	0.167
43	A	8	5	1.00	25	0.200
44	A	5	5	1.00	25	0.200
45	A	7	4	1.00	25	0.160
46	A	4	3	1.00	23	0.130
47	A	7	4	1.00	22	0.182
48	A	7	6	1.00	25	0.240
49	A	8	5	1.00	25	0.200
50	A	5	4	1.00	25	0.160
51	A	8	5	1.00	25	0.200
52	A	20	7	1.00	23	0.304
53	A	5	5	1.00	23	0.217
54	A	21	7	1.00	23	0.304
55	A	4	3	1.00	21	0.143
56	A	19	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	6	1.00	23	0.261
58	A	20	7	1.00	23	0.304
59	A	11	8	1.00	23	0.348
60	A	21	9	1.00	23	0.391
61	A	7	6	1.00	25	0.240
62	A	7	6	1.00	25	0.240
63	A	7	6	1.00	23	0.261
64	A	7	6	1.00	22	0.273
65	A	7	6	1.00	25	0.240
66	A	7	7	1.00	25	0.280
67	A	7	6	1.01	25	0.240
68	A	7	6	1.00	25	0.240
69	A	7	6	1.00	25	0.240
70	A	7	6	1.00	25	0.240
71	A	7	6	1.00	25	0.240
72	A	7	6	1.00	23	0.261
73	A	7	6	1.00	22	0.273
74	A	7	6	1.00	25	0.240
75	A	8	7	1.00	25	0.280
76	A	7	6	1.00	25	0.240
77	A	7	6	1.00	25	0.240
78	A	7	6	1.00	25	0.240
79	A	11	7	1.00	29	0.241
80	A	10	7	1.00	29	0.241
81	A	8	7	1.00	29	0.241
82	A	8	7	1.00	27	0.259
83	A	16	11	1.00	26	0.423
84	A	16	11	1.00	29	0.379
85	A	24	12	1.00	29	0.414
86	A	0	0	0.00	0	0.000
87	A	13	4	1.00	26	0.154
88	A	10	4	1.00	26	0.154
89	A	7	4	1.00	24	0.167
90	A	6	3	1.00	26	0.115
91	A	8	3	1.00	26	0.115
92	A	10	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
93	A	1	1	1.00	19	0.053
94	A	2	2	1.00	24	0.083
95	A	2	2	1.00	26	0.077
96	A	2	2	1.00	30	0.067
97	A	1	1	1.00	21	0.048
98	A	2	2	1.00	26	0.077
99	A	2	2	1.00	28	0.071
100	A	2	2	1.00	32	0.062
101	A	1	1	1.00	18	0.056
102	A	3	3	1.00	23	0.130
103	A	3	3	1.00	25	0.120
104	A	3	3	1.00	29	0.103
105	A	1	1	1.00	19	0.053
106	A	2	2	1.00	24	0.083
107	A	2	2	1.00	26	0.077
108	A	2	2	1.00	30	0.067
109	A	1	1	1.00	19	0.053
110	A	2	2	1.00	24	0.083
111	A	2	2	1.00	26	0.077
112	A	2	2	1.00	30	0.067
113	A	1	1	1.00	21	0.048
114	A	2	2	1.00	26	0.077
115	A	2	2	1.00	28	0.071
116	A	2	2	1.00	32	0.062
117	A	1	1	1.00	21	0.048
118	A	2	2	1.00	26	0.077
119	A	2	2	1.00	28	0.071
120	A	2	2	1.00	32	0.062
121	A	1	1	1.00	18	0.056
122	A	4	3	0.94	23	0.130
123	A	4	3	0.94	25	0.120
124	A	4	3	1.00	29	0.103
125	A	1	1	1.00	18	0.056
126	A	3	3	1.00	23	0.130
127	A	3	3	1.00	25	0.120
128	A	3	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	1	1	1.00	19	0.053
130	A	2	2	1.00	24	0.083
131	A	2	2	1.00	26	0.077
132	A	2	2	1.00	30	0.067
133	A	1	1	1.00	21	0.048
134	A	2	2	1.00	26	0.077
135	A	2	2	1.00	28	0.071
136	A	2	2	1.00	32	0.062
137	A	1	1	1.00	18	0.056
138	A	1	1	1.00	23	0.043
139	A	1	1	1.00	25	0.040
140	A	2	2	1.00	29	0.069
141	A	4	2	1.00	29	0.069
142	A	5	3	1.00	29	0.103
143	A	6	3	1.00	29	0.103
144	A	3	3	1.00	59	0.051
145	A	5	3	1.00	31	0.097
146	A	5	3	1.00	29	0.103
147	A	5	3	1.00	27	0.111
148	A	5	3	1.00	26	0.115
149	A	8	5	1.00	29	0.172
150	A	5	3	1.00	29	0.103
151	A	5	3	1.00	29	0.103
152	A	10	4	1.00	31	0.129
153	A	7	4	1.00	29	0.138
154	A	2	2	1.00	22	0.091
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=163

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+\right.$$

```
[Out] a*d^5*x+1/4*d^4*(5*a*e+b*d)*x^4+1/7*d^3*(c*d^2+5*e*(2*a*e+b*d))*x^7+1/2*d^2
*e*(c*d^2+2*e*(a*e+b*d))*x^10+5/13*d*e^2*(2*c*d^2+e*(a*e+2*b*d))*x^13+1/16*
e^3*(10*c*d^2+e*(a*e+5*b*d))*x^16+1/19*e^4*(b*e+5*c*d)*x^19+1/22*c*e^5*x^22
```

Rubi [A] time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7)
/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*b*
d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(5*c
*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^3\right)^5\left(a+bx^3+cx^6\right)\,dx &= \int \left(ad^5+d^4(bd+5ae)x^3+d^3\left(cd^2+5e(bd+2ae)\right)x^6+5d^2e\left(cd^2+2e(bd\right.\right.\\ &= ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\left(cd^2+5e(bd+2ae)\right)x^7+\frac{1}{2}d^2e\left(cd^2+2e(bc\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 164, normalized size = 1.01

$$\frac{5}{13}de^2x^{13}\left(ae^2+2bde+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2ae^2+2bde+cd^2\right)+\frac{1}{16}e^3x^{16}\left(ae^2+5bde+10cd^2\right)+\frac{1}{7}d^3x^7\left(10ae^2+5\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*a*e^2)*x^7
)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d*e^2*(2*c*d^2 + 2*b*
d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*d*e + a*e^2)*x^16)/16 + (e^4*(
5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

fricas [A] time = 0.69, size = 182, normalized size = 1.12

$$\frac{1}{22}x^{22}e^5c+\frac{5}{19}x^{19}e^4dc+\frac{1}{19}x^{19}e^5b+\frac{5}{8}x^{16}e^3d^2c+\frac{5}{16}x^{16}e^4db+\frac{1}{16}x^{16}e^5a+\frac{10}{13}x^{13}e^2d^3c+\frac{10}{13}x^{13}e^3d^2b+\frac{5}{13}x^{13}e^4da+\frac{1}{2}x^{10}ed$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*e^5*c + 5/19*x^19*e^4*d*c + 1/19*x^19*e^5*b + 5/8*x^16*e^3*d^2*c
+ 5/16*x^16*e^4*d*b + 1/16*x^16*e^5*a + 10/13*x^13*e^2*d^3*c + 10/13*x^13*e
^3*d^2*b + 5/13*x^13*e^4*d*a + 1/2*x^10*e*d^4*c + x^10*e^2*d^3*b + x^10*e^3
*d^2*a + 1/7*x^7*d^5*c + 5/7*x^7*e*d^4*b + 10/7*x^7*e^2*d^3*a + 1/4*x^4*d^5
*b + 5/4*x^4*e*d^4*a + x*d^5*a
```

giac [A] time = 0.34, size = 173, normalized size = 1.06

$$\frac{1}{22}cx^{22}e^5+\frac{5}{19}cdx^{19}e^4+\frac{1}{19}bx^{19}e^5+\frac{5}{8}cd^2x^{16}e^3+\frac{5}{16}bdx^{16}e^4+\frac{1}{16}ax^{16}e^5+\frac{10}{13}cd^3x^{13}e^2+\frac{10}{13}bd^2x^{13}e^3+\frac{5}{13}adx^{13}e^4+\frac{1}{2}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/22*c*x^22*e^5 + 5/19*c*d*x^19*e^4 + 1/19*b*x^19*e^5 + 5/8*c*d^2*x^16*e^3
+ 5/16*b*d*x^16*e^4 + 1/16*a*x^16*e^5 + 10/13*c*d^3*x^13*e^2 + 10/13*b*d^2*
x^13*e^3 + 5/13*a*d*x^13*e^4 + 1/2*c*d^4*x^10*e + b*d^3*x^10*e^2 + a*d^2*x^
10*e^3 + 1/7*c*d^5*x^7 + 5/7*b*d^4*x^7*e + 10/7*a*d^3*x^7*e^2 + 1/4*b*d^5*x
^4 + 5/4*a*d^4*x^4*e + a*d^5*x
```

maple [A] time = 0.00, size = 169, normalized size = 1.04

$$\frac{c\,e^5x^{22}}{22}+\frac{\left(e^5b+5d\,e^4c\right)x^{19}}{19}+\frac{\left(e^5a+5d\,e^4b+10d^2e^3c\right)x^{16}}{16}+\frac{\left(5d\,e^4a+10d^2e^3b+10d^3e^2c\right)x^{13}}{13}+\frac{\left(10d^2e^3a+10d^3e^2b\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^5*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/22*c*e^5*x^22+1/19*(b*e^5+5*c*d*e^4)*x^19+1/16*(a*e^5+5*b*d*e^4+10*c*d^2*
e^3)*x^16+1/13*(5*a*d*e^4+10*b*d^2*e^3+10*c*d^3*e^2)*x^13+1/10*(10*a*d^2*e^
3+10*b*d^3*e^2+5*c*d^4*e)*x^10+1/7*(10*a*d^3*e^2+5*b*d^4*e+c*d^5)*x^7+1/4*(
5*a*d^4*e+b*d^5)*x^4+a*d^5*x
```

maxima [A] time = 0.81, size = 166, normalized size = 1.02

$$\frac{1}{22}ce^5x^{22}+\frac{1}{19}\left(5\,cde^4+be^5\right)x^{19}+\frac{1}{16}\left(10\,cd^2e^3+5\,bde^4+ae^5\right)x^{16}+\frac{5}{13}\left(2\,cd^3e^2+2\,bd^2e^3+ade^4\right)x^{13}+\frac{1}{2}\left(cd^4e+2\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b*
d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1/2
```

```
*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 10*a
*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

mupad [B] time = 1.60, size = 158, normalized size = 0.97

$$x^4\left(\frac{b\,d^5}{4}+\frac{5\,a\,e\,d^4}{4}\right)+x^{19}\left(\frac{b\,e^5}{19}+\frac{5\,c\,d\,e^4}{19}\right)+x^7\left(\frac{c\,d^5}{7}+\frac{5\,b\,d^4\,e}{7}+\frac{10\,a\,d^3\,e^2}{7}\right)+x^{16}\left(\frac{5\,c\,d^2\,e^3}{8}+\frac{5\,b\,d\,e^4}{16}+\frac{a\,e^5}{16}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^5*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^5)/4 + (5*a*d^4*e)/4) + x^19*((b*e^5)/19 + (5*c*d*e^4)/19) + x^7*
((c*d^5)/7 + (10*a*d^3*e^2)/7 + (5*b*d^4*e)/7) + x^16*((a*e^5)/16 + (5*c*d^
2*e^3)/8 + (5*b*d*e^4)/16) + (c*e^5*x^22)/22 + a*d^5*x + (d^2*e*x^10*(2*a*e
^2 + c*d^2 + 2*b*d*e))/2 + (5*d*e^2*x^13*(a*e^2 + 2*c*d^2 + 2*b*d*e))/13
```

sympy [A] time = 0.10, size = 187, normalized size = 1.15

$$ad^5x+\frac{ce^5x^{22}}{22}+x^{19}\left(\frac{be^5}{19}+\frac{5cde^4}{19}\right)+x^{16}\left(\frac{ae^5}{16}+\frac{5bde^4}{16}+\frac{5cd^2e^3}{8}\right)+x^{13}\left(\frac{5ade^4}{13}+\frac{10bd^2e^3}{13}+\frac{10cd^3e^2}{13}\right)+x^{10}\left(ad^2e^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) + x**16*(a*e
**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*d*e**4/13 + 10*b*d**
2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**2*e**3 + b*d**3*e**2 + c*d**4*
e/2) + x**7*(10*a*d**3*e**2/7 + 5*b*d**4*e/7 + c*d**5/7) + x**4*(5*a*d**4*e
/4 + b*d**5/4)
```

3.2 $\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=135

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+$$

```
[Out] a*d^4*x+1/4*d^3*(4*a*e+b*d)*x^4+1/7*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^7+1/5*d*e
*(2*c*d^2+e*(2*a*e+3*b*d))*x^10+1/13*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^13+1/16*
e^3*(b*e+4*c*d)*x^16+1/19*c*e^4*x^19
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*d
+ a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) dx = \int \left(ad^4+d^3(bd+4ae)x^3+d^2\left(cd^2+4bde+6ae^2\right)x^6+2de\left(2cd^2+e(3bd+2\right.\right.$$
$$= ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7+\frac{1}{5}de\left(2cd^2+e(3bd+\right.$$

Mathematica [A] time = 0.04, size = 135, normalized size = 1.00

$$\frac{1}{13}e^2x^{13}\left(ae^2+4bde+6cd^2\right)+\frac{1}{5}dex^{10}\left(2ae^2+3bde+2cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*(6*c*d^2 + 4*b*d*e +
 a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

fricas [A] time = 0.90, size = 147, normalized size = 1.09

$$\frac{1}{19}x^{19}e^4c+\frac{1}{4}x^{16}e^3dc+\frac{1}{16}x^{16}e^4b+\frac{6}{13}x^{13}e^2d^2c+\frac{4}{13}x^{13}e^3db+\frac{1}{13}x^{13}e^4a+\frac{2}{5}x^{10}ed^3c+\frac{3}{5}x^{10}e^2d^2b+\frac{2}{5}x^{10}e^3da+\frac{1}{7}x^7d^4c+\frac{4}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*e^4*c + 1/4*x^16*e^3*d*c + 1/16*x^16*e^4*b + 6/13*x^13*e^2*d^2*c
+ 4/13*x^13*e^3*d*b + 1/13*x^13*e^4*a + 2/5*x^10*e*d^3*c + 3/5*x^10*e^2*d^2
*b + 2/5*x^10*e^3*d*a + 1/7*x^7*d^4*c + 4/7*x^7*e*d^3*b + 6/7*x^7*e^2*d^2*a
 + 1/4*x^4*d^4*b + x^4*e*d^3*a + x*d^4*a
```

giac [A] time = 0.34, size = 141, normalized size = 1.04

$$\frac{1}{19}\,cx^{19}e^4+\frac{1}{4}\,cdx^{16}e^3+\frac{1}{16}\,bx^{16}e^4+\frac{6}{13}\,cd^2x^{13}e^2+\frac{4}{13}\,bdx^{13}e^3+\frac{1}{13}\,ax^{13}e^4+\frac{2}{5}\,cd^3x^{10}e+\frac{3}{5}\,bd^2x^{10}e^2+\frac{2}{5}\,adx^{10}e^3+\frac{1}{7}\,cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/19*c*x^19*e^4 + 1/4*c*d*x^16*e^3 + 1/16*b*x^16*e^4 + 6/13*c*d^2*x^13*e^2
+ 4/13*b*d*x^13*e^3 + 1/13*a*x^13*e^4 + 2/5*c*d^3*x^10*e + 3/5*b*d^2*x^10*e
^2 + 2/5*a*d*x^10*e^3 + 1/7*c*d^4*x^7 + 4/7*b*d^3*x^7*e + 6/7*a*d^2*x^7*e^2
 + 1/4*b*d^4*x^4 + a*d^3*x^4*e + a*d^4*x
```

maple [A] time = 0.00, size = 136, normalized size = 1.01

$$\frac{c\,e^4x^{19}}{19}+\frac{\left(e^4b+4d\,e^3c\right)x^{16}}{16}+\frac{\left(e^4a+4d\,e^3b+6e^2d^2c\right)x^{13}}{13}+\frac{\left(4d\,e^3a+6e^2d^2b+4c\,d^3e\right)x^{10}}{10}+\frac{\left(6a\,d^2e^2+4b\,d^3e+c\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^4*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/19*c*e^4*x^19+1/16*(b*e^4+4*c*d*e^3)*x^16+1/13*(a*e^4+4*b*d*e^3+6*c*d^2*e
^2)*x^13+1/10*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^10+1/7*(6*a*d^2*e^2+4*b*d
^3*e+c*d^4)*x^7+1/4*(4*a*d^3*e+b*d^4)*x^4+a*d^4*x
```

maxima [A] time = 0.72, size = 135, normalized size = 1.00

$$\frac{1}{19}\,ce^4x^{19}+\frac{1}{16}\left(4\,cde^3+be^4\right)x^{16}+\frac{1}{13}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^{13}+\frac{1}{5}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^{10}+\frac{1}{7}\left(cd^4+4\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7*(
c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)*x^
4
```

mupad [B] time = 0.06, size = 130, normalized size = 0.96

$$x^4\,\left(\frac{b\,d^4}{4}+a\,e\,d^3\right)+x^{16}\,\left(\frac{b\,e^4}{16}+\frac{c\,d\,e^3}{4}\right)+x^7\,\left(\frac{c\,d^4}{7}+\frac{4\,b\,d^3\,e}{7}+\frac{6\,a\,d^2\,e^2}{7}\right)+x^{13}\,\left(\frac{6\,c\,d^2\,e^2}{13}+\frac{4\,b\,d\,e^3}{13}+\frac{a\,e^4}{13}\right)+\frac{c\,e^4\,x}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^4*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^4)/4 + a*d^3*e) + x^16*((b*e^4)/16 + (c*d*e^3)/4) + x^7*((c*d^4)/
7 + (6*a*d^2*e^2)/7 + (4*b*d^3*e)/7) + x^13*((a*e^4)/13 + (6*c*d^2*e^2)/13
+ (4*b*d*e^3)/13) + (c*e^4*x^19)/19 + a*d^4*x + (d*e*x^10*(2*a*e^2 + 2*c*d^
2 + 3*b*d*e))/5
```

sympy [A] time = 0.09, size = 151, normalized size = 1.12

$$ad^4x+\frac{ce^4x^{19}}{19}+x^{16}\left(\frac{be^4}{16}+\frac{cde^3}{4}\right)+x^{13}\left(\frac{ae^4}{13}+\frac{4bde^3}{13}+\frac{6cd^2e^2}{13}\right)+x^{10}\left(\frac{2ade^3}{5}+\frac{3bd^2e^2}{5}+\frac{2cd^3e}{5}\right)+x^7\left(\frac{6ad^2e^2}{7}+\frac{4bd}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**13*(a*e**4
/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*e**3/5 + 3*b*d**2*e*
*2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7 + 4*b*d**3*e/7 + c*d**4/7) + x
**4*(a*d**3*e + b*d**4/4)
```

3.3 $\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx$

Optimal. Leaf size=103

$$\frac{1}{10}ex^{10}\left(e(ae+3bd)+3cd^2\right)+\frac{1}{7}dx^7\left(3e(ae+bd)+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

```
[Out] a*d^3*x+1/4*d^2*(3*a*e+b*d)*x^4+1/7*d*(c*d^2+3*e*(a*e+b*d))*x^7+1/10*e*(3*c
*d^2+e*(a*e+3*b*d))*x^10+1/13*e^2*(b*e+3*c*d)*x^13+1/16*c*e^3*x^16
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{10}ex^{10}\left(e(ae+3bd)+3cd^2\right)+\frac{1}{7}dx^7\left(3e(ae+bd)+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 + (c
*e^3*x^16)/16
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx = \int \left(ad^3+d^2(bd+3ae)x^3+d\left(cd^2+3e(bd+ae)\right)x^6+e\left(3cd^2+e(3bd+ae\right.\right.$$
$$= ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3e(bd+ae)\right)x^7+\frac{1}{10}e\left(3cd^2+e(3bd+\right.$$

Mathematica [A] time = 0.03, size = 104, normalized size = 1.01

$$\frac{1}{10}ex^{10}\left(ae^2+3bde+3cd^2\right)+\frac{1}{7}dx^7\left(3ae^2+3bde+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^7)/7
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 +
(c*e^3*x^16)/16
```

fricas [A] time = 0.80, size = 112, normalized size = 1.09

$$\frac{1}{16}x^{16}e^3c+\frac{3}{13}x^{13}e^2dc+\frac{1}{13}x^{13}e^3b+\frac{3}{10}x^{10}ed^2c+\frac{3}{10}x^{10}e^2db+\frac{1}{10}x^{10}e^3a+\frac{1}{7}x^7d^3c+\frac{3}{7}x^7ed^2b+\frac{3}{7}x^7e^2da+\frac{1}{4}x^4d^3b+\frac{3}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^3*c + 3/13*x^13*e^2*d*c + 1/13*x^13*e^3*b + 3/10*x^10*e*d^2*c +
 3/10*x^10*e^2*d*b + 1/10*x^10*e^3*a + 1/7*x^7*d^3*c + 3/7*x^7*e*d^2*b + 3/
7*x^7*e^2*d*a + 1/4*x^4*d^3*b + 3/4*x^4*e*d^2*a + x*d^3*a
```

giac [A] time = 0.30, size = 109, normalized size = 1.06

$$\frac{1}{16}cx^{16}e^3+\frac{3}{13}cdx^{13}e^2+\frac{1}{13}bx^{13}e^3+\frac{3}{10}cd^2x^{10}e+\frac{3}{10}bdx^{10}e^2+\frac{1}{10}ax^{10}e^3+\frac{1}{7}cd^3x^7+\frac{3}{7}bd^2x^7e+\frac{3}{7}adx^7e^2+\frac{1}{4}bd^3x^4+\frac{3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/16*c*x^16*e^3 + 3/13*c*d*x^13*e^2 + 1/13*b*x^13*e^3 + 3/10*c*d^2*x^10*e +
 3/10*b*d*x^10*e^2 + 1/10*a*x^10*e^3 + 1/7*c*d^3*x^7 + 3/7*b*d^2*x^7*e + 3/
7*a*d*x^7*e^2 + 1/4*b*d^3*x^4 + 3/4*a*d^2*x^4*e + a*d^3*x
```

maple [A] time = 0.00, size = 103, normalized size = 1.00

$$\frac{c\,e^3x^{16}}{16}+\frac{\left(e^3b+3cd\,e^2\right)x^{13}}{13}+\frac{\left(e^3a+3bd\,e^2+3d^2ec\right)x^{10}}{10}+\frac{\left(3ad\,e^2+3b\,d^2e+c\,d^3\right)x^7}{7}+a\,d^3x+\frac{\left(3a\,d^2e+b\,d^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^3*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2+3*c*d^2*e
)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2*e+b*d^3)*x^4+a*d^3*
x
```

maxima [A] time = 0.65, size = 102, normalized size = 0.99

$$\frac{1}{16}ce^3x^{16}+\frac{1}{13}\left(3\,cde^2+be^3\right)x^{13}+\frac{1}{10}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^{10}+\frac{1}{7}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^7+ad^3x+\frac{1}{4}\left(bd^3+3\,ad\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*e
^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/4*
(b*d^3 + 3*a*d^2*e)*x^4
```

mupad [B] time = 0.04, size = 102, normalized size = 0.99

$$x^4\left(\frac{b\,d^3}{4}+\frac{3\,a\,e\,d^2}{4}\right)+x^{13}\left(\frac{b\,e^3}{13}+\frac{3\,c\,d\,e^2}{13}\right)+x^7\left(\frac{c\,d^3}{7}+\frac{3\,b\,d^2\,e}{7}+\frac{3\,a\,d\,e^2}{7}\right)+x^{10}\left(\frac{3\,c\,d^2\,e}{10}+\frac{3\,b\,d\,e^2}{10}+\frac{a\,e^3}{10}\right)+\frac{c\,e^3\,x^1}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^3*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^3)/4 + (3*a*d^2*e)/4) + x^13*((b*e^3)/13 + (3*c*d*e^2)/13) + x^7*
((c*d^3)/7 + (3*a*d*e^2)/7 + (3*b*d^2*e)/7) + x^10*((a*e^3)/10 + (3*b*d*e^2
)/10 + (3*c*d^2*e)/10) + (c*e^3*x^16)/16 + a*d^3*x
```

sympy [A] time = 0.09, size = 117, normalized size = 1.14

$$ad^3x+\frac{ce^3x^{16}}{16}+x^{13}\left(\frac{be^3}{13}+\frac{3cde^2}{13}\right)+x^{10}\left(\frac{ae^3}{10}+\frac{3bde^2}{10}+\frac{3cd^2e}{10}\right)+x^7\left(\frac{3ade^2}{7}+\frac{3bd^2e}{7}+\frac{cd^3}{7}\right)+x^4\left(\frac{3ad^2e}{4}+\frac{bd^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) + x**10*(a*e
**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e**2/7 + 3*b*d**2*e/7
 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)
```

3.4 $\int \left(d+ex^3\right)^2 \left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=73

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

```
[Out] a*d^2*x+1/4*d*(2*a*e+b*d)*x^4+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/10*e*(b*e+2*c
*d)*x^10+1/13*c*e^2*x^13
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^2\left(a+bx^3+cx^6\right)\,dx=\int \left(ad^2+d(bd+2ae)x^3+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^9+ce^2x^{12}\right)$$
$$=ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{7}x^7\left(ae^2+2bde+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

fricas [A] time = 0.83, size = 76, normalized size = 1.04

$$\frac{1}{13}x^{13}e^2c+\frac{1}{5}x^{10}edc+\frac{1}{10}x^{10}e^2b+\frac{1}{7}x^7d^2c+\frac{2}{7}x^7edb+\frac{1}{7}x^7e^2a+\frac{1}{4}x^4d^2b+\frac{1}{2}x^4eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/7*x^7*d^2*c + 2/7*x^
7*e*d*b + 1/7*x^7*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + x*d^2*a
```

giac [A] time = 0.33, size = 76, normalized size = 1.04

$$\frac{1}{13}\,cx^{13}e^2+\frac{1}{5}\,cdx^{10}e+\frac{1}{10}\,bx^{10}e^2+\frac{1}{7}\,cd^2x^7+\frac{2}{7}\,bdx^7e+\frac{1}{7}\,ax^7e^2+\frac{1}{4}\,bd^2x^4+\frac{1}{2}\,adx^4e+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/7*c*d^2*x^7 + 2/7*b*
d*x^7*e + 1/7*a*x^7*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + a*d^2*x
```

maple [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{c\,e^2x^{13}}{13}+\frac{\left(b\,e^2+2cde\right)x^{10}}{10}+\frac{\left(a\,e^2+2deb+c\,d^2\right)x^7}{7}+a\,d^2x+\frac{\left(2dea+b\,d^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^2*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/4
*(2*a*d*e+b*d^2)*x^4+a*d^2*x
```

maxima [A] time = 0.75, size = 69, normalized size = 0.95

$$\frac{1}{13}\,ce^2x^{13}+\frac{1}{10}\left(2\,cde+be^2\right)x^{10}+\frac{1}{7}\left(cd^2+2\,bde+ae^2\right)x^7+\frac{1}{4}\left(bd^2+2\,ade\right)x^4+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e^
2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

mupad [B] time = 0.04, size = 70, normalized size = 0.96

$$x^7\left(\frac{c\,d^2}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^2}{7}\right)+x^4\left(\frac{b\,d^2}{4}+\frac{a\,e\,d}{2}\right)+x^{10}\left(\frac{b\,e^2}{10}+\frac{c\,d\,e}{5}\right)+\frac{c\,e^2\,x^{13}}{13}+a\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^2*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^4*((b*d^2)/4 + (a*d*e)/2) + x
^10*((b*e^2)/10 + (c*d*e)/5) + (c*e^2*x^13)/13 + a*d^2*x
```

sympy [A] time = 0.08, size = 75, normalized size = 1.03

$$ad^2x+\frac{ce^2x^{13}}{13}+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right)+x^7\left(\frac{ae^2}{7}+\frac{2bde}{7}+\frac{cd^2}{7}\right)+x^4\left(\frac{ade}{2}+\frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(a*e**2/7 +
 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)
```

3.5 $\int \left(d+ex^3\right) \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=42

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

```
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1407}

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \left(d+ex^3\right)\left(a+bx^3+cx^6\right) dx &= \int \left(ad+(bd+ae)x^3+(cd+be)x^6+cex^9\right) dx \\ &= adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10} \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

fricas [A] time = 0.91, size = 40, normalized size = 0.95

$$\frac{1}{10}x^{10}ec+\frac{1}{7}x^7dc+\frac{1}{7}x^7eb+\frac{1}{4}x^4db+\frac{1}{4}x^4ea+xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c + 1/7*x^7*d*c + 1/7*x^7*e*b + 1/4*x^4*d*b + 1/4*x^4*e*a + x*d
*a
```

giac [A] time = 0.33, size = 43, normalized size = 1.02

$$\frac{1}{10}cx^{10}e + \frac{1}{7}cdx^7 + \frac{1}{7}bx^7e + \frac{1}{4}bdx^4 + \frac{1}{4}ax^4e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e + 1/7*c*d*x^7 + 1/7*b*x^7*e + 1/4*b*d*x^4 + 1/4*a*x^4*e + a*d
*x
```

maple [A] time = 0.00, size = 37, normalized size = 0.88

$$\frac{ce\,x^{10}}{10} + \frac{(be+cd)\,x^7}{7} + \frac{(ae+bd)\,x^4}{4} + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)*(c*x^6+b*x^3+a),x)
```

```
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

maxima [A] time = 0.55, size = 36, normalized size = 0.86

$$\frac{1}{10}cex^{10} + \frac{1}{7}(cd+be)x^7 + \frac{1}{4}(bd+ae)x^4 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

mupad [B] time = 0.04, size = 38, normalized size = 0.90

$$\frac{c\,e\,x^{10}}{10} + \left(\frac{b\,e}{7} + \frac{c\,d}{7}\right)x^7 + \left(\frac{a\,e}{4} + \frac{b\,d}{4}\right)x^4 + a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((a*e)/4 + (b*d)/4) + x^7*((b*e)/7 + (c*d)/7) + a*d*x + (c*e*x^10)/10
```

sympy [A] time = 0.07, size = 39, normalized size = 0.93

$$adx + \frac{cex^{10}}{10} + x^7\left(\frac{be}{7} + \frac{cd}{7}\right) + x^4\left(\frac{ae}{4} + \frac{bd}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4)
```

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-\right.}{\sqrt{3}d^{2/3}e^{7/3}}$$

```
[Out] -(-b*e+c*d)*x/e^2+1/4*c*x^4/e+1/3*(a*e^2-b*d*e+c*d^2)*ln(d^(1/3)+e^(1/3)*x)
/d^(2/3)/e^(7/3)-1/6*(a*e^2-b*d*e+c*d^2)*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/
3)*x^2)/d^(2/3)/e^(7/3)-1/3*(a*e^2-b*d*e+c*d^2)*arctan(1/3*(d^(1/3)-2*e^(1/
3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(7/3)*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1411, 388, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-\right.}{\sqrt{3}d^{2/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)*ArcTan[(d
^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c*d
^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c*d^2
 - b*d*e + a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3
)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx &= \frac{cx^4}{4e}+\frac{\int \frac{4ae-(4cd-4be)x^3}{d+ex^3}\,dx}{4e}\\
&= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}-\left(-a-\frac{d(cd-be)}{e^2}\right)\int \frac{1}{d+ex^3}\,dx\\
&= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{2/3}}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}}\\
&= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}+\frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}-\frac{\left(cd^2-bde+ae^2\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x-}}{6d^{2/3}e^{7/3}}\\
&= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}+\frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}-\frac{\left(cd^2-bde+ae^2\right)\log\left(d^{2/3}-\sqrt[3]{}\right.}{6d^{2/3}e^{7/3}}\\
&= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}-\frac{\left(cd^2-bde+ae^2\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{7/3}}+\frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{}\right.}{3d^{2/3}e^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 176, normalized size = 0.94

$$\frac{-\frac{2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}}+\frac{4\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}}+12\sqrt[3]{e}x(be-cd)}{12e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*Sqrt[3]*(c*d^2 + e*(-(b
*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (4*(c*d^
2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (2*(c*d^2 + e*(-(
b*d) + a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(12*e
^(7/3))
```

fricas [A] time = 0.90, size = 465, normalized size = 2.47

$$\left[\frac{3\,cd^2e^2x^4 + 6\sqrt{\frac{1}{3}}\left(cd^3e - bd^2e^2 + ade^3\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}\,\log\left(\frac{2\,dex^3 - 3\left(d^2e\right)^{\frac{1}{3}}dx - d^2 + 3\sqrt{\frac{1}{3}}\left(2\,dex^2 + \left(d^2e\right)^{\frac{2}{3}}x - \left(d^2e\right)^{\frac{1}{3}}d\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}}{ex^3+d}\right) -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*c*d^2*e^2*x^4 + 6*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt(-
(d^2*e)^(1/3)/e)*log((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(
2*d*e*x^2 + (d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x
^3 + d)) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2
/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*
x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3), 1/12*(3*c*d^2*e
^2*x^4 + 12*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt((d^2*e)^(1/3)/e)
*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/
e)/d^2) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/
3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x
 + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3)]
```

giac [A] time = 0.37, size = 173, normalized size = 0.92

$$-\frac{\sqrt{3}\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{3\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(cd^2 - bde + ae^2\right)e^{(-1)}\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right.\right.}{6\left(-de^2\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^
(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) - 1/6*(c*d^2 - b*d*e + a*e^
2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3)
 - 1/3*(c*d^2*e^2 - b*d*e^3 + a*e^4)*(-d*e^(-1))^(1/3)*e^(-4)*log(abs(x - (
-d*e^(-1))^(1/3)))/d + 1/4*(c*x^4*e^3 - 4*c*d*x*e^2 + 4*b*x*e^3)*e^(-4)
```

maple [B] time = 0.01, size = 313, normalized size = 1.66

$$\frac{c\,x^4}{4e}+\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}+\frac{a\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{a\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{\sqrt{3}\,bd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}-\frac{bd\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d),x)
```

```
[Out] 1/4*c*x^4/e+1/e*b*x-1/e^2*c*d*x+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a-1/3/e
^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*d*b+1/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*
c*d^2-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a+1/6/e^2/(d/e)^(
2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*d*b-1/6/e^3/(d/e)^(2/3)*ln(x^2-(d/e)
^(1/3)*x+(d/e)^(2/3))*c*d^2+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(d/e)^(1/3)*x-1))*a-1/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e
)^(1/3)*x-1))*d*b+1/3/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x-1))*c*d^2
```

maxima [A] time = 1.52, size = 169, normalized size = 0.90

$$\frac{cex^4-4\,(cd-be)x}{4\,e^2}+\frac{\sqrt{3}\left(cd^2-bde+ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\left(cd^2-bde+ae^2\right)\log\left(x^2-x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c*e*x^4 - 4*(c*d - b*e)*x)/e^2 + 1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*a
rctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(1/3))/(e^3*(d/e)^(2/3)) - 1/6*
(c*d^2 - b*d*e + a*e^2)*log(x^2 - x*(d/e)^(1/3) + (d/e)^(2/3))/(e^3*(d/e)^(
2/3)) + 1/3*(c*d^2 - b*d*e + a*e^2)*log(x + (d/e)^(1/3))/(e^3*(d/e)^(2/3))
```

mupad [B] time = 0.27, size = 165, normalized size = 0.88

$$x\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)+\frac{c\,x^4}{4\,e}+\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}}+\frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(c\,d^2-b\,d\right.}{3\,d^{2/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^4)/(4*e) + (log(e^(1/3)*x + d^(1/3))*(a*e^2 + c*
d^2 - b*d*e))/(3*d^(2/3)*e^(7/3)) + (log(3^(1/2)*d^(1/3)*1i + 2*e^(1/3)*x -
 d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3
)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2
)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3))
```

sympy [A] time = 0.90, size = 175, normalized size = 0.93

$$\frac{cx^4}{4e}+x\left(\frac{b}{e}-\frac{cd}{e^2}\right)+\mathrm{RootSum}\left(27t^3d^2e^7-a^3e^6+3a^2bde^5-3a^2cd^2e^4-3ab^2d^2e^4+6abcd^3e^3-3ac^2d^4e^2+b^3d^3e^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + x*(b/e - c*d/e**2) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6
+ 3*a**2*b*d*e**5 - 3*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**3*
e**3 - 3*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c**2*
d**5*e - c**3*d**6, Lambda(_t, _t*log(3*_t*d*e**2/(a*e**2 - b*d*e + c*d**2)
 + x)))
```

3.7 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2} \, dx$

Optimal. Leaf size=213

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+$$

```
[Out] c*x/e^2+1/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)-1/9*(4*c*d^2-e*(2*a*e+b*d
))*ln(d^(1/3)+e^(1/3)*x)/d^(5/3)/e^(7/3)+1/18*(4*c*d^2-e*(2*a*e+b*d))*ln(d^
(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(5/3)/e^(7/3)+1/9*(4*c*d^2-e*(2*a*e+
b*d))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(5/3)/e^(7/3)*3^(
1/2)
```

Rubi [A] time = 0.23, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1409, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + ((4*c*d^2 -
 e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqr
t[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*
x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(18*d^(5/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\int \frac{cd^2-e(bd+2ae)-3cdex^3}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{1}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}e^2} - \frac{\left(4cd^2-e(bd+2ae)\right)\int}{9d^{5/3}}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{\left(4cd^2-e(bd+2ae\right.}{18}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{\left(4cd^2-e(bd+2ae\right.}{}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} + \frac{\left(4cd^2-e(bd+2ae)\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}\,d^{5/3}e^{7/3}} - \frac{\left(4cd^2-e(bd+2ae\right.}{9d^{5}}$$

Mathematica [A] time = 0.19, size = 199, normalized size = 0.93

$$\frac{\frac{6\sqrt[3]{e}x\left(e(ae-bd)+cd^2\right)}{d\left(d+ex^3\right)} + \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} - \frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(4cd^2-e(2a\right.}{d^{5/3}}}{18e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^3))
+ (2*Sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))
/Sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*x
])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2])/d^(5/3))/(18*e^(7/3))
```

fricas [A] time = 0.97, size = 697, normalized size = 3.27

$$\left[\frac{18\,cd^3e^2x^4 - 3\sqrt{\frac{1}{3}}\left(4\,cd^4e - bd^3e^2 - 2\,ad^2e^3 + \left(4\,cd^3e^2 - bd^2e^3 - 2\,ade^4\right)x^3\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3 - 3\left(d^2e\right)^{\frac{1}{3}}dx - d^2 + 3\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*c*d^3*e^2*x^4 - 3*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^3
+ (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log((2*
d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(2/3
)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + (4*c*d^3 - b*
d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(
d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 - b*d^2*e - 2*a*d
*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x + (d^2*
e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e^4*x^3 + d^4*e^3
), 1/18*(18*c*d^3*e^2*x^4 - 6*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^
3 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan
(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2)
 + (4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d
^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 -
 b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*l
og(d*e*x + (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e
^4*x^3 + d^4*e^3)]
```

giac [A] time = 0.38, size = 199, normalized size = 0.93

$$cxe^{(-2)} + \frac{\sqrt{3}\left(4\,cd^2 - bde - 2\,ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{9\left(-de^2\right)^{\frac{2}{3}}d} + \frac{\left(4\,cd^2 - bde - 2\,ae^2\right)e^{(-1)}\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x\right.}{18\left(-de^2\right)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) + 1/9*sqrt(3)*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(1/3*sqrt(3)*(2*
x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/((-d*e^2)^(2/3)*d) + 1/18*
(4*c*d^2 - b*d*e - 2*a*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-
1))^(2/3))/((-d*e^2)^(2/3)*d) + 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*(-d*e^(-1))
^(1/3)*e^(-2)*log(abs(x - (-d*e^(-1))^(1/3)))/d^2 + 1/3*(c*d^2*x - b*d*x*e
+ a*x*e^2)*e^(-2)/((x^3*e + d)*d)
```

maple [A] time = 0.01, size = 345, normalized size = 1.62

$$\frac{ax}{3\left(e\,x^3+d\right)d}-\frac{bx}{3\left(e\,x^3+d\right)e}+\frac{cdx}{3\left(e\,x^3+d\right)e^2}+\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de}+\frac{2a\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de}-\frac{a\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right.}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x)
```

```
[Out] c*x/e^2+1/3/d*x/(e*x^3+d)*a-1/3/e*x/(e*x^3+d)*b+1/3/e^2*d*x/(e*x^3+d)*c+2/9
/e/d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a+1/9/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*
b-4/9/e^3*d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c-1/9/e/d/(d/e)^(2/3)*ln(x^2-(d/e
)^(1/3)*x+(d/e)^(2/3))*a-1/18/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2
/3))*b+2/9/e^3*d/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c+2/9/e/d/(d
/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a+1/9/e^2/(d/e)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b-4/9/e^3*d/(d/e)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c
```

maxima [A] time = 1.60, size = 204, normalized size = 0.96

$$\frac{\left(cd^2-bde+ae^2\right)x}{3\left(de^3x^3+d^2e^2\right)}+\frac{cx}{e^2}-\frac{\sqrt{3}\left(4\,cd^2-bde-2\,ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9\,de^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\left(4\,cd^2-bde-2\,ae^2\right)\log\left(x^2-x\left(\frac{d}{e}\right)^{\frac{1}{3}}\right.}{18\,de^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^3 + d^2*e^2) + c*x/e^2 - 1/9*sqrt(3)
*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(
1/3))/(d*e^3*(d/e)^(2/3)) + 1/18*(4*c*d^2 - b*d*e - 2*a*e^2)*log(x^2 - x*(d
/e)^(1/3) + (d/e)^(2/3))/(d*e^3*(d/e)^(2/3)) - 1/9*(4*c*d^2 - b*d*e - 2*a*e
^2)*log(x + (d/e)^(1/3))/(d*e^3*(d/e)^(2/3))
```

mupad [B] time = 1.80, size = 187, normalized size = 0.88

$$\frac{c\,x}{e^2}+\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}+\frac{x\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d\left(e^3\,x^3+d\,e^2\right)}+\frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\right.}{9\,d^{5/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^2,x)
```

```
[Out] (c*x)/e^2 + (log(e^(1/3)*x + d^(1/3))*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3)) + (x*(a*e^2 + c*d^2 - b*d*e))/(3*d*(d*e^2 + e^3*x^3)) + (log(3^
(1/2)*d^(1/3)*1i + 2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a*e^2 -
 4*c*d^2 + b*d*e))/(9*d^(5/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3
)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3))
```

sympy [A] time = 1.68, size = 206, normalized size = 0.97

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{3d^2e^2+3de^3x^3}+\text{RootSum}\left(729t^3d^5e^7-8a^3e^6-12a^2bde^5+48a^2cd^2e^4-6ab^2d^2e^4+48abcd^3e^3-96ac^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d**2*e**2 + 3*d*e**3*x**3) + Root
Sum(729*_t**3*d**5*e**7 - 8*a**3*e**6 - 12*a**2*b*d*e**5 + 48*a**2*c*d**2*e
**4 - 6*a*b**2*d**2*e**4 + 48*a*b*c*d**3*e**3 - 96*a*c**2*d**4*e**2 - b**3*
d**3*e**3 + 12*b**2*c*d**4*e**2 - 48*b*c**2*d**5*e + 64*c**3*d**6, Lambda(_
t, _t*log(9*_t*d**2*e**2/(2*a*e**2 + b*d*e - 4*c*d**2) + x)))
```

3.8 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3} \, dx$

Optimal. Leaf size=242

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\right.}{}$$

```
[Out] 1/6*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^2-1/18*(7*c*d^2-e*(5*a*e+b*d))*x/
d^2/e^2/(e*x^3+d)+1/27*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(1/3)+e^(1/3)*x)/d^(8/3
)/e^(7/3)-1/54*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)
*x^2)/d^(8/3)/e^(7/3)-1/27*(2*c*d^2+e*(5*a*e+b*d))*arctan(1/3*(d^(1/3)-2*e^
(1/3)*x)/d^(1/3)*3^(1/2))/d^(8/3)/e^(7/3)*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1409, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*d + 5*a*e))*ArcTan[(
d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9*Sqrt[3]*d^(8/3)*e^(7/3)) + ((
2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) -
 ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(54*d^(8/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^3}\,dx &= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\int \frac{cd^2-e(bd+5ae)-6cdex^3}{\left(d+ex^3\right)^2}\,dx}{6de^2}\\
&= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\int \frac{1}{d+ex^3}\,dx}{9d^2e^2}\\
&= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{27d^{8/3}e^2}+\frac{\left(2cd^2\right.}{}\\
&= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{27d^{8/3}e^{7/3}}-\frac{\left(2\right.}{}\\
&= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{27d^{8/3}e^{7/3}}-\frac{\left(2\right.}{}\\
&= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}-\frac{\left(2cd^2+e(bd+5ae)\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{9\sqrt{3}\,d^{8/3}e^{7/3}}+\frac{\left(2\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 209, normalized size = 0.86

$$\frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5ae+bd)+2cd^2\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e(5ae+bd)+2cd^2\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2}\right.}{54d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*x^3) + a*e
*(8*d + 5*e*x^3))))/(d + e*x^3)^2 - 2*Sqrt[3]*(2*c*d^2 + e*(b*d + 5*a*e))*A
rcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] + 2*(2*c*d^2 + e*(b*d + 5*a*e))*
Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*e^(7/3))
```

fricas [B] time = 0.56, size = 941, normalized size = 3.89

$$\left[-\frac{3\left(7cd^4e^2-bd^3e^3-5ad^2e^4\right)x^4-3\sqrt{\frac{1}{3}}\left(2cd^5e+bd^4e^2+5ad^3e^3+\left(2cd^3e^3+bd^2e^4+5ade^5\right)x^6+2\left(2cd^4e^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 3*sqrt(1/3)*(2*c*d^
5*e + b*d^4*e^2 + 5*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 +
 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log(
(2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(
2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + ((2*c*d^2*
e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3
*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*
x + (d^2*e)^(1/3)*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 +
 b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)
^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)
*x)/(d^4*e^5*x^6 + 2*d^5*e^4*x^3 + d^6*e^3), -1/54*(3*(7*c*d^4*e^2 - b*d^3*
e^3 - 5*a*d^2*e^4)*x^4 - 6*sqrt(1/3)*(2*c*d^5*e + b*d^4*e^2 + 5*a*d^3*e^3 +
 (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 + 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5
*a*d^2*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x
- (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2) + ((2*c*d^2*e^2 + b*d*e^3 + 5
*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 +
5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)
*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^
2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x
+ (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)*x)/(d^4*e^5*x^6
+ 2*d^5*e^4*x^3 + d^6*e^3)]
```

giac [A] time = 0.40, size = 224, normalized size = 0.93

$$-\frac{\sqrt{3}\left(2\,cd^{2}+bde+5\,ae^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{27\,\left(-de^{2}\right)^{\frac{2}{3}}\,d^{2}}-\frac{\left(2\,cd^{2}+bde+5\,ae^{2}\right)e^{(-1)}\log\left(x^{2}+\left(-de^{(-1)}\right)^{\frac{1}{3}}\,x+\left(-d\right.\right.}{54\,\left(-de^{2}\right)^{\frac{2}{3}}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*c*d^2 + b*d*e + 5*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(
-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/((-d*e^2)^(2/3)*d^2) - 1/54*(2*c*d^2
+ b*d*e + 5*a*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3)
)/((-d*e^2)^(2/3)*d^2) - 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*(-d*e^(-1))^(1/3)
*e^(-2)*log(abs(x - (-d*e^(-1))^(1/3)))/d^3 - 1/18*(7*c*d^2*x^4*e - b*d*x^4
*e^2 - 5*a*x^4*e^3 + 4*c*d^3*x + 2*b*d^2*x*e - 8*a*d*x*e^2)*e^(-2)/((x^3*e
+ d)^2*d^2)
```

maple [A] time = 0.01, size = 362, normalized size = 1.50

$$\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d^{2}e}+\frac{5a\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d^{2}e}-\frac{5a\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{54\left(\frac{d}{e}\right)^{\frac{2}{3}}d^{2}e}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d\,e^{2}}+\frac{b\ln\left(x\right.}{27\left(\frac{d}{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x)
```

```
[Out] (1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^2)/d/e^2*x
)/(e*x^3+d)^2+5/27/e/d^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a+1/27/e^2/d/(d/e)^(
2/3)*ln(x+(d/e)^(1/3))*b+2/27/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c-5/54/e/d^
2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a-1/54/e^2/d/(d/e)^(2/3)*ln
(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b-1/27/e^3/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x
+(d/e)^(2/3))*c+5/27/e/d^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^
(1/3)*x-1))*a+1/27/e^2/d/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1
/3)*x-1))*b+2/27/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*
x-1))*c
```

maxima [A] time = 1.69, size = 240, normalized size = 0.99

$$-\frac{\left(7\,cd^{2}e-bde^{2}-5\,ae^{3}\right)x^{4}+2\left(2\,cd^{3}+bd^{2}e-4\,ade^{2}\right)x}{18\left(d^{2}e^{4}x^{6}+2\,d^{3}e^{3}x^{3}+d^{4}e^{2}\right)}+\frac{\sqrt{3}\left(2\,cd^{2}+bde+5\,ae^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{27\,d^{2}e^{3}\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\left(2\,cd^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*c*d^2*e - b*d*e^2 - 5*a*e^3)*x^4 + 2*(2*c*d^3 + b*d^2*e - 4*a*d*e
^2)*x)/(d^2*e^4*x^6 + 2*d^3*e^3*x^3 + d^4*e^2) + 1/27*sqrt(3)*(2*c*d^2 + b*
```

```
d*e + 5*a*e^2)*arctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(1/3))/(d^2*e^3
*(d/e)^(2/3)) - 1/54*(2*c*d^2 + b*d*e + 5*a*e^2)*log(x^2 - x*(d/e)^(1/3) +
(d/e)^(2/3))/(d^2*e^3*(d/e)^(2/3)) + 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*log(x
 + (d/e)^(1/3))/(d^2*e^3*(d/e)^(2/3))
```

mupad [B] time = 0.29, size = 221, normalized size = 0.91

$$\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}}-\frac{\frac{x\,\left(2\,c\,d^2+b\,d\,e-4\,a\,e^2\right)}{9\,d\,e^2}-\frac{x^4\,\left(-7\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{18\,d^2\,e}}{d^2+2\,d\,e\,x^3+e^2\,x^6}+\frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^3,x)
```

```
[Out] (log(e^(1/3)*x + d^(1/3))*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
 - ((x*(2*c*d^2 - 4*a*e^2 + b*d*e))/(9*d*e^2) - (x^4*(5*a*e^2 - 7*c*d^2 + b
*d*e))/(18*d^2*e))/(d^2 + e^2*x^6 + 2*d*e*x^3) + (log(3^(1/2)*d^(1/3)*1i +
2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/
(27*d^(8/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
```

sympy [A] time = 5.23, size = 246, normalized size = 1.02

$$\frac{x^4\left(5ae^3+bde^2-7cd^2e\right)+x\left(8ade^2-2bd^2e-4cd^3\right)}{18d^4e^2+36d^3e^3x^3+18d^2e^4x^6}+\operatorname{RootSum}\left(19683t^3d^8e^7-125a^3e^6-75a^2bde^5-150a^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

```
[Out] (x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d**2*e - 4*c
*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**4*x**6) + RootSum(19
683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2*b*d*e**5 - 150*a**2*c*d**2*e*
*4 - 15*a*b**2*d**2*e**4 - 60*a*b*c*d**3*e**3 - 60*a*c**2*d**4*e**2 - b**3*
d**3*e**3 - 6*b**2*c*d**4*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, Lambda(_t,
 _t*log(27*_t*d**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))
```

3.9 $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} dx$

Optimal. Leaf size=132

$$-\frac{(ace+b^2(-e)+bcd)\log(a+bx^3+cx^6)}{6c^3}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

```
[Out] 1/3*(-b*e+c*d)*x^3/c^2+1/6*e*x^6/c-1/6*(a*c*e-b^2*e+b*c*d)*ln(c*x^6+b*x^3+a
)/c^3-1/3*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b
^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$-\frac{(ace+b^2(-e)+bcd)\log(a+bx^3+cx^6)}{6c^3}-\frac{(3abce-2ac^2d+b^2cd+b^3(-e))\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e +
 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^3*Sqrt[b^2 - 4*a
*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{cd-be}{c^2}+\frac{ex}{c}-\frac{a(cd-be)+\left(bcd-b^2e+ace\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^3\right)$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\operatorname{Subst}\left(\int \frac{a(cd-be)+\left(bcd-b^2e+ace\right)x}{a+bx+cx^2}\,dx, x, x^3\right)}{3c^2}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(bcd-b^2e+ace\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^3}+\frac{\left(b^2cd-2ac^2d-b^3\right.}{}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(bcd-b^2e+ace\right)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{\left(b^2cd-2ac^2d-b^3e+3ab\right.}{}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}-\frac{\left(bcd-b^2e+ace\right.}{}$$

Mathematica [A] time = 0.07, size = 126, normalized size = 0.95

$$\frac{\left(-ace+b^2e-bcd\right)\log\left(a+bx^3+cx^6\right)+\frac{2\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2cx^3(cd-be)+c^2ex^6}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*c*
d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

fricas [A] time = 1.84, size = 430, normalized size = 3.26

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)ex^6+2\left(\left(b^2c^2-4\,ac^3\right)d-\left(b^3c-4\,abc^2\right)e\right)x^3+\sqrt{b^2-4\,ac}\left(\left(b^2c-2\,ac^2\right)d-\left(b^3-3\,abc\right)e\right)\log\left(\right.}{6\left(b^2c^3-4\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b
*c^2)*e)*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*
log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))
/(c*x^6 + b*x^3 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4), 1/6*((b^2*c^2 - 4*a*c^3
```

```
)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 - 2*sqrt(-b
^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x^3 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 1.00, size = 131, normalized size = 0.99

$$\frac{cx^6e + 2\,cdx^3 - 2\,bx^3e}{6\,c^2} - \frac{\left(bcd - b^2e + ace\right)\log\left(cx^6 + bx^3 + a\right)}{6\,c^3} + \frac{\left(b^2cd - 2\,ac^2d - b^3e + 3\,abce\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(c*x^6*e + 2*c*d*x^3 - 2*b*x^3*e)/c^2 - 1/6*(b*c*d - b^2*e + a*c*e)*log
(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arc
tan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [B] time = 0.01, size = 260, normalized size = 1.97

$$\frac{e\,x^6}{6c} - \frac{be\,x^3}{3c^2} + \frac{d\,x^3}{3c} + \frac{abe\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2} - \frac{2ad\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\,c} - \frac{b^3e\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\,c^3} + \frac{b^2d\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\,c^2} - \frac{ae}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*x^6/c-1/3/c^2*b*e*x^3+1/3/c*d*x^3-1/6/c^2*ln(c*x^6+b*x^3+a)*a*e+1/6/c
^3*ln(c*x^6+b*x^3+a)*b^2*e-1/6/c^2*ln(c*x^6+b*x^3+a)*b*d+1/c^2/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*b*e-2/3/c/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*d-1/3/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^3*e+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.40, size = 3586, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3*(d/(3*c) - (b*e)/(3*c^2)) + (e*x^6)/(6*c) - (log(a + b*x^3 + c*x^6)*(3*
b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c
^4 - 9*b^2*c^3)) - (atan((4*c^6*(4*a*c - b^2)^(3/2)*(x^3*((b*((b^5*c^3*d^3
- b^8*e^3 - 2*a*b^3*c^4*d^3 + a^2*b*c^5*d^3 + a^3*c^5*d^2*e - 3*b^6*c^2*d^2
*e - 8*a^2*b^4*c^2*e^3 + 4*a^3*b^2*c^3*e^3 + 5*a*b^6*c*e^3 + 3*b^7*c*d*e^2
+ 9*a*b^4*c^3*d^2*e - 12*a*b^5*c^2*d*e^2 - 4*a^3*b*c^4*d*e^2 - 7*a^2*b^2*c^
```

```
4*d^2*e + 14*a^2*b^3*c^3*d*e^2)/c^6 - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d^2 + 1
2*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*e^2 -
24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3*c^7*d
 - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
 + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*
c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((45*b^3*c^7*d - 45*
b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*
a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3
))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) -
 (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e -
 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^4
 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c -
b^2)^(1/2)) + (3*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e +
 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(4*c^3*(4*a*c - b
^2)*(36*a*c^4 - 9*b^2*c^3))))/(4*a^2*c) - ((2*a*c - b^2)*((((((45*b^3*c^7*d
 - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
 + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1
/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36
*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d -
15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d
^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*
e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3
*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3
*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4
 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^
2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*
e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b
*c*e)^3)/(4*c^6*(4*a*c - b^2)^(3/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) - (b*
((a*b^7*e^3 - a*b^4*c^3*d^3 - 4*a^2*b^5*c*e^3 - 2*a^4*b*c^3*e^3 + a^4*c^4*d
*e^2 + a^2*b^2*c^4*d^3 + 5*a^3*b^3*c^2*e^3 - 3*a*b^6*c*d*e^2 + 3*a*b^5*c^2*
d^2*e + 2*a^3*b*c^4*d^2*e - 6*a^2*b^3*c^3*d^2*e + 9*a^2*b^4*c^2*d*e^2 - 7*a
^3*b^2*c^3*d*e^2)/c^6 + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*a*b^5*c
^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30*a*b^4*
c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a
*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3
*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e +
 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*
b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*
e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a*
b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*
b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b^3*e + 2*
a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a*b*(b^3*e
 + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 1
2*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*
(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) - (3
*a*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e + 12*a^2*c^2*e -
3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*c^3*(4*a*c - b^2)*(36*a*c^4 -
9*b^2*c^3))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^8*d - 72*a*b^2*c^7*
d + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b
^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a
*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^
3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^
2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)) + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*
```

```
a*b^5*c^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30
*a*b^4*c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d
 + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^
2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*
b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c
^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c
- b^2)^(1/2)) - (a*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^3)/(2*c^6*(4
*a*c - b^2)^(3/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(b^9*e^3 + 8*a^3*c^6*d
^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 + 3*b^7*c^2*d^2*e - 12*a^2*b^2*c^5*d^3 +
 27*a^2*b^5*c^2*e^3 - 27*a^3*b^3*c^3*e^3 - 9*a*b^7*c*e^3 - 3*b^8*c*d*e^2 -
21*a*b^5*c^3*d^2*e + 24*a*b^6*c^2*d*e^2 - 36*a^3*b*c^5*d^2*e + 48*a^2*b^3*c
^4*d^2*e - 63*a^2*b^4*c^3*d*e^2 + 54*a^3*b^2*c^4*d*e^2))*(b^3*e + 2*a*c^2*d
 - b^2*c*d - 3*a*b*c*e))/(3*c^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 55.47, size = 620, normalized size = 4.70

$$x^3\left(-\frac{be}{3c^2}+\frac{d}{3c}\right)+\left(-\frac{\sqrt{-4ac+b^2}\left(3abce-2ac^2d-b^3e+b^2cd\right)}{6c^3\left(4ac-b^2\right)}-\frac{ace-b^2e+bcd}{6c^3}\right)\log\left(x^3+\frac{2a^2ce-ab^2e+abcd-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] x**3*(-b*e/(3*c**2) + d/(3*c)) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**
2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)
/(6*c**3))*log(x**3 + (2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(-sqrt(-
4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c -
 b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(-sqrt(-4*a*c +
b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2))
- (a*c*e - b**2*e + b*c*d)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b*
*2*c*d)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d
)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*log(x**3 + (
2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e
 - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*
e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c*
*2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d
)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)) + e*x**6/(6*c)
```

3.10 $\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=97

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

```
[Out] 1/3*e*x^3/c+1/6*(-b*e+c*d)*ln(c*x^6+b*x^3+a)/c^2+1/3*(2*a*c*e-b^2*e+b*c*d)*
arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 773, 634, 618, 206, 628}

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/
(6*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x(d+ex)}{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= \frac{ex^3}{3c} + \frac{\mathrm{Subst}\left(\int \frac{-ae+(cd-be)x}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}$$
$$= \frac{ex^3}{3c} + \frac{(cd-be)\,\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2} - \frac{\left(bcd-b^2e+2ace\right)\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x\right.}{6c^2}$$
$$= \frac{ex^3}{3c} + \frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2} + \frac{\left(bcd-b^2e+2ace\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right.}{3c^2}$$
$$= \frac{ex^3}{3c} + \frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} + \frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.96

$$\frac{\frac{2\left(-2ace+b^2e-bcd\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + (cd-be)\log\left(a+bx^3+cx^6\right) + 2cex^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2
 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c^2
)
```

fricas [A] time = 1.36, size = 305, normalized size = 3.14

$$\left[\frac{2\left(b^2c-4\,ac^2\right)ex^3 + \left(bcd-\left(b^2-2\,ac\right)e\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right) + \left(\left(b^2c-4\,ac^2\right)d-\right.}{6\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*
c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + 2*(
b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 1.07, size = 95, normalized size = 0.98

$$\frac{x^3e}{3\,c} + \frac{(cd-be)\log\left(cx^6+bx^3+a\right)}{6\,c^2} - \frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*e/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

maple [A] time = 0.00, size = 175, normalized size = 1.80

$$\frac{e\,x^3}{3c}-\frac{2ae\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c}+\frac{b^2e\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c^2}-\frac{bd\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c}-\frac{be\ln\left(c\,x^6+b\,x^3+a\right)}{6c^2}+\frac{d\ln\left(c\,x^6+b\right.}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*e*x^3/c-1/6/c^2*ln(c*x^6+b*x^3+a)*b*e+1/6/c*ln(c*x^6+b*x^3+a)*d-2/3/c/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*e+1/3/c^2/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/3/c/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.95, size = 2624, normalized size = 27.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] (e*x^3)/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d -
 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*(4*a*c - b^2)^(3/2)
*(x^3*((b*((b^2*c^3*d^3 - b^5*e^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - 3*b^3*c
^2*d^2*e + 2*a*b^3*c*e^3 + 3*b^4*c*d*e^2 + 2*a*b*c^3*d^2*e - 4*a*b^2*c^2*d*
e^2)/c^3 - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^
3*e^2 - 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4
*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((45*b^2*c^5*d - 45*b^3*c^4*e +
36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*
c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*b^
2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*b^2*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(4*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((45*b^2*c^5*d
 - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3
*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/
```

```
(6*c^2*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e +
 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) + (b^2*(2*a*c*e - b^2*e + b*c*d)^3)/(4*c^3*(4*a*c - b^2)^(3/2)
) - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^3*e^2 -
 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4*e + 36
*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*
e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e)
)/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) + (b*((a^2*b^2*c*e^3 - a*b^4*e^3
 + a^2*c^3*d^2*e + a*b*c^3*d^3 + 3*a*b^3*c*d*e^2 - 3*a*b^2*c^2*d^2*e - 2*a^
2*b*c^2*d*e^2)/c^3 - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^
2 + 12*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d
 - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((36*a^2*c^5*e + 72*a*b*c^5*d -
72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*
b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c -
b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*
b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*a*b*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^5*e
 + 72*a*b*c^5*d - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d -
 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d)
)/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e
 + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^2 + 12
*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d - 72*
a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c
*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e
))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b
^2)^(1/2)) + (a*b*(2*a*c*e - b^2*e + b*c*d)^3)/(2*c^3*(4*a*c - b^2)^(3/2)))
)/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(8*a^3*c^3*e^3 - b^6*e^3 + b^3*c^3*d^3 -
3*b^4*c^2*d^2*e - 12*a^2*b^2*c^2*e^3 + 6*a*b^4*c*e^3 + 3*b^5*c*d*e^2 + 6*a*
b^2*c^3*d^2*e - 12*a*b^3*c^2*d*e^2 + 12*a^2*b*c^3*d*e^2))*(2*a*c*e - b^2*e
+ b*c*d))/(3*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 17.49, size = 434, normalized size = 4.47

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(-\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)+2acd+3\ldots}{2ace-b^2e+bcd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) -
(b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-sqrt(-4*a*c + b**2)*
(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2))
+ 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c*
*2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + (
sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b
*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b**2)*(2*
a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)) + 2
*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(
```

```
4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + e*x**
3/(3*c)
```

3.11 $\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=72

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c-1/3*(-b*e+2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c} + \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6c}$$
$$= \frac{e\log\left(a+bx^3+cx^6\right)}{6c} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3c}$$
$$= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^3+cx^6\right)}{6c}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.99

$$\frac{e\log\left(a+bx^3+cx^6\right) - \frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

fricas [A] time = 1.09, size = 216, normalized size = 3.00

$$\left[\frac{\left(b^2-4\,ac\right)e\log\left(cx^6+bx^3+a\right) - \sqrt{b^2-4\,ac}\,(2\,cd-be)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\left(b^2c-4\,ac^2\right)}, \frac{\left(b^2-4\,ac\right)e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a
*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c*x^
6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 1.21, size = 70, normalized size = 0.97

$$\frac{e\log\left(cx^6+bx^3+a\right)}{6\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

maple [A] time = 0.00, size = 99, normalized size = 1.38

$$-\frac{be\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c} + \frac{2d\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}} + \frac{e\ln\left(c\,x^6+b\,x^3+a\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b
^2)^(1/2))*d-1/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*e*
b/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.63, size = 1632, normalized size = 22.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] - (log(a + b*x^3 + c*x^6)*(3*b^2*e - 12*a*c*e))/(2*(36*a*c^2 - 9*b^2*c)) -
(atan((b*(4*a*c - b^2)^(3/2)*(a*c*d*e^2 - a*b*e^3 - ((3*b^2*e - 12*a*c*e)*(
((3*b^2*e - 12*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 1
2*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 -
12*a*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) + ((((b*e - 2*c*d)*(72*a*b*c^2*e -
36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*
(4*a*c - b^2)^(1/2)) + (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*
a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(2*(36*a*c^2 - 9*b^2*
c)*(4*a*c - b^2))))/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*
d*e^2)) - (4*x^3*((b*(b^2*e^3 + c^2*d^2*e + ((3*b^2*e - 12*a*c*e)*(6*c^3*d^
2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e
 - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^
2 - 18*b*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) - 2*b*c*d*e^2 - ((((b*e - 2*c*d
)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2
- 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^2*c^2*(3*b^2*e - 12*a*c*e)*(b
*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(
6*c*(4*a*c - b^2)^(1/2)) - (3*b^2*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(
4*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2))))/(4*a^2*c) - ((2*a*c - b^2)*(((3*b^2
*e - 12*a*c*e)*(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*
b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^
2*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c -
b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b^2*(b*e - 2*c*d)^3)/(4*(4*a*c -
b^2)^(3/2)) + ((b*e - 2*c*d)*(6*c^3*d^2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2
*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/
(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^2 - 18*b*c^2*d*e))/(6*c*(4*a*c - b^2)
^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/(b^3*e^3 - 8*
c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2) + ((2*a*c - b^2)*(4*a*c - b^2)*((
(3*b^2*e - 12*a*c*e)*(((b*e - 2*c*d)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c
^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) +
 (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*a*c^2 - 9*b^2*c)*(4*a*
c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) + ((b*e - 2*c*d)*(((3*b^2*e - 12
*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*
```

```
a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 - 12*a*c^2*d*e))
/(6*c*(4*a*c - b^2)^(1/2)) - (a*b*(b*e - 2*c*d)^3)/(2*(4*a*c - b^2)^(3/2)))
)/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2)))*(b*e - 2*
c*d))/(3*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 6.55, size = 287, normalized size = 3.99

$$\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\left(4ac-b^2\right)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\left(4ac-b^2\right)}\right)+2ae+3b^2\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\left(4ac-b^2\right)}\right)-bd}{be-2cd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3
 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2
))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

3.12 $\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

```
[Out] d*ln(x)/a-1/6*d*ln(c*x^6+b*x^3+a)/a+1/3*(-2*a*e+b*d)*arctanh((2*c*x^3+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^3\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx, x, x^3\right)}{3a}\\
&= \frac{d\log(x)}{a}-\frac{d\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a}+\frac{(-bd+2ae)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}-\frac{(-bd+2ae)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 80, normalized size = 1.03

$$
\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

fricas [A] time = 1.43, size = 240, normalized size = 3.08

$$
\left[-\frac{\left(b^2-4\,ac\right)d\log\left(cx^6+bx^3+a\right)-6\left(b^2-4\,ac\right)d\log(x)+\sqrt{b^2-4\,ac}\,(bd-2\,ae)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-\left(2\,cx^3\right.}{cx^6+bx^3+a}\right.}{6\left(ab^2-4\,a^2c\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c -
(2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c), -
1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 1.05, size = 76, normalized size = 0.97

$$
-\frac{d\log\left(cx^6+bx^3+a\right)}{6\,a}+\frac{d\log\left(|x|\right)}{a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arcta
n((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

maple [A] time = 0.01, size = 106, normalized size = 1.36

$$-\frac{bd \arctan\left(\frac{2c x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\, a}+\frac{2e \arctan\left(\frac{2c x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}+\frac{d \ln(x)}{a}-\frac{d \ln\left(c x^6+b x^3+a\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/a*d*ln(x)-1/6*d*ln(c*x^6+b*x^3+a)/a+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3
+b)/(4*a*c-b^2)^(1/2))*e-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-
b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.76, size = 4149, normalized size = 53.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^3 + c*x^6)*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 -
36*a^2*c)) - (atan((48*a^4*x^3*(4*a*c - b^2)^2*((((((3*b^2*d - 12*a*c*d)*(
((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*
a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*
a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*
a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 -
 36*a^2*c)) - ((2*a*e - b*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12
*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 -
 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 3
6*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(3*b^2*d - 12*a*c*d))
/(2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*
c*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*
a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4
*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e
))/(2*(9*a*b^2 - 36*a^2*c)) + 7*c^4*d*e^2))/(6*a*(4*a*c - b^2)^(1/2)) - (((
(2*a*e - b*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b
^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c
^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378
*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2)))
)/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^
2*c^4)*(2*a*e - b*d)^2)/(72*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(2*a*e
```

```
 - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 3
78*a*b^2*c^4)*(2*a*e - b*d)^3)/(432*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
(3/2)))*(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2*c*d + 2*a^2*b*c*e))/(16
*a^4*c^3*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) - ((c^3*e^4 - ((3*b
^2*d - 12*a*c*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2
*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*
a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*
b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c)
) + 7*c^4*d*e^2))/(2*(9*a*b^2 - 36*a^2*c)) + ((((2*a*e - b*d)*(((2*a*e - b*
d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a
^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(
1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/
(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2))
 + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d)^2)/(72
*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2
 - 36*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c
*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d
- 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12
*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^
2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a
*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(10
8*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^
3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4
*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((108*b^4*c^
3 - 378*a*b^2*c^4)*(2*a*e - b*d)^4)/(1296*a^4*(4*a*c - b^2)^2))*(4*b^5*d -
2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2*d + 4*a^2*b^2*c*e))/(16
*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e))
))/(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e - 12*a^2*b*c^3*d*e^2) -
 (3*(4*a*c - b^2)^(3/2)*(c^3*d*e^3 + ((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*
(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 1
2*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3
*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e
 - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(2*(9*a*b^2 - 36*a^2*
c)) - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(
27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a
*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d
*e))/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(2*(9*a*b^2 - 3
6*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^
2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6
*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4
*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) + ((2
*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b
^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^
2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(2*a*e
- b*d))/(6*a*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(2*a*e - b*d)^4)/(48*a^3*(4*a*
c - b^2)^2))*(4*b^5*d - 2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2
*d + 4*a^2*b^2*c*e))/(c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e
- 12*a^2*b*c^3*d*e^2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) + (3*(
4*a*c - b^2)^2*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d
)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(
9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(3*b^2*d - 12
*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9
*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(27*b^3*c^3*d -
 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*
c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))/(6*a*(4*a
*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(((2*a*e - b*d
)*(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d -
 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c
```

```
^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2
)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a
*e - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(6*a*(4*a*c - b^2)^
(1/2)) + ((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(27*b
^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2
 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))
/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(6*a*(4*a*c - b^2)^
(1/2)) - (b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d)^3)/(16*a^2*(9*a*b^2 -
36*a^2*c)*(4*a*c - b^2)^(3/2)))*(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2
*c*d + 2*a^2*b*c*e))/(c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e
- 12*a^2*b*c^3*d*e^2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)))*(2*a*
e - b*d))/(3*a*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

3.13 $\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

```
[Out] -1/3*d/a/x^3-(-a*e+b*d)*ln(x)/a^2+1/6*(-a*e+b*d)*ln(c*x^6+b*x^3+a)/a^2-1/3*
(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+
b^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(3*a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a*e
)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-acd-abe+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^3\right)$$
$$= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2d-acd-abe+c(bd-ae)x}{a+bx+cx^2}\,dx, x, x^3\right)}{3a^2}$$
$$= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2}+\frac{\left(b^2d-2acd-abe\right.}{}$$
$$= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(b^2d-2acd-abe\right)\text{Subst}}{}$$
$$= -\frac{d}{3ax^3}-\frac{\left(b^2d-2acd-abe\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+\right.}{6a^2}$$

Mathematica [C] time = 0.05, size = 130, normalized size = 1.16

$$\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{-\#1^3ace\log(x-\#1)+\#1^3bcd\log(x-\#1)-abe\log(x-\#1)-acd\log(x-\#1)+b^2d\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}+\frac{\log(x)(ae-b}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*d/(a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6
& , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Log[
x - #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

fricas [A] time = 2.32, size = 385, normalized size = 3.44

$$\left[\frac{\left(abe-\left(b^2-2\,ac\right)d\right)\sqrt{b^2-4\,ac}\,x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\left(\left(b^3-4\,abc\right)d-\left(ab^2-4\,a^2c\right)e\right)x^3\,\text{lo}}{6\left(a^2b^2-4\,a^3c\right)x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*
c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a))
 + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6
*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)
*d)/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-b^2 +
 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^3
```

```
 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6*((b^3 -
 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^
2*b^2 - 4*a^3*c)*x^3)]
```

giac [A] time = 1.07, size = 128, normalized size = 1.14

$$\frac{(bd-ae)\log\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{(bd-ae)\log\left(|x|\right)}{a^2}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bdx^3-ax^3e-ad}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(b*d - a*e)*log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 +
1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqr
t(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*x^3*e - a*d)/(a^2*x^3)
```

maple [A] time = 0.01, size = 191, normalized size = 1.71

$$-\frac{be\arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a}-\frac{2cd\arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a}+\frac{b^2d\arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a^2}+\frac{e\ln(x)}{a}-\frac{e\ln\left(cx^6+bx^3+a\right)}{6a}-\frac{bd\ln(x)}{a^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a*d/x^3+1/a*ln(x)*e-1/a^2*ln(x)*b*d-1/6/a*ln(c*x^6+b*x^3+a)*e+1/6/a^2*
ln(c*x^6+b*x^3+a)*b*d-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2
)^(1/2))*b*e-2/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*
c*d+1/3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 9.57, size = 7282, normalized size = 65.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (log(x)*(a*e - b*d))/a^2 - (log(((((((((a*e - b*d + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d))
/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a*c*d))/a + (9*b^2*c^3*(a*b + 4*b
^2*x^3 - 14*a*c*x^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4
*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) - (3*c^5*d*x^3*(11*b^2*d - 14*a*b*e
 + 14*a*c*d))/a^2 + (9*b*c^4*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(a*e - b*d
 + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) +
 (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e - 12*b*d))
/a^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(
```

```
1/2)))/(6*a^2) + (c^6*d^3*(a*e - b*d))/a^4 - (c^7*d^4*x^3)/a^4)*(((((((b*d
- a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*
b^2*c^3*(a*b*e - b^2*d + a*c*d))/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a
*c*d))/a - (9*b^2*c^3*(a*b + 4*b^2*x^3 - 14*a*c*x^3)*(b*d - a*e + a^2*(-(a*
b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) + (
3*c^5*d*x^3*(11*b^2*d - 14*a*b*e + 14*a*c*d))/a^2 - (9*b*c^4*d*(3*a*b*e - 3
*b^2*d + a*c*d))/a^2)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(
4*a*c - b^2)))^(1/2)))/(6*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3
+ (c^6*d^2*x^3*(7*a*e - 12*b*d))/a^3)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) - (c^6*d^3*(a*e - b*d))/a^4
+ (c^7*d^4*x^3)/a^4))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(
36*a^3*c - 9*a^2*b^2)) - d/(3*a*x^3) - (atan((48*a^8*x^3*((((((((((18*a^3*b
^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d
- 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3
*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a
^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*
d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 +
((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12
*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*
a^2*(4*a*c - b^2)^(1/2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/
(2*(36*a^3*c - 9*a^2*b^2)) - (((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 2
52*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b
^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b
^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*
b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a
*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*
d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2
*c^4)*(a*b*e - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*
b*c*d))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a
*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (((7*a^2*c^6*d^2*e - 12*a*b*c^6*d^3)/a
^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18
*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^
3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^
4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))
/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d
))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b
^2)^(1/2)) - ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)
^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(432*a^10*(4*a*c - b^2)
^(3/2)*(36*a^3*c - 9*a^2*b^2)))*(4*b^5*d - 7*a^3*c^2*e - 4*a*b^4*e - 16*a*b
^3*c*d + 9*a^2*b*c^2*d + 15*a^2*b^2*c*e))/(16*a^4*c^3*(49*a^3*c*e^2 - 12*b^
4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a
^2*b*c*d*e)) - (((((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5
*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^
2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c
*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*
b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12
*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d)
)/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e
 - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(72*
a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((7*a^2*c^6*d^2*e - 12*a*b*c
^6*d^3)/a^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^
4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a
^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*
d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*
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a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 1
2*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (c^7*d^4)/a^4 - ((108*a^4*b^4*c^
3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)^4)/(1296*a^12*(4*a*c - b^2)^
2) + (((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((
108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a
*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*
(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d
+ 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c
- b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 1
2*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5
*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252
*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2
*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3
*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b
^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)))*(a*b*e - b^2*d + 2*a*c*d))/(6
*a^2*(4*a*c - b^2)^(1/2)))*(8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b
^2*c^2*d + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(64*a^4*c^3*(4
*a*c - b^2)^(1/2)*(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2
 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e)))*(4*a*c - b^2)^2)/(8*a^
3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^
3*e^3 + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a
^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c^4*d*e^2) + (3*a^4*(4*a*c - b^2)^2*((((((((27
*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(
3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))
*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(a*b*e
 - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(4*a*(
4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c
*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((9*a^3*b*c^5*d^2 - 27*a^2*
b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^
3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c
*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2
*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/
(6*a^2*(4*a*c - b^2)^(1/2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d
))/(2*(36*a^3*c - 9*a^2*b^2)) + (((a^2*c^6*d^3 - 9*a*b^2*c^5*d^3 + 9*a^2*b*
c^5*d^2*e)/a^4 + (((9*a^3*b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d
*e)/a^4 + (((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 +
(27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c
 - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3
*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a
^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
- (((((((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*
a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9
*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3
*c^3*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c
*d))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a
*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^2
*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 - (b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(16*a^5*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2)))*(4*b^5*d
 - 7*a^3*c^2*e - 4*a*b^4*e - 16*a*b^3*c*d + 9*a^2*b*c^2*d + 15*a^2*b^2*c*e)
)/(c^3*(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3
*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e)*(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*
b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^3*e^3 + 3*a*b^5*c^3*d^2*e + 12
*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c
^4*d*e^2)) - (3*a^4*(4*a*c - b^2)^(3/2)*((b*c^6*d^4 - a*c^6*d^3*e)/a^4 - ((
(a^2*c^6*d^3 - 9*a*b^2*c^5*d^3 + 9*a^2*b*c^5*d^2*e)/a^4 + (((9*a^3*b*c^5*d^
2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c^4*d - 27
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*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e
 + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2
*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b
^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a
*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((((((27*a
^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*
b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(
a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(a*b*e -
 b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(4*a*(4*
a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2
*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3
*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(36*a^3*c - 9*a^2
*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a
^2*b^2)) + (((((((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a
^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 + (9*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d -
3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((9*a^3
*b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c
^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d -
3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d
- 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e
- b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)))*(a*b*e - b^2*d + 2*a*c*d))
/(6*a^2*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^4)/(48*a^
7*(4*a*c - b^2)^2))*(8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b^2*c^2*
d + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(4*c^3*(49*a^3*c*e^2
- 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2
 - 97*a^2*b*c*d*e)*(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*
b^2*c^5*d^3 + a^3*b^3*c^3*e^3 + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12
*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c^4*d*e^2)))*(a*b*e -
b^2*d + 2*a*c*d))/(3*a^2*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.14 $\int \frac{x^4\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

<u>**Optimal**</u>. Leaf size=723

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2ace+b^2(-e)+bc\ldots}{\sqrt{b^2-4ac}}\right.}{\ldots}$$

```
[Out] 1/2*e*x^2/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*e
+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)
^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)
^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d-b*
e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3
))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-
4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-
4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(2*a*c*e-b^2
*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-
1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2)
)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

<u>**Rubi**</u> [A] time = 1.81, antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1502, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2ace+b^2(-e)+bc\ldots}{\sqrt{b^2-4ac}}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/
(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*
c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)
/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqr
t[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)
*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e - (b
*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])
/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{ex^2}{2c} - \frac{\int \frac{x\left(2ae-2(cd-be)x^3\right)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{ex^2}{2c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-\cdots}}}{2c}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\cdots}{\frac{\left(b-\sqrt{b^2-4ac}\right.}{2^{2/3}}}}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{\cdots}}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2a\cdots}{\sqrt{b^2-4ac}}\right.}{3\,2^{2/\cdots}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2a\cdots}{\sqrt{b^2-4ac}}\right.}{3\,2^{2/\cdots}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\cdots}{\cdots}\right.}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b+\sqrt{b^2-4\cdots}}}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.12

$$\frac{3ex^2-2\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.01, size = 70, normalized size = 0.10

$$\frac{e\,x^2}{2c}-\frac{\left((be-cd)\,\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^4+\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)ae\right)\ln\left(-\,\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)+\right.}{3c\left(2\,\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^5c+\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*e*x^2/c-1/3/c*sum(((b*e-c*d)*_R^4+_R*a*e)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_
R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex^2}{2\,c}-\frac{-\int\frac{(cd-be)x^4-aex}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3 + a), x)/
c
```

mupad [B] time = 42.01, size = 13112, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a
*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a
^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d
^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d
^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*
a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*
a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*
c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3
- 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*
e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3
*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c - b^2)*(b*
e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e -
 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
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3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(2/3))/18 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*
e - b^2*e + b*c*d))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d
^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b
^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*
a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2
*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((2^(1/3)*((2^
(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*
c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5
*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*
d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 +
 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2
*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2
 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 +
5*a*b*c^2*d*e))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c
^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b
^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^
4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2
*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b
^2)^3))^(2/3))/18 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d
))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a
^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*
b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e +
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30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e -
96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + (e*x^2)/(2*c) + log(- (2^(1/3)*((2
^(2/3)*(3^(1/2)*1i - 1)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*
a*c*e^2 - 2*b*c*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2
*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*
c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d
*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b
^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*
b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-
(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3
*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^
2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*
c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3
*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*
a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*
d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i + 1)*(
-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^
3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e
^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5
*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^
3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*
b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a
^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/
2 - 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*
a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3
*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e
+ 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e -
 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log(- (2^(1/3)*((2^(2/3)*(3^(1/2)
*1i - 1)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c
*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 1
6*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^
4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^
5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 +
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96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 +
 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(
-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a
^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d
^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d
^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*
a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*
a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*(4*a*c - b^2)
*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d
*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i + 1)*(-(b^8*e^3 + 16*
a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*
d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*
d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96
*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6
*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c
*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8
*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*
a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3
 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 +
5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*
d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2
*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3
*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 -
 48*a^2*b^2*c^7)))^(1/3) - log(- (2^(1/3)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^
2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) - (27*2^(
1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 -
 b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*
b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6
*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*
e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*
a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^
3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^
5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c
^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^
2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2
 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b
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^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 + (9*a*(4*a*c - b^2)*(b*e - c*d)*(b
^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*
e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i - 1)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b
^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*
c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c
^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^
2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*
b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2
*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*e^3 + 16*a^4*c
^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 -
 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e
 + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*
b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*
c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7
)))^(1/3) - log(- (2^(1/3)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(4*a*c -
b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(3
^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 -
b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a
*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41
*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*
(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^
4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^
2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*
a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*
a*c - b^2)^3))^(1/3))/12 + (9*a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*
d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2
*d*e))/c^2)*(3^(1/2)*1i - 1)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^
5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a
^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72
*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4
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*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e
 + b*c*d))/c^2)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^
3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d
^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^
2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3
*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2
))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

3.15 $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=718

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right.}{}$$

```
[Out] e*x/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*e+(-2*a
*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2)
)^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2)
)^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a
*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*
(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d-b*e+(-2*
a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(c*
d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+
b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+
b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(2*a*c*e-b^2*e+b*c
*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ar
ctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d
-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] time = 1.46, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1502, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d -
b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e - (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+c}}{2c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}}}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4a\right.}}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\mathrm{l}}{3\sqrt[3]{2}\,c^{4/3}}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\mathrm{l}}{3\sqrt[3]{2}\,c^{4/3}}$$

$$= \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b+\sqrt{}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.12

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^3+d\right)x^3}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 67, normalized size = 0.09

$$\frac{ex}{c}+\frac{\left((-be+cd)\operatorname{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^3-ae\right)\ln\left(-\operatorname{RootOf}\left(\_Z^6c+\_Z^3b+a\right)+x\right)}{3c\left(2\operatorname{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^5c+\operatorname{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/c*e*x+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=Root
Of(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex}{c}-\frac{-\int\frac{(cd-be)x^3-ae}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 30.15, size = 11453, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d
^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e
+ 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/3)*
((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)
^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3
*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^
2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^
3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2
*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*
a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*
e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*
a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^
2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6
*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*
```

```
a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e
^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^
2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*
c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c
^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^
2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3
*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e
+ 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3
*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 + log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^
3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3
*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b
^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48
*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
- 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 +
 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2
*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^
3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2
*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2
+ 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^
2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b
^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3
- b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b
*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32
*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3
- 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2
*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2
- 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*d*
x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((
b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(
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4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c
^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(
1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 +
 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
 b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2*e^3
 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a*b*
c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 +
2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4
*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d
*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2
*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^
2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2
*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d
^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3
- b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e +
 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e
^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*
d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e
^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c -
b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))
^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3
*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2
*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*
b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3)
)^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a
^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*
b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^
2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*
c - b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2*
e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a
*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2
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+ 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^
2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b
^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4
*(4*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3
+ 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c
*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*
c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^
3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e
^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*
e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*
c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c
^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*
d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6
)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)*(81*
a*c^3*d*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b
^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48
*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
+ 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 -
 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2
*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2
*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2
+ 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^
2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b
^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d
*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*
b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a
*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b
^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*
c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2
*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*
b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2
*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^
2*c^6)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)
*(81*a*c^3*d*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*
c - b^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^
```

```
2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^
2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b
^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2
 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
 - b^2)^3))^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2
*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*
e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^
4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(c^4*(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3
+ a^2*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d
*e^2 + 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*
e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b
^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^
6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 2
7*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^
2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 -
b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e
 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3
- 9*a*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*e^3 - 16*a^2*c^5*d^
3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a
^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^
5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a
^2*b^2*c^6)))^(1/3) + (e*x)/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.16 $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\ldots\right.}{6\,2\ldots}$$

```
[Out] -1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-4
*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/
3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^
2)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b-(-4
*a*c+b^2)^(1/2))^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b+(-
4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-
4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*arctan(1/3*(
1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(b*e-2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A] time = 0.73, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\ldots\right.}{6\,2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sq
rt[3]*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/
3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*
c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\,2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\,2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\,2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right.}{3}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3e\log(x-\#1)+d\log(x-\#1)}{2\#1^4c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
+ 2*c*#1^4) & ]/3
```

fricas [B] time = 93.49, size = 13607, normalized size = 21.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4
*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*
d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c
^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c +
4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(2*(a*
b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d - (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a
^3*b*c^4)*e)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^
4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^
3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c
```

```
 + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*
c^7)) - sqrt(3)*((b^3*c^2 - 4*a*b*c^3)*d^3*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^
2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)
*e^4))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b
^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^
3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(
a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(
a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 -
4*a^2*c^3))^(1/3)*sqrt((2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c
^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c
 + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^
2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 + (1/2)^(2/3)*(((a*b^
6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 -
12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12
*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*
b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a
^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 1
2*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) - ((b^4*c^3 - 4*a*b^2*c^4)*d^
5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*
a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a
^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*
a^4*b*c^2)*e^5)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*
c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e
^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d
^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4
*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(
a*b^2*c^2 - 4*a^2*c^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b
^4*c^2 + 2*a*b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4
*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3
*c - 4*a^3*b*c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 - ((a
*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 +
 32*a^4*c^5)*d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 -
 2*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a
*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*
c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*
a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b
*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*
b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*
a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2
*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4
*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))/(b*c^4*d^7 - 2*(b^
2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 +
2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2
*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)
*e^7)) - (1/2)^(1/3)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d
 - (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*x*sqrt((b^2*c^4*d^6 - 12*a
*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*
c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*
a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) - sqrt(3)*((b^3*c^2 - 4*a*b*c^3
)*d^3*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e
^3 - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e^4)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a
*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(
a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(
7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a
^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a
^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3) - sqrt(3)*(b*c^3*
d^5 + 10*a*b*c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2
```

```
)*d^2*e^3 + (a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5))/(b*c^3*d^5
+ 10*a*b*c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^
2*e^3 + (a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5)) - 2/3*sqrt(3)*(
1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqr
t((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(
a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 -
 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^
6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^
2 - 4*a^2*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a
^2*b^2*c^4 + 16*a^3*c^5)*d - (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*
sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 -
2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^
4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)
*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) + sqrt(
3)*((b^3*c^2 - 4*a*b*c^3)*d^3*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*
b^3*c - 4*a^2*b*c^2)*d*e^3 - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e^4))*(-(c^2
*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 -
12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^
2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2
*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 -
 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(
1/3)*sqrt((2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c
^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^
2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b
*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 - (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^
2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^
3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*
e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^
3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e
^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5
 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) + ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3
*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*
e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36
*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^
5)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^
2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3
*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a
^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a
^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4
*a^2*c^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*
b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b
^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*
c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 + ((a*b^5*c^3 - 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*
d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c
^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e
+ 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3
+ 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5
 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 +
 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2
*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a
^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 -
 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*
b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64
*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))/(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c
^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^
4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^
2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)) - (1/2
)^(1/3)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d - (a*b^5*c^2
```

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 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e
+ 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3
+ 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5
 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 +
 48*a^4*b^2*c^6 - 64*a^5*c^7)) + sqrt(3)*((b^3*c^2 - 4*a*b*c^3)*d^3*e - 6*(
a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 - (a*b^4 -
 6*a^2*b^2*c + 8*a^3*c^2)*e^4)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b
^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6
*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2
 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^
3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 -
64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3) + sqrt(3)*(b*c^3*d^5 + 10*a*b*
c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^2*e^3 + (
a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5))/(b*c^3*d^5 + 10*a*b*c^2*
d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^2*e^3 + (a*b^
3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5)) - 1/6*(1/2)^(1/3)*(-(c^2*d^3
 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a
*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*
c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*
a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)
*log(2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^
5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3
*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*
e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 + (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4*
c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48
*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*
(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*
(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (
a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*
a^4*b^2*c^6 - 64*a^5*c^7)) - ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 -
 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 -
(a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b
^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x)*
(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*
d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 +
 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3
*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6
*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c
^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*b^2*c^
3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b^4*c -
 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*c^2)*d
^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 - ((a*b^5*c^3 - 8*a^2*b^
3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e
+ 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a
*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7
*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^
2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^
4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 -
 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4
)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3
*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c
+ 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c
^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)) - 1/6*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*
d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^
5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*
e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d
*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c
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^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*log(2*(b
*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5
*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a
*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^
3*b^2 - 2*a^4*c)*e^7)*x^2 - (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*
a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*
c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^
3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^
2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 -
 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c
^6 - 64*a^5*c^7)) + ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*
c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c
+ 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 +
32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x)*(-(c^2*d^
3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*
a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b
*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^
3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12
*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3
) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*b^2*c^3 - 24*a^
2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b^4*c - 3*a^2*b^
2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^4 -
(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 + ((a*b^5*c^3 - 8*a^2*b^3*c^4 + 1
6*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e + 3*(a^2*
b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3
+ 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*
c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4
*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6
 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^
3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2
 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2
*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c
^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*
b^2*c^2 - 4*a^2*c^3))^(1/3)) + 1/3*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a
*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(
a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(
7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a
^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a
^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*log((1/2)^(2/3)*(
(b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4
*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64
*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (
a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5 - ((a*b^6*c^3 - 12*a^2*b^4*c^4 + 4
8*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^
2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^
3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^
2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 -
 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c
^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*
c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e
^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d
^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4
*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(
a*b^2*c^2 - 4*a^2*c^3))^(2/3) + 2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e
+ (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*
(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5
 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x) + 1/3*(1/2)^(1
/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*
```

```
c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c
^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2
*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2
*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a
^2*c^3))^(1/3)*log((1/2)^(2/3)*((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3
 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2
- (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3
*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5 +
((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*
c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^4*d^6
- 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*
a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c -
 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4
 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2
 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e +
 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 +
 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5
+ (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 +
48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3) + 2*(b*c^4*d^
7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b
^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 +
6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 -
 2*a^4*c)*e^7)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^3 + d\right)x}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 49, normalized size = 0.08

$$\frac{\left(\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^4 e + \mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right) + x\right)}{6\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^5 c + 3\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^3 + d\right)x}{cx^6 + bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 24.56, size = 7457, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 +
 a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 -
b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(
4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(
2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c -
 b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^
2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a
^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 2
4*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c
 - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*
c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c
^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3
 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*
d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^
2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*
e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4
*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3
+ b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^
2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*
c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (
2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4
*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3
*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2
*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e
^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a
^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27
*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^
2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
- a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 +
 b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) - log(c*x*(
b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^(1/3)*(3^(1/2)*1i - 1)*((a*b^5*e^
3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a
*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^
```

```
2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*
(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^(2/3)*(3^(1/2)*1i + 1)*(27*c^3*x*(
4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c
^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2
*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c
^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2
 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*
d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/12 - 45*a^2
*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 10
8*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 + 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3
 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a
^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^
2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48
*a^3*b^2*c^4)))^(1/3) - log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^
(1/3)*(3^(1/2)*1i - 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2
*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*
c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
)/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^(
2/3)*(3^(1/2)*1i + 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*
d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((a
*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^
2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c
^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))
^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a
*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*
a*c - b^2)^3))^(1/3))/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^
3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 +
1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*
c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a
^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5
- a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log(c*x*(b*e - c*d
)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*((a*b^5*e^3 + 16*a^
2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*
e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c
^3*e^3 - 108*a^2*c^4*d^2*e - (2^(2/3)*(3^(1/2)*1i - 1)*(27*c^3*x*(4*a*c - b
^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) + (27*2^(1/3)*a*b*c^3*(3^
(1/2)*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 -
8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16
*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c
```

```
- b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 +
b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*
b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/12 - 45*a^2*b^2*c^2*
e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c
^3*d*e^2))/36)*((3^(1/2)*1i)/2 - 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^
2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*
e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*
c^4)))^(1/3) + log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^
(1/2)*1i + 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
- a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 +
 b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*
(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e - (2^(2/3)*(3^(
1/2)*1i - 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a
*b*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3
+ 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b
^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2
+ 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4
)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3
*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*
b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2
)^3))^(1/3))/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e -
 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 - 1/2)*((a*
b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2
 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^
2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c
^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

3.17 $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{\ }\right.}{}$$

```
[Out] 1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3
)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2
)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)
^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)
*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b
^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4
*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*(1
-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(b*e-2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] time = 0.65, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{\ }\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(2/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4}$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,d}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right.}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right.}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e+\frac{2cd-}{\sqrt{b^2-}}\right.}{}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3e\log(x-\#1)+d\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1^
2 + 2*c*#1^5) & ]/3
```

fricas [B] time = 27.69, size = 14094, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4
*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 +
4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a
^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a
^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^
5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*((a
^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c
 - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*x*sqrt(-(12*a^4*b*c*d
*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2
 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*
```

```
c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - sqrt(3)*((b^5*c^2 - 6*a
*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d
^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2
*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (
a^3*b^4 - 4*a^4*b^2*c)*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2
*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*
c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
 + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*
a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) - (1/2)^(1/6)*(sqrt(3)*((a^2*b^6*
c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a
^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^
4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*
b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^
3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5
*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3
+ 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a
*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*
c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 -
 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*
c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2
*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^
3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^
2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/
(a^2*b^2*c - 4*a^3*c^2))^(2/3)*sqrt((2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7
 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2
+ 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 +
 (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^
(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c
 - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 3
2*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a
^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((
a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^
6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*
d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^
2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3
*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e
 + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6
- (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5
*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d
^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3
)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b
^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d
*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*
d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^
2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3
*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^
3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c
 - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^
3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*
e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(
12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 +
6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 +
 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/
(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c -
```

```
4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b
*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^
2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*
b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - 2*sqrt(3)*(a^4*b*e^7 - (b^2*
c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2
*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*
a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d
*e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e -
 (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*
e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5
- 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) + 2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c
*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*arctan
(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c
^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^
6*c^4)*e^2)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^
3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 -
 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c +
 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^
7*c^5)) + sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2
 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*
b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a
^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5)*x)*((b*c*d^3 -
 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 -
a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^
2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*
a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a
^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3)
- (1/2)^(1/6)*(sqrt(3)*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*
e^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2
*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^
3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^
2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) +
 sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2
*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^
3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c
- 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2*
e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6
 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^
5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*
d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
+ 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3)*sqrt((2*(a^
4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c +
6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3
*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b
^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c
^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*
(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3
*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 -
 2*(a^4*b^3 - 4*a^5*b*c)*e^5 + ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^
3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64
*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
- 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
+ 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a
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^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(
-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6
+ 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2
 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4
)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c
 - 4*a^3*c^2))^(2/3) - (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*
b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a
^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
- 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
+ 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a
^7*c^5)) + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2
- 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10
*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d
^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3
- (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2
- 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a
^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*
(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3
- 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2
*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*
c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6
)) + 2*sqrt(3)*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)
*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c
^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*
d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^
7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2
 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4
+ (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - 1/6*(1/2
)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(
12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 +
6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 +
 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/
(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c -
 4*a^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2
- a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c +
 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17
*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c
- 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c
^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3
*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4
*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((a^2*b^7*c - 12
*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b
^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (
a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4
*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*
b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^
3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^
4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3)*(((a^2*b^5*c
^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^
5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*
b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
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3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)
*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c
^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 +
 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*
d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e
^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 -
 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c
+ 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 -
 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(
1/3)) - 1/6*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*
a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4
*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^
3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^
4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5
)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d
^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^
2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4
 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2
)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5
*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 +
 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*
(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 +
((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*
b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*
c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*
c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b
^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*
c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2
*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^
6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d
^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)
*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) - (1/2)^(1
/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3
*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)
*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*
c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*
c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b
^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*
c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) + ((b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4
*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*
e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*
d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(
-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6
+ 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2
 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4
)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c
 - 4*a^3*c^2))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 +
 (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 -
 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^
2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(
a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*
c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*d^2*e^
3 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^
2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x + (1/2)^(1/3)*((b^4*c
 - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*
```

```
b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 - ((a^2*b^5*c - 8*
a^3*b^3*c^2 + 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*
e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c
^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)
*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)
*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((
b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c
*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c
^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^
3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c
^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2
))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c
- 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3
 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 -
8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c +
6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7
*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*d^2*e^3 + a^3*b*e^
5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c + a^2*c^
2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x + (1/2)^(1/3)*((b^4*c - 6*a*b^2*c
^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*b^2*c - 4*a^
3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + ((a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12
*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*
(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2
*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a
^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*
a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4
*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b
*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3
*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*
b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.01, size = 47, normalized size = 0.07

$$\frac{\left(\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^3 e + d\right) \ln\left(-\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right) + x\right)}{6\,\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^5 c + 3\,\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 18.96, size = 7469, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(a + b*x^3 + c*x^6),x)
```

```
[Out] log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*
e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3
 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1/3)*(81*c^3*x*(4*a*c - b^2
)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^
4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*
e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2
*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b^5
*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3
- 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e
- 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2
*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(
2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^3*e^
3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(-(b^5*c*d^3 + a^2*b^4*e^3
+ 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 +
b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6
*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 -
48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4
- b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^
2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d
^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^
2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c
*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*
a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1
/3)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^
2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2
*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3
*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*
a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c -
 b^2)^3))^(1/3))/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3
*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(
-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3
*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d
^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^
```

2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2
*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4
+ a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) +
(2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a
*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^
2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5*d^3 - 9*b^2*c^4*d^3 - 9*a*b^
3*c^2*e^3 + 36*a^2*b*c^3*e^3 - 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i + 1
)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i -
1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*
e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(
1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36 + 27*a*b^2*c^3*d*e^2))/12
)*((3^(1/2)*1i)/2 - 1/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*
b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2
*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3)
+ log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d
^2*e^2 - 4*b*c^2*d^3*e) + (2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5*c*d^3 + a^2*b^4*
e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^
3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e
- 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5*d^
3 - 9*b^2*c^4*d^3 - 9*a*b^3*c^2*e^3 + 36*a^2*b*c^3*e^3 - 108*a^2*c^4*d*e^2
+ (2^(1/3)*(3^(1/2)*1i + 1)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(
2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 +
16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c
- b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^
2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a
^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 +
a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^
2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3
*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^
4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e +
3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36
+ 27*a*b^2*c^3*d*e^2))/12)*((3^(1/2)*1i)/2 - 1/2)*(-(b^5*c*d^3 + a^2*b^4*e
^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3
- b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e
- 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2
- 48*a^4*b^2*c^3)))^(1/3) - log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e
^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) + (2^(2/3)*(3^(1/2)*1i +
1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b
*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c

```
^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^
2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b
^2)^3))^(1/3)*(9*b^2*c^4*d^3 - 36*a*c^5*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^
3*e^3 + 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*c^3*x*(4*a*c - b^
2)^2*(a*e - b*d) + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(
b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d
^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2
*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*
c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3)
)^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 +
 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(
4*a*c - b^2)^3))^(2/3))/36 - 27*a*b^2*c^3*d*e^2))/12)*((3^(1/2)*1i)/2 + 1/2
)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*
c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^
3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2
*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 -
a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3) - log(3*c^2*x*(2*c^3*d
^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e)
 + (2^(2/3)*(3^(1/2)*1i + 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 -
8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(9*b^2*c^4*d^3 - 36*a*c^5*d^3 + 9*a
*b^3*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i
- 1)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1
i + 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^
3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2
))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c
^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36 - 27*a*b^2*c^3*d*e^2))
/12)*((3^(1/2)*1i)/2 + 1/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8
*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1
/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

3.18 $\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=653

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\right.}{}$$

```
[Out] -d/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(d+(-
2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/
3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3
)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3
)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/
2))*2^(1/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^(1/3)*c
^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2
^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2
^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))
*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)+
1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))
*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/3)
```

Rubi [A] time = 1.18, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1504, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (
b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d + (b
*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*
a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4
*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{ax} - \frac{\int \frac{x\left(bd-ae+cdx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{d}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}$$

$$= -\frac{d}{ax} + \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt[3]{b+\cdots}}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}\cdots}}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2\cdots}}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\cdots\right.}{3\,2^{2/3}a\sqrt[3]{\cdots}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\cdots\right.}{3\,2^{2/3}a\sqrt[3]{\cdots}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4a\cdots}}}}{\sqrt{3}}\right.}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.13

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{3a}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3+d}{\left(cx^6+bx^3+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2), x)
```

maple [C] time = 0.01, size = 70, normalized size = 0.11

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{4}cd+\left(-ae+bd\right)\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)+\right.}{3a\left(2\,\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{5}c+\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a*sum((c*d*_R^4+(-a*e+b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_
Z^6*c+_Z^3*b+a))-1/a*d/x
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 38.02, size = 11174, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*
a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2
*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*
a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*
a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3
*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*
b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3
+ b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^
3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a
^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c
*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2
*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7*d^3 -
a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*
c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*
d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e -
24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^
```

```
(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9
*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/6 +
36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^
3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/18
+ a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2
*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5
*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 4
8*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2)))^(1/3) + log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^
3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b
^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*
e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^
4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c
*d^2 + 6*a^2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 -
 a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b
*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5
*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e -
 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e -
9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*
(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^
3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3
 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b
^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*
c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3)
)^(1/3))/6 + 36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a
^8*b^2*c^5*d^3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^
4*d*e^2))/18 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((b^7*d^3 - a^3*b^4*e
^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 +
 a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 4
8*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^
3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c +
48*a^6*b^2*c^2)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*(-(b^7*d^3 - a^3*b^
4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^
3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2
+ 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3
*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b
```

```
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9*c^6
*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 - (2^(2/
3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^
2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) -
(27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^
4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^
3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2
+ 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3
*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7*
d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*
a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^
2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^
2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2
*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3)
)/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/
36 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^
3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^
3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*
b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*
e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e
 + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12
*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*(-(b^7
*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32
*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a
^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d
^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^
2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
)*(36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5
*d^3 - (2^(2/3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a
^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^
2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^7
*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32
*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a
^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d
^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^
2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
))/4)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*
c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2
*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^
2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c
*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27
*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
```

```
3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b
^2)^3))^(1/3))/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2
*c^4*d*e^2))/36 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((3^(1/2)*1i)/2 +
1/2)*((b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^
2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c
*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a
^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*
b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64
*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(a^7*c^4*e*x*(a*e^2
+ c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*(-(b^7*d^3 - a^3*b^4*e^3 + b
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*
c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*
e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9*c^6*d^3 - 10
8*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2^(2/3)*(3^(1/
2)*1i - 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2
 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) + (27*2^(1/
3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3 + b
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*
c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*
e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7*d^3 - a^3
*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3
*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e
^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*
a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^
2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/12 + 10
8*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/36)*((3^(
1/2)*1i)/2 - 1/2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3
 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e
- 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a
^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*
(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(a^7*c^
4*e*x*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*(-(b^7*d^3 - a^
3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^
3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*
e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24
*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a
```

```
^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9
*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2
^(2/3)*(3^(1/2)*1i - 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2
+ a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e
) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^
3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^
3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*
e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24
*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a
^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(
b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 -
 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 +
3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*
c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2
*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(
1/3))/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^
2))/36)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^
2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3
*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^
2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3)
 - d/(a*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.19 $\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=655

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\cdots\right.}{}$$

```
[Out] -1/2*d/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))
*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)
+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2
))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*
2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*
c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^
2)^(1/2))*2^(2/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln(2^(
1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1
/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)
*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^
(2/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^
(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^
(1/3))*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] time = 1.11, antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1504, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(2*a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2
 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3
)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3
)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{2ax^2} - \frac{\int \frac{2(bd-ae)+2cdx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}}{2a}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{}}}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\sqrt[3]{}}}{3\sqrt[3]{2}\,a\left(b+\sqrt{b^2-4a}\right.}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\,a\,(}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\,a\,(}$$

$$= -\frac{d}{2ax^2} + \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{}}{\sqrt[3]{b+\sqrt{b^2}}}}{\sqrt{3}}\right.}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.14

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3a}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3+d}{\left(cx^6+bx^3+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3), x)
```

maple [C] time = 0.01, size = 68, normalized size = 0.10

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{3}cd+ae-bd\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)+x\right)}{3a\left(2\,\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{5}c+\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)^{2}b\right)}-\frac{d}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+
_Z^3*b+a))-1/2/a*d/x^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 37.90, size = 13466, normalized size = 20.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
+ 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^
3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a
^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 +
 b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*
a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 4
1*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
```

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2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^
4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b
^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5
*(4*a*c - b^2)^3))^(2/3))/18 + 36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 1
08*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*
b^3*c^4*d*e^2))/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*
d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4
*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^
2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3
*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
+ 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) +
log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
- 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^
3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a
^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 -
 b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*
a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 4
1*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^
4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b
^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5
*(4*a*c - b^2)^3))^(2/3))/18 + 36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 1
08*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*
b^3*c^4*d*e^2))/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*
d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4
*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^
2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3
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*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
+ 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*
b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) -
d/(2*a*x^2) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^
4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c
^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^
3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e
- 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
 + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(108*a^9*c^6*d^2*e - 72*a^8*b*c^6*d^3 -
 36*a^10*c^5*e^3 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*
(a*b*e - b^2*d + a*c*d) + (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c -
b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3
)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3
*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*
b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d
^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/
4)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2
*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e
 + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9
*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 -
 9*a^6*b^5*c^4*d^3 + 54*a^7*b^3*c^5*d^3 + 9*a^9*b^2*c^4*e^3 + 108*a^9*b*c^5
*d*e^2 + 27*a^7*b^4*c^4*d^2*e - 135*a^8*b^2*c^5*d^2*e - 27*a^8*b^3*c^4*d*e^
2))/12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a
*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8*d^3 - a^3*b^5*
e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 -
 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2
- 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b
^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^
3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
)))^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*
c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2
*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*
d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^
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2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e +
 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b
^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(108*a^9*c^6*d^2*e - 72*a^8*b*c^6*d^3 - 3
6*a^10*c^5*e^3 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a
*b*e - b^2*d + a*c*d) + (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^
2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b
^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^
7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2
*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2
*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)
*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c
^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^
2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e -
 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 - 9
*a^6*b^5*c^4*d^3 + 54*a^7*b^3*c^5*d^3 + 9*a^9*b^2*c^4*e^3 + 108*a^9*b*c^5*d
*e^2 + 27*a^7*b^4*c^4*d^2*e - 135*a^8*b^2*c^5*d^2*e - 27*a^8*b^3*c^4*d*e^2)
)/12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b
^2*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8*d^3 - a^3*b^5*e^
3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 1
6*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 -
48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5
*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*
c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2))
)^(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^
4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e
^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*
e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27
*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 7
2*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
))/(a^5*(4*a*c - b^2)^3))^(1/3)*(36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a
^9*c^6*d^2*e + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b
*e - b^2*d + a*c*d) - (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)
^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2
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*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e
 + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9
*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)*(
(b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3
*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*
e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5
*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3
*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 + 9*a
^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e
^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2))/
12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2
*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*d^3 - a^3*b^5*e^3
+ 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*
a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48
*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c
*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^
2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^
(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*
d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^
2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a
^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*
a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(a^5*(4*a*c - b^2)^3))^(1/3)*(36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e
 - b^2*d + a*c*d) - (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2
*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c
^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^
2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e -
 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)*((b
^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) +
8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d
^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e
+ 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)
```

```
^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a
^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 + 9*a^6
*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e^2
 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2))/12
 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2*d
^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*d^3 - a^3*b^5*e^3 +
16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^
5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a
^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d
^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*
d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
2*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1
/3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.20 $\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{x^6}{6}+\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^6/6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1-x)x^2}{1-x+x^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(-x+\frac{x}{1-x+x^2}\right)dx,x,x^3\right)\\ &= -\frac{x^6}{6}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^6}{6}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^6}{6}+\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\ &= -\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$-\frac{x^6}{6}+\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/6*x^6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

fricas [A] time = 0.72, size = 37, normalized size = 0.80

$$-\frac{1}{6}x^6+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

giac [A] time = 0.42, size = 37, normalized size = 0.80

$$-\frac{1}{6}x^6+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

maple [A] time = 0.00, size = 38, normalized size = 0.83

$$-\frac{x^6}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 0.99, size = 37, normalized size = 0.80

$$-\frac{1}{6}x^6+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

mupad [B] time = 0.06, size = 39, normalized size = 0.85

$$\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^8*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^
6/6
```

sympy [A] time = 0.14, size = 42, normalized size = 0.91

$$-\frac{x^6}{6}+\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

3.21 $\int \frac{x^5(1-x^3)}{1-x^3+x^6} \, dx$

Optimal. Leaf size=31

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3*x^3-2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1474, 773, 618, 204}

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^3/3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^3}{3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^3}{3}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\ &= -\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/3*x^3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 0.50, size = 24, normalized size = 0.77

$$-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

giac [A] time = 0.58, size = 24, normalized size = 0.77

$$-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

maple [A] time = 0.00, size = 25, normalized size = 0.81

$$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^3+1)/(x^6-x^3+1),x)
[Out] -1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 0.96, size = 24, normalized size = 0.77

$$-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

mupad [B] time = 0.04, size = 26, normalized size = 0.84

$$-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(x^3 - 1))/(x^6 - x^3 + 1),x)
[Out] - (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^3/3
```

sympy [A] time = 0.12, size = 32, normalized size = 1.03

$$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
[Out] -x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.22 $\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[Out] -1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2\left(1-x^3\right)}{1-x^3+x^6}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)$$
$$= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)$$
$$= -\frac{1}{6}\log\left(1-x^3+x^6\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

fricas [A] time = 0.61, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

giac [A] time = 0.57, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9} - \frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^3+1)/(x^6-x^3+1),x)
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 0.95, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x^3 - 1))/(x^6 - x^3 + 1),x)
[Out] - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.23 $\int \frac{1-x^3}{x\left(1-x^3+x^6\right)} \, dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx, x, x^3\right)\\
&= \log(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) & ]/3
```

fricas [A] time = 0.86, size = 34, normalized size = 0.83

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

giac [A] time = 0.59, size = 35, normalized size = 0.85

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)
```

maxima [A] time = 0.97, size = 38, normalized size = 0.93

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

mupad [B] time = 1.86, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

sympy [A] time = 0.15, size = 41, normalized size = 1.00

$$\log(x)-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

3.24 $\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3+2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1474, 800, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1-x}{x^2\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{-1+x-x^2}\right)dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+x-x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-2x^3\right)\\
&= -\frac{1}{3x^3}+\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 45, normalized size = 1.45

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
[Out] -1/3*1/x^3 - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) & ]/3
```

fricas [A] time = 0.80, size = 28, normalized size = 0.90

$$-\frac{2\sqrt{3}\,x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+3}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3
```

giac [A] time = 0.45, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

maple [A] time = 0.01, size = 25, normalized size = 0.81

$$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^4/(x^6-x^3+1),x)
[Out] -2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3
```

maxima [A] time = 0.96, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

mupad [B] time = 0.04, size = 26, normalized size = 0.84

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^4*(x^6 - x^3 + 1)),x)
[Out] (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)
```

sympy [A] time = 0.14, size = 36, normalized size = 1.16

$$-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
[Out] -2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

3.25 $\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=418

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/4*x^4+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I
*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1
/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A] time = 0.54, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1502, 12, 1374, 200, 31, 634, 617, 204, 628}

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^4/4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3
]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((
1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3
+ I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt
[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2
^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
+ (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1
/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4} - \frac{1}{4}\int -\frac{4x^3}{1-x^3+x^6}\,dx$$
$$= -\frac{x^4}{4} + \int \frac{x^3}{1-x^3+x^6}\,dx$$
$$= -\frac{x^4}{4} - \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx + \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$
$$= -\frac{x^4}{4} + \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\right.}{}$$
$$= -\frac{x^4}{4} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right.}{}$$
$$= -\frac{x^4}{4} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right.}{}$$
$$= -\frac{x^4}{4} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1\right.}{9\sqrt[3]{2}\left(1-i\right.}$$

Mathematica [**C**] time = 0.01, size = 47, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/4*x^4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

fricas [**B**] time = 1.02, size = 1036, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/4*x^4 + 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(2*18^(2/
3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*co
s(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) +
 2))^2 + 18*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*s
qrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) +
2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1
/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*
x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3
*arctan(sqrt(3) + 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt
(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/108*(
```

```
6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*sqrt(3)*co
s(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 -
 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(
sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) +
2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1
/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqr
t(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sq
rt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(c
os(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/27*(
18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*
sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*c
os(2/3*arctan(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 -
108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*c
os(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*
12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos
(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3
)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/
6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 -
 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(
2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*
log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*
12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18
*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^
(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)
*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt
(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)
*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

giac [B] time = 0.58, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*
pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*p
i)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i +
 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(
2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^
4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos
(2/9*pi) + sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(
3)*i + 1)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9
*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi
) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan
(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*
(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3
- 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt
(3)*sin(4/9*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x
 + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9
*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*s
in(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*p
i) + cos(2/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)
- 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*
sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((s
qrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$-\frac{x^4}{4}+\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/4*x^4+1/3*sum(1/(2*_R^5-_R^2)*_R^3*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\,x^4+\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

mupad [B] time = 0.65, size = 332, normalized size = 0.79

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{x^4}{4}-\frac{2^{2/3}\,\ln\left(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - x^4/4 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^
(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(-
3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3
)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/
3))/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x
- (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2
)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3
^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(
5/6)*1i))/36
```

sympy [A] time = 0.18, size = 31, normalized size = 0.07

$$-\frac{x^4}{4}-\mathrm{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-1458t^4+9t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**
4 + 9*_t + x)))
```

3.26 $\int \frac{x^4(1-x^3)}{1-x^3+x^6} \, dx$

Optimal. Leaf size=382

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log}{3}$$

```
[Out] -1/2*x^2+1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/
2))/(1-I*3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2
))^(1/3))*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*3^(
1/2))^(1/3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1
-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(1/2)-
1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*3^(1/2
)+1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*
2^(1/3)/(1+I*3^(1/2))^(1/3)*3^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1502, 12, 1375, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^2/2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1
- I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))
/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2
^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])
^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 -
I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqr
t[3]*(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*
Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^4\left(1-x^3\right)}{1-x^3+x^6}\,dx = -\frac{x^2}{2}-\frac{1}{2}\int -\frac{2x}{1-x^3+x^6}\,dx$$
$$= -\frac{x^2}{2}+\int \frac{x}{1-x^3+x^6}\,dx$$
$$= -\frac{x^2}{2}-\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$
$$= -\frac{x^2}{2}+\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\cdots$$
$$= -\frac{x^2}{2}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\cdots}}{2\sqrt{3}}\cdots$$
$$= -\frac{x^2}{2}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2}\cdots\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{\cdots}}\cdots$$
$$= -\frac{x^2}{2}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\cdots}{3\cdots}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.13

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/2*x^2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

fricas [B] time = 1.14, size = 1588, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) -
2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
```

```
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3
) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sq
rt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) -
2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)) - 1
/2*x^2 - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^
(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2
/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(s
qrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 864*cos(2/3*a
rctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*s
qrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3
) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2
/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^
(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqr
t(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*
sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
 - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)
*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sq
rt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin
(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*
arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sq
rt(3) - 2))^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) -
2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(
2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(s
qrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sq
rt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1
/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/
3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) -
2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/
3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arc
tan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12
*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sq
rt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^
(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(
2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin
(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))
*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*s
in(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(
3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/
3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)
```

giac [B] time = 0.69, size = 817, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi
)^2 + 5*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 1
```

```
0*cos(4/9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqr
t(3)*sin(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*co
s(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^
5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*
pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(
2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*s
in(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*si
n(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9
*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi)
- 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/9*pi)^2 -
sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*
cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9
*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4
/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*s
in(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*
pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sq
rt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 +
sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*
cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) - cos(2/9*pi)
^2 + sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1)
 - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin
(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1
/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
- cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*
x + x^2 + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.12

$$-\frac{x^2}{2}+\frac{\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/2*x^2+1/3*sum(1/(2*_R^5-_R^2)*_R*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}x^2+\int\frac{x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.28, size = 309, normalized size = 0.81

$$\frac{\ln\left(x+\left(81\,x-\frac{27\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\left(81\,x-\frac{27\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1}{486}\right.\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - x^2/2 -
```

```
(2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/
6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 -
 (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)
^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.19, size = 32, normalized size = 0.08

$$-\frac{x^2}{2} - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**
5 - 27*_t**2 + x)))
```

3.27 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=378

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\right.}{3\sqrt{3}}$$

```
[Out] -x-1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(1
-I*3^(1/2))^(2/3)+1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/
3))*3^(1/2))/(1+I*3^(1/2))^(2/3)+1/9*I*2^(2/3)*ln(-2^(1/3)*x+(1-I*3^(1/2))^
(1/3))/(1-I*3^(1/2))^(2/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(
1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(2/3)/(1-I*3^(1/2))^(2/3)*3^(1/2)-1/9*I*
2^(2/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(2/3)*3^(1/2)+1/18
*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*2^(2/3
)/(1+I*3^(1/2))^(2/3)*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1502, 1347, 200, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\right.}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*
Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqr
t[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/
3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/
3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]
*(1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x + \int \frac{1}{1-x^3+x^6}\,dx$$
$$= -x - \frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$
$$= -x + \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} + \frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}\,x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}}$$
$$= -x + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} - \frac{i\int \frac{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+2x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{3\sqrt[3]{2}\sqrt{3}(1-i\sqrt{3})^{2/3}}$$
$$= -x + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} - \frac{i\log\left((1-i\sqrt{3})^{2/3}+\sqrt[3]{2(}\right.}{3\sqrt[3]{2}\sqrt{3}(1-}$$
$$= -x - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} + \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i\log(}{3\sqrt{3}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.12

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

fricas [B] time = 0.99, size = 1030, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 2/27*18^(2/3)*12
^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) -
 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2
/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)
) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3
```

```
)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(
2)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arcta
n(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3
) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*
cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2
)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) -
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
+ 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - x
```

giac [B] time = 0.63, size = 632, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
 - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqr
t(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*
sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*p
i))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi
))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 +
sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9
*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9
*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*
sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4
/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*
cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*p
i)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi)
+ cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/
18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3
)*sin(1/9*pi) + cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x +
x^2 + 1) - x
```

maple [C] time = 0.01, size = 41, normalized size = 0.11

$$-x+\frac{\ln\left(-\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -x+1/3*sum(1/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x+\int\frac{1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.38, size = 330, normalized size = 0.87

$$-x+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(3+\sqrt{3}\right.}{12}\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 - x + (log(x + (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^
(1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12
)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^
(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i
+ 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 24, normalized size = 0.06

$$-x-\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(729t^4+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x))
)
```

3.28 $\int \frac{x(1-x^3)}{1-x^3+x^6} \, dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/}\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] 1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^(
1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(1/
2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))
^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*
ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3
)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3
+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*
3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/}\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + (
(3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(1-x^3\right)}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$=-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\cdots}{9\ 2\cdots}$$

$$=-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\int\cdots}{\cdots}$$

$$=-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\cdots}{\cdots}$$

$$=\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\cdots}\right.}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 2
*#1^4) & ]
```

fricas [B] time = 1.22, size = 1583, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) +
2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*a
rctan(sqrt(3) + 2))^2 + 36*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(-1/432*(6*1
8^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*arcta
n(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan
(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18
^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos
(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3)
 + 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) +
2))))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/
3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*ar
ctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 +
108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqr
t(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)
)^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) +
2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arc
tan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(
3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)
*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*c
os(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1
/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(
sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 3
6*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)
*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(s
qrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) +
 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(
3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
+ 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) +
 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*a
rctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqr
t(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(s
qrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 + 12*(18^(2/3)*12^(2/3)*x*co
s(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arc
tan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 +
 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1
/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/
3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) +
```

```
2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 -
 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3
*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(
sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 1/108*(18^(2/3)*12^(
1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arcta
n(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^
(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*
12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*ar
ctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^
2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))
 + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*co
s(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2
))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*
arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)
```

giac [B] time = 0.58, size = 821, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*p
i) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*
sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 -
 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)
^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/9*pi)*sin(
2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi
)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 1
0*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2
*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)
*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/
9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(
4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*
sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin
(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1/18*
(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)
^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2
 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + 2*cos(
2/9*pi)^2 - 2*sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x +
 x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi
)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 - 4*sqrt(3)*cos(1/9*pi)*sin
(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) +
cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.11

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{4}-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(x^3-1)x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.26, size = 281, normalized size = 0.68

$$\frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\left(-3-\sqrt{3}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/
3))/18 + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)^(1/3)
)/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6
)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)
*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(-
 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/
3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1
/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)
^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5
/6)*1i))/36
```

sympy [A] time = 0.18, size = 22, normalized size = 0.05

$$-\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))
```

3.29 $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^
(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(1
/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2)
)^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18
*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/
3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(
3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I
*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) +
((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\int}{9\sqrt[3]{2}\,(}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\int}{18}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log}{}$$

$$= -\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.14

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
 2*#1^5) & ]
```

fricas [B] time = 1.12, size = 1031, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(-18^(2/3)*12^(1/6)*
sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*1
8^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 2/27*18^(2/3)*1
2^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
 + 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*a
rctan(sqrt(3) + 2))^2 - 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3)
 + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*si
n(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) +
 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(
1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sq
rt(2)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*ar
ctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqr
t(3) + 2))^2))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(
3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
+ 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) +
 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) +
 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arct
an(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(
3) + 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))
 + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/216*(18^(1/3)*1
2^(5/6)*sqrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(s
qrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1
/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)
*sqrt(3)*x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2))
) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3
)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(2*18^(2/3)*12^(1/6)*sqrt(3)*x*
sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) +
 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 1/1
08*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1
/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) +
 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*s
in(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

giac [B] time = 0.72, size = 637, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*
pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*
sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)
^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*si
n(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin
(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*
sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i +
 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos
(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^
4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1
/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqr
t(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi)
)*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(
1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin
(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*
pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.11

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.30, size = 319, normalized size = 0.78

$$\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9i}{2}\right)\left(-36-\sqrt{3}\,12i\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12i\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9i}{2}\right)\left(-36+\sqrt{3}\,12i\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12i\right)^{1/3}}{18}-2^{2/3}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
```

```
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(- 3^
(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3) +
3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1
i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6
)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/108)*(3^(1/2
)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5
/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)
/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(729t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t +
x)))
```

3.30 $\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/}\right.}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2)
)*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*
3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(
1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-
1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))
^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3
))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/
(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/3)
```

Rubi [A] time = 0.28, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1504, 1374, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/}\right.}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x} - \int \frac{x^4}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x} + \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{x} + -\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\, 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\, 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \cdots$$

$$= -\frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\, 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\, 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + -\frac{(-3 \cdots}{\cdots}$$

$$= -\frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\, 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\, 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i \cdots}{\cdots}$$

$$= -\frac{1}{x} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\, 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\, 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\cdots\right.}{9\, 2^{2/3}\cdots}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/
3
```

fricas [B] time = 1.32, size = 1598, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*s
in(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
- 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*
12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*
x*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3)
- 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3
```

```
) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3
*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(s
qrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(
2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)
*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*1
2^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arc
tan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2
 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(
2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*
cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arct
an(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(
3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)
) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*x*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*
sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(
3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 864*cos(2/3*arcta
n(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(
3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) -
2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*a
rctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
 - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(
2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2
))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^
(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3
)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arct
an(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3
) - 2))^4)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) +
 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(2/3
)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(sqrt
(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(
3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)
*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))
^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))
- (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(
1/6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(
1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3)
 - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*a
rctan(sqrt(3) - 2))^2 + 36*x^2) + (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arct
an(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*
arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2)
)^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(
1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 108)/x
```

giac [B] time = 0.71, size = 829, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
```

```
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^5 -
 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*sin
(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)*
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin(
1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/9
*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(10*sqrt(3)
*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqr
t(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)
^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*si
n(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*
sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
+ 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/x
```

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$-\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^4\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^4*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

mupad [B] time = 0.40, size = 313, normalized size = 0.75

$$\frac{\ln\left(-x+\left(162\,x+\frac{27\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(-x-\left(162\,x+\frac{27\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\left(-\frac{1}{162}+\dots\right.\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - 1/x
```

```
 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 - (2^(1/3)*
3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12
 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3
^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1
i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*l
og(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(
3^(1/3) - 3^(5/6)*1i))/36
```

sympy [**A**] time = 0.19, size = 31, normalized size = 0.07

$$-\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t*
*2 + x))) - 1/x
```

3.31 $\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\,\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\sqrt[3]{2}\,\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I
*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1
/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A] time = 0.36, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1504, 12, 1374, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\,\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\sqrt[3]{2}\,\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx &= -\frac{1}{2x^2}-\frac{1}{2}\int \frac{2x^3}{1-x^3+x^6}\,dx \\
&= -\frac{1}{2x^2}-\int \frac{x^3}{1-x^3+x^6}\,dx \\
&= -\frac{1}{2x^2}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx \\
&= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\cdots \\
&= -\frac{1}{2x^2}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\cdots \\
&= -\frac{1}{2x^2}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\cdots \\
&= -\frac{1}{2x^2}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\cdots}{9\sqrt[3]{2}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]
/3
```

fricas [B] time = 1.17, size = 1062, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2))*log(-2*18^(2/3)
*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(
2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 18*x^2) - 8*18^(2/3)*12^(1/6)*x^2*arctan(1/216*(18^(1/3)*12^(5/6)*sq
rt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) -
2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1
/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*
x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3
*arctan(sqrt(3) - 2)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqr
```

```
t(3) - 2)) + 18^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/1
08*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 108*sqrt(3
)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))
^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arc
tan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3)
 - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12
^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(
sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan
(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))
/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2)) + 4*(
18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1
/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) -
 2))^2 - 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)
*x - 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(1
8^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/
6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt
(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*
(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3
)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) -
 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2)) + (18^(2/3)*12^(1/6)*sqrt(3)*x^
2*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(
3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*
18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(
2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 18*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) - 2)
) + 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/
6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*a
rctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 +
3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 54)/x^2
```

giac [B] time = 0.64, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*s
qrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9*
pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^4 -
 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(2
/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) +
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin(
1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(1
/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(8*sqrt(3)*
cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9
*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9
*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^
4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)
^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$-\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3 \ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2\right)}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^3/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^3*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^2}-\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.40, size = 332, normalized size = 0.79

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{1}{2\,x^2}-\frac{2^{2/3}\,\ln\left(x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(36 - 3^(1/2)*12i)^
(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2
)*12i + 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3 -
3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3 - 3^
(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3))/12
)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(
2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^
(1/3)*1i)/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*
log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1
/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

sympy [A] time = 0.20, size = 32, normalized size = 0.08

$$-\mathrm{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-1458t^4-9t+x\right)\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
+ x))) - 1/(2*x**2)
```

3.32 $\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[Out] 1/6*ln(x^6-x^3+1)+1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1468, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(-2+x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{-2+x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 37, normalized size = 1.03

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
[Out] -(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

fricas [**A**] time = 0.96, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

giac [**A**] time = 0.46, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

maple [**A**] time = 0.00, size = 33, normalized size = 0.92

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-2)/(x^6-x^3+1),x)
[Out] 1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [**A**] time = 0.98, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

mupad [B] time = 1.84, size = 34, normalized size = 0.94

$$\frac{\ln\left(x^6 - x^3 + 1\right)}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 - 2))/(x^6 - x^3 + 1),x)
[Out] log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3
```

sympy [A] time = 0.13, size = 37, normalized size = 1.03

$$\frac{\log\left(x^6 - x^3 + 1\right)}{6} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3
```

3.33 $\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx$

<u>**Optimal**</u>. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

<u>**Rubi**</u> **[A]** time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 618</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 628</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

<u>Rule 634</u>

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

<u>Rule 800</u>

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

<u>Rule 1474</u>

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)dx,x,x^3\right)\\
&= \log(x)+\frac{1}{3}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

fricas [A] time = 1.05, size = 34, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

giac [A] time = 0.54, size = 35, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{3}+\ln(x)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)
```

maxima [A] time = 0.99, size = 38, normalized size = 0.97

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)-\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)+\frac{1}{3}\log\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

mupad [B] time = 1.85, size = 36, normalized size = 0.92

$$\ln(x)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

sympy [A] time = 0.15, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^{3}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

3.34 $\int \frac{1+x^3}{x-x^4+x^7} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1594, 1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^3}{x-x^4+x^7}\,dx &= \int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)dx,x,x^3\right) \\
&= \log(x)+\frac{1}{3}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx,x,x^3\right) \\
&= \log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right) \\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right) \\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

fricas [A] time = 1.05, size = 34, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

giac [A] time = 0.41, size = 35, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{\sqrt{3}\, \arctan\left(\frac{\left(2 x^{3}-1\right) \sqrt{3}}{3}\right)}{3}+\ln(x)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^7-x^4+x),x)
```

```
[Out] 1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)-1/6*ln(x^6-x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{5}-2\, x^{2}}{x^{6}-x^{3}+1}\, dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")
```

```
[Out] -integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.92

$$\ln(x)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\, \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\, \sqrt{3}\, x^{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x - x^4 + x^7),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

sympy [A] time = 0.14, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\, \mathrm{atan}\left(\frac{2\sqrt{3}\, x^{3}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**7-x**4+x),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

3.35 $\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=396

$$\frac{2x\left(d+ex^3\right)^{5/2}\left(667ae^2-58bde+16cd^2\right)}{11339e^2}+\frac{30dx\left(d+ex^3\right)^{3/2}\left(667ae^2-58bde+16cd^2\right)}{124729e^2}+\frac{54d^2x\sqrt{d+ex^3}\left(667ae^2\right.}{124729e}$$

```
[Out] 30/124729*d*(667*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2+2/11339*(66
7*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(5/2)/e^2-2/667*(-29*b*e+8*c*d)*x*(e
*x^3+d)^(7/2)/e^2+2/29*c*x^4*(e*x^3+d)^(7/2)/e+54/124729*d^2*(667*a*e^2-58*
b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+54/124729*3^(3/4)*d^3*(667*a*e^2-58*b
*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2))
)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*
((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)
^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1411, 388, 195, 218}

$$\frac{2x\left(d+ex^3\right)^{5/2}\left(667ae^2-58bde+16cd^2\right)}{11339e^2}+\frac{30dx\left(d+ex^3\right)^{3/2}\left(667ae^2-58bde+16cd^2\right)}{124729e^2}+\frac{54d^2x\sqrt{d+ex^3}\left(667ae^2\right.}{124729e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*Sqrt[d + e*x^3])/(124729*e^2) +
 (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(3/2))/(124729*e^2)
+ (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(5/2))/(11339*e^2) - (
2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7
/2))/(29*e) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*
a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^
2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d
^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/
(124729*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3)
 + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)dx = \frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}+\frac{2\int \left(d+ex^3\right)^{5/2}\left(\frac{29ae}{2}-\left(4cd-\frac{29be}{2}\right)x^3\right)dx}{29e}$$

$$= -\frac{2(8cd-29be)x\left(d+ex^3\right)^{7/2}}{667e^2}+\frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}-\frac{1}{667}\left(-667a-\frac{2d(8c}{}\right.$$

$$= \frac{2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex^3\right)^{5/2}}{11339}-\frac{2(8cd-29be)x\left(d+ex^3\right)^{7/2}}{667e^2}+\frac{2cx}{}$$

$$= \frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{124729}+\frac{2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex\right.}{11339}$$

$$= \frac{54d^2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729}+\frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+e\right.}{124729}$$

$$= \frac{54d^2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729}+\frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+e\right.}{124729}$$

Mathematica [C] time = 0.18, size = 103, normalized size = 0.26

$$\frac{x\sqrt{d+ex^3}\left(\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)\left(29d^2e(23ae-2bd)+16cd^4\right)}{\sqrt{\frac{ex^3}{d}+1}}-2\left(d+ex^3\right)^3\left(-29be+8cd-23cex^3\right)\right)}{667e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^3*(8*c*d - 29*b*e - 23*c*e*x^3) + ((16*c
*d^4 + 29*d^2*e*(-2*b*d + 23*a*e))*Hypergeometric2F1[-5/2, 1/3, 4/3, -((e*x
^3)/d)])/Sqrt[1 + (e*x^3)/d]))/(667*e^2)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ce^2x^{12} + \left(2\,cde + be^2\right)x^9 + \left(cd^2 + 2\,bde + ae^2\right)x^6 + \left(bd^2 + 2\,ade\right)x^3 + ad^2\right)\sqrt{ex^3 + d}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((c*e^2*x^12 + (2*c*d*e + b*e^2)*x^9 + (c*d^2 + 2*b*d*e + a*e^2)*x^
6 + (b*d^2 + 2*a*d*e)*x^3 + a*d^2)*sqrt(e*x^3 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

maple [B] time = 0.26, size = 1070, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] c*(2/29*e^2*x^13*(e*x^3+d)^(1/2)+122/667*d*e*x^10*(e*x^3+d)^(1/2)+1562/1133
9*d^2*x^7*(e*x^3+d)^(1/2)+810/124729*d^3/e*x^4*(e*x^3+d)^(1/2)-1296/124729*
d^4/e^2*x*(e*x^3+d)^(1/2)-864/124729*I*d^5/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x
+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1
/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(
-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*
e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(2/23*e^2*x^10*(e*x^3+d)^(1/2)+98/391*
d*e*x^7*(e*x^3+d)^(1/2)+974/4301*d^2*x^4*(e*x^3+d)^(1/2)+162/4301*d^3/e*x*(
e*x^3+d)^(1/2)+108/4301*I*d^4/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^
2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((
x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)
))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2
)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e
*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(
1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d
*e^2)^(1/3)))^(1/2)))+a*(2/17*e^2*x^7*(e*x^3+d)^(1/2)+74/187*d*e*x^4*(e*x^3
+d)^(1/2)+106/187*d^2*x*(e*x^3+d)^(1/2)-54/187*I*d^3*3^(1/2)/e*(-d*e^2)^(1/
3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d
*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(
-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2
)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3
)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^3+d\right)^{5/2}\,\left(c\,x^6+b\,x^3+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6), x)
```

sympy [A] time = 9.01, size = 400, normalized size = 1.01

$$\frac{ad^{\frac{5}{2}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2ad^{\frac{3}{2}}ex^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a\sqrt{d}\,e^2x^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{bd^{\frac{5}{2}}x^4\Gamma\left(\frac{4}{3}\right)\cdots}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(5/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + 2*a*d**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,)
, e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + a*sqrt(d)*e**2*x**7*gamma(7/3)
*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*
d**(5/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/
d)/(3*gamma(7/3)) + 2*b*d**(3/2)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3
,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*sqrt(d)*e**2*x**10*gamma(
10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)
) + c*d**(5/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar
(I*pi)/d)/(3*gamma(10/3)) + 2*c*d**(3/2)*e*x**10*gamma(10/3)*hyper((-1/2, 1
0/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)) + c*sqrt(d)*e**2*x
**13*gamma(13/3)*hyper((-1/2, 13/3), (16/3,), e*x**3*exp_polar(I*pi)/d)/(3*
gamma(16/3))
```

3.36 $\int \left(d+ex^3\right)^{3/2} \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=356

$$\frac{2x\left(d+ex^3\right)^{3/2}\left(391ae^2-46bde+16cd^2\right)}{4301e^2}+\frac{18dx\sqrt{d+ex^3}\left(391ae^2-46bde+16cd^2\right)}{21505e^2}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^2\left(\sqrt[3]{d}+\right.}{}$$

```
[Out] 2/4301*(391*a*e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2-2/391*(-23*b*e+8
*c*d)*x*(e*x^3+d)^(5/2)/e^2+2/23*c*x^4*(e*x^3+d)^(5/2)/e+18/21505*d*(391*a*
e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+18/21505*3^(3/4)*d^2*(391*a*e^
2-46*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^
(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2
)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*
x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1411, 388, 195, 218}

$$\frac{2x\left(d+ex^3\right)^{3/2}\left(391ae^2-46bde+16cd^2\right)}{4301e^2}+\frac{18dx\sqrt{d+ex^3}\left(391ae^2-46bde+16cd^2\right)}{21505e^2}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^2\left(\sqrt[3]{d}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*Sqrt[d + e*x^3])/(21505*e^2) + (2
*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^(3/2))/(4301*e^2) - (2*(8*
c*d - 23*b*e)*x*(d + e*x^3)^(5/2))/(391*e^2) + (2*c*x^4*(d + e*x^3)^(5/2))/
(23*e) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2
)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3
) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(2150
5*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(
1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)dx = \frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}+\frac{2\int \left(d+ex^3\right)^{3/2}\left(\frac{23ae}{2}-\left(4cd-\frac{23be}{2}\right)x^3\right)dx}{23e}$$

$$= -\frac{2(8cd-23be)x\left(d+ex^3\right)^{5/2}}{391e^2}+\frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}-\frac{1}{391}\left(-391a-\frac{2d(8cd}{\ldots}\right.$$

$$= \frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{4301}-\frac{2(8cd-23be)x\left(d+ex^3\right)^{5/2}}{391e^2}+\frac{2cx\ldots}{\ldots}$$

$$= \frac{18d\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505}+\frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\left(d+ex^3\right)^{3\ldots}}{4301}$$

$$= \frac{18d\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505}+\frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\left(d+ex^3\right)^{3\ldots}}{4301}$$

Mathematica [C] time = 0.15, size = 101, normalized size = 0.28

$$\frac{x\sqrt{d+ex^3}\left(\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)\left(23de(17ae-2bd)+16cd^3\right)}{\sqrt{\frac{ex^3}{d}+1}}-2\left(d+ex^3\right)^2\left(-23be+8cd-17cex^3\right)\right)}{391e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^2*(8*c*d - 23*b*e - 17*c*e*x^3) + ((16*c
*d^3 + 23*d*e*(-2*b*d + 17*a*e))*Hypergeometric2F1[-3/2, 1/3, 4/3, -((e*x^3
)/d)])/Sqrt[1 + (e*x^3)/d]))/(391*e^2)
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cex^9+(cd+be)x^6+(bd+ae)x^3+ad\right)\sqrt{ex^3+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^9 + (c*d + b*e)*x^6 + (b*d + a*e)*x^3 + a*d)*sqrt(e*x^3 + d
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

maple [B] time = 0.04, size = 1010, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] c*(2/23*e*x^10*(e*x^3+d)^(1/2)+52/391*d*x^7*(e*x^3+d)^(1/2)+54/4301*d^2/e*x
^4*(e*x^3+d)^(1/2)-432/21505*d^3/e^2*x*(e*x^3+d)^(1/2)-288/21505*I*d^4/e^3*
3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1
/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)
^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e
+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^
2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2
*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+b*(2/17*e*x^7*
(e*x^3+d)^(1/2)+40/187*d*x^4*(e*x^3+d)^(1/2)+54/935*d^2/e*x*(e*x^3+d)^(1/2)
+36/935*I*d^3/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3
^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)
/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/
2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e
)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/
2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-
d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2
)))+a*(2/11*e*x^4*(e*x^3+d)^(1/2)+28/55*d*x*(e*x^3+d)^(1/2)-18/55*I*d^2*3^(
1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/
3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^
(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+
1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2
)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*
(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^3+d\right)^{3/2}\,\left(c\,x^6+b\,x^3+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6), x)
```

sympy [A] time = 5.92, size = 257, normalized size = 0.72

$$\frac{ad^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a\sqrt{d}\,ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{bd^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b\sqrt{d}\,ex^7\Gamma\left(\frac{7}{3}\right){}_2F}{3\Gamma(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + a*sqrt(d)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e
*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*d**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*sqrt(d)*e
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*g
amma(10/3)) + c*d**(3/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3
*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c*sqrt(d)*e*x**10*gamma(10/3)*hyper((
-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3))
```

3.37 $\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)dx$

Optimal. Leaf size=316

$$\frac{2x\sqrt{d+ex^3}\left(187ae^2-34bde+16cd^2\right)}{935e^2}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(187ae^2-34bde+16\right.}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+}}$$

```
[Out] -2/187*(-17*b*e+8*c*d)*x*(e*x^3+d)^(3/2)/e^2+2/17*c*x^4*(e*x^3+d)^(3/2)/e+2
/935*(187*a*e^2-34*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+2/935*3^(3/4)*d*(1
87*a*e^2-34*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3
)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1
+3^(1/2)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e
^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1411, 388, 195, 218}

$$\frac{2x\sqrt{d+ex^3}\left(187ae^2-34bde+16cd^2\right)}{935e^2}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(187ae^2-34bde+16\right.}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*x*Sqrt[d + e*x^3])/(935*e^2) - (2*(8*c
*d - 17*b*e)*x*(d + e*x^3)^(3/2))/(187*e^2) + (2*c*x^4*(d + e*x^3)^(3/2))/(
17*e) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 +
Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) +
e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*e^(7
/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)^2]*Sqrt[d + e*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)dx = \frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}+\frac{2\int \sqrt{d+ex^3}\left(\frac{17ae}{2}-\left(4cd-\frac{17be}{2}\right)x^3\right)dx}{17e}$$

$$= -\frac{2(8cd-17be)x\left(d+ex^3\right)^{3/2}}{187e^2}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}-\frac{1}{187}\left(-187a-\frac{2d(8cd-}{e^2}\right.$$

$$= \frac{2}{935}\left(187a+\frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3}-\frac{2(8cd-17be)x\left(d+ex^3\right)^{3/2}}{187e^2}+\frac{2}{}$$

$$= \frac{2}{935}\left(187a+\frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3}-\frac{2(8cd-17be)x\left(d+ex^3\right)^{3/2}}{187e^2}+\frac{2}{}$$

Mathematica [C] time = 0.13, size = 98, normalized size = 0.31

$$\frac{x\sqrt{d+ex^3}\left(\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)\left(17e(11ae-2bd)+16cd^2\right)}{\sqrt{\frac{ex^3}{d}+1}}-2\left(d+ex^3\right)\left(-17be+8cd-11cex^3\right)\right)}{187e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)*(8*c*d - 17*b*e - 11*c*e*x^3) + ((16*c*d
^2 + 17*e*(-2*b*d + 11*a*e))*Hypergeometric2F1[-1/2, 1/3, 4/3, -((e*x^3)/d)
])/Sqrt[1 + (e*x^3)/d]))/(187*e^2)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6+bx^3+a\right)\sqrt{ex^3+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

maple [B] time = 0.04, size = 956, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] c*(2/17*x^7*(e*x^3+d)^(1/2)+6/187*d/e*x^4*(e*x^3+d)^(1/2)-48/935*d^2/e^2*x*
(e*x^3+d)^(1/2)-32/935*I*d^3/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^
(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x
-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^
(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(
-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*
e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2
),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(
1/3)/e)/e)^(1/2)))+b*(2/11*x^4*(e*x^3+d)^(1/2)+6/55*d/e*x*(e*x^3+d)^(1/2)+4
/55*I*d^2/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/
2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/
(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-
d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1
/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*
3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^
2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+
a*(2/5*x*(e*x^3+d)^(1/2)-2/5*I*d*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2
)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*(
(x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)
)^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)
/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1
/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)
^(1/3)/e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\,x^3+d}\,\left(c\,x^6+b\,x^3+a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6), x)
```

sympy [A] time = 3.45, size = 124, normalized size = 0.39

$$\frac{a\sqrt{d}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{b\sqrt{d}\,x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{c\sqrt{d}\,x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*sqrt(d)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d)
/(3*gamma(4/3)) + b*sqrt(d)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x*
*3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + c*sqrt(d)*x**7*gamma(7/3)*hyper((-1/
2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3))
```

3.38 $\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$

Optimal. Leaf size=278

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(55ae^2-22bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2\cdots}{\cdots}$$

```
[Out] -2/55*(-11*b*e+8*c*d)*x*(e*x^3+d)^(1/2)/e^2+2/11*c*x^4*(e*x^3+d)^(1/2)/e+2/
165*(55*a*e^2-22*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d
^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+
e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1411, 388, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(55ae^2-22bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

```
[Out] (-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3])/(55*e^2) + (2*c*x^4*Sqrt[d + e*x^3]
)/(11*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 22*b*d*e + 55*a*e^2)*(d^(1/3) +
 e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])
*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*
x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*e^(7/
3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)
^2]*Sqrt[d + e*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
```

```
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\int \frac{\frac{11ae}{2}-\left(4cd-\frac{11be}{2}\right)x^3}{\sqrt{d+ex^3}}\,dx}{11e}$$

$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e} - \frac{1}{55}\left(-55a - \frac{2d(8cd-11be)}{e^2}\right)\int \frac{1}{\sqrt{d+ex^3}}$$

$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\sqrt{2+\sqrt{3}}\,\left(16cd^2-22bde+55ae^2\right)\left(\sqrt[3]{d}\right.}{55\sqrt[4]{3}}$$

Mathematica [C] time = 0.09, size = 98, normalized size = 0.35

$$\frac{x\left(\sqrt{\frac{ex^3}{d}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\left(11e(5ae-2bd)+16cd^2\right)-2\left(d+ex^3\right)\left(-11be+8cd-5cex^3\right)\right)}{55e^2\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

```
[Out] (x*(-2*(d + e*x^3)*(8*c*d - 11*b*e - 5*c*e*x^3) + (16*c*d^2 + 11*e*(-2*b*d
+ 5*a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)
]))/(55*e^2*Sqrt[d + e*x^3])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

maple [B] time = 0.03, size = 907, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x)
```

```
[Out] c*(2/11/e*x^4*(e*x^3+d)^(1/2)-16/55*d/e^2*x*(e*x^3+d)^(1/2)-32/165*I*d^2/e^
3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^
(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^
2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)
/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*
e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3
/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+b*(2/5/e*x*(
e*x^3+d)^(1/2)+4/15*I*d/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)
/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*
e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)
*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^
2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^
(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*
3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/
e)/e)^(1/2)))-2/3*I*a*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1
/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)
^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I
*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(
1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3
)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1
/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6 + b\,x^3 + a}{\sqrt{e\,x^3 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2), x)
```

sympy [A] time = 2.90, size = 119, normalized size = 0.43

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2}, \frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(
d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_pol
ar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((1/2, 7/3), (1
0/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3))
```

3.39 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2x\left(ae^2-bde+cd^2\right)}{3de^2\sqrt{d+ex^3}}-\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,\left(16cd^2-5e(ae+2bd)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right.\right.}{15\sqrt[4]{3}\,de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,\sqrt{d+ex^3}}$$

```
[Out] 2/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(1/2)+2/5*c*x*(e*x^3+d)^(1/2)/e^2
-2/45*(16*c*d^2-5*e*(a*e+2*b*d))*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d
^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3
)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1409, 388, 218}

$$\frac{2x\left(ae^2-bde+cd^2\right)}{3de^2\sqrt{d+ex^3}}-\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,\left(16cd^2-5e(ae+2bd)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right.\right.}{15\sqrt[4]{3}\,de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) + (2*c*x*Sqrt[d + e
*x^3])/(5*e^2) - (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(2*b*d + a*e))*(d^(1/
3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt
[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1
/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*d
*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{3/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{3de^2\sqrt{d+ex^3}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+ae)\right)-\frac{3}{2}cdex^3}{\sqrt{d+ex^3}}\,dx}{3de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{3de^2\sqrt{d+ex^3}} + \frac{2cx\sqrt{d+ex^3}}{5e^2} - \frac{\left(16cd^2-5e(2bd+ae)\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx}{15de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{3de^2\sqrt{d+ex^3}} + \frac{2cx\sqrt{d+ex^3}}{5e^2} - \frac{2\sqrt{2+\sqrt{3}}\left(16cd^2-5e(2bd+ae)\right)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right.}{15\sqrt[4]{3}\,de^{7/3}\sqrt{}}$$

Mathematica [C] time = 0.10, size = 102, normalized size = 0.35

$$\frac{x\left(\sqrt{\frac{ex^3}{d}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\left(5e(ae+2bd)-16cd^2\right)+2\left(5e(ae-bd)+cd\left(8d+3ex^3\right)\right)\right)}{15de^2\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (x*(2*(5*e*(-(b*d) + a*e) + c*d*(8*d + 3*e*x^3)) + (-16*c*d^2 + 5*e*(2*b*d
+ a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])
)/(15*d*e^2*Sqrt[d + e*x^3])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)\sqrt{ex^3+d}}{e^2x^6+2\,dex^3+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^2*x^6 + 2*d*e*x^3 + d^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6+bx^3+a}{\left(ex^3+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

maple [B] time = 0.05, size = 934, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x)
```

```
[Out] c*(2/3/e^2*d*x/((x^3+d/e)*e)^(1/2)+2/5/e^2*x*(e*x^3+d)^(1/2)+32/45*I*d/e^3*
3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1
/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)
^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e
+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^
2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2
*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+b*(-2/3/e*x/((
x^3+d/e)*e)^(1/2)-4/9*I/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)
/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*
e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)
*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^
2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^
(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*
3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/
e)/e)^(1/2)))+a*(2/3/d*x/((x^3+d/e)*e)^(1/2)-2/9*I/d*3^(1/2)*(-d*e^2)^(1/3)
/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e
^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/
2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e
^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/
(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/
2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6 + b\,x^3 + a}{{\left(e\,x^3 + d\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2), x)
```

sympy [A] time = 15.07, size = 119, normalized size = 0.41

$$\frac{ax\Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| {\frac{ex^3 e^{i\pi}}{d}} \right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{4}{3}, \frac{3}{2} \\ \frac{7}{3} \end{matrix}\middle| {\frac{ex^3 e^{i\pi}}{d}} \right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| {\frac{ex^3 e^{i\pi}}{d}} \right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(3/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((3/2, 7/3),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(10/3))
```

3.40 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{5/2}}\,dx$

Optimal. Leaf size=309

$$-\frac{2x\left(-7ae^2-2bde+11cd^2\right)}{27d^2e^2\sqrt{d+ex^3}}+\frac{2x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+\right.}{27\sqrt[4]{3}\,d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}\right.}}}$$

```
[Out] 2/9*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(3/2)-2/27*(-7*a*e^2-2*b*d*e+11*c
*d^2)*x/d^2/e^2/(e*x^3+d)^(1/2)+2/81*(16*c*d^2+e*(7*a*e+2*b*d))*(d^(1/3)+e^
(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*
x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^2/e^(7/3)
/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1409, 385, 218}

$$-\frac{2x\left(-7ae^2-2bde+11cd^2\right)}{27d^2e^2\sqrt{d+ex^3}}+\frac{2x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+\right.}{27\sqrt[4]{3}\,d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - (2*(11*c*d^2 -
2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(
16*c*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)
*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)
*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^2*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(
1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{5/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{9de^2\left(d+ex^3\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+7ae)\right)-\frac{9}{2}cdex^3}{\left(d+ex^3\right)^{3/2}}\,dx}{9de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{9de^2\left(d+ex^3\right)^{3/2}} - \frac{2\left(11cd^2-2bde-7ae^2\right)x}{27d^2e^2\sqrt{d+ex^3}} - \frac{\left(4\left(-\frac{9}{2}cd^2e+\frac{1}{4}e\left(2cd^2-e(2bd+7a\right.\right.\right.}{27d^2e^3}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{9de^2\left(d+ex^3\right)^{3/2}} - \frac{2\left(11cd^2-2bde-7ae^2\right)x}{27d^2e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(16cd^2+e(2bd+7ae)\right)}{27}$$

Mathematica [C] time = 0.14, size = 129, normalized size = 0.42

$$\frac{x\left(d+ex^3\right)\sqrt{\frac{ex^3}{d}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\left(e(7ae+2bd)+16cd^2\right)-2x\left(e\left(bd\left(d-2ex^3\right)-ae\left(10d+7ex^3\right)\right)+cd^2\right.}{27d^2e^2\left(d+ex^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (-2*x*(c*d^2*(8*d + 11*e*x^3) + e*(b*d*(d - 2*e*x^3) - a*e*(10*d + 7*e*x^3)
)) + (16*c*d^2 + e*(2*b*d + 7*a*e))*x*(d + e*x^3)*Sqrt[1 + (e*x^3)/d]*Hyper
geometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(27*d^2*e^2*(d + e*x^3)^(3/2))
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(cx^6+bx^3+a\right)\sqrt{ex^3+d}}{e^3x^9+3\,de^2x^6+3\,d^2ex^3+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^3*x^9 + 3*d*e^2*x^6 + 3*d^2
*e*x^3 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6+bx^3+a}{\left(ex^3+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

maple [B] time = 0.05, size = 1005, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x)
```

```
[Out] c*(2/9*d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2-22/27/e^2*x/((x^3+d/e)*e)^(1/2)-
32/81*I/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)
*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-
3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*
e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2
)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^
(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)
^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+b*
(-2/9*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+4/27/e/d*x/((x^3+d/e)*e)^(1/2)-4/81
*I/e^2/d*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-
d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2
*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2
)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(
e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/
2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1
/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+a*(2/
9/d*x/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+14/27/d^2*x/((x^3+d/e)*e)^(1/2)-14/81
*I/d^2*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-
d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2
*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2
)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(
e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/
2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1
/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6 + b\,x^3 + a}{\left(e\,x^3 + d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2), x)
```

sympy [A] time = 86.88, size = 119, normalized size = 0.39

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(5
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(5/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((7/3, 5/2),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(5/2)*gamma(10/3))
```

3.41 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{7/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{135d^2e^2\left(d+ex^3\right)^{3/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(91ae^2+14bde+16cd^2\right)}{405d^3e^2\sqrt{d+ex^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^2\cdots}{((\cdots}}}{\cdots}$$

```
[Out] 2/15*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(5/2)-2/135*(-13*a*e^2-2*b*d*e+1
7*c*d^2)*x/d^2/e^2/(e*x^3+d)^(3/2)+2/405*(91*a*e^2+14*b*d*e+16*c*d^2)*x/d^3
/e^2/(e*x^3+d)^(1/2)+2/1215*(91*a*e^2+14*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x
)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/
3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^3/e^(7/3)/(e*x^3
+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] time = 0.32, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1409, 385, 199, 218}

$$\frac{2x\left(91ae^2+14bde+16cd^2\right)}{405d^3e^2\sqrt{d+ex^3}}-\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{135d^2e^2\left(d+ex^3\right)^{3/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}\cdots}{((1+\cdots}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(17*c*d^2 -
 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 14
*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{7/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+13ae)\right)-\frac{15}{2}cdex^3}{\left(d+ex^3\right)^{5/2}}\,dx}{15de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\left(17cd^2-2bde-13ae^2\right)x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{\left(16cd^2+14bde+91ae^2\right)\int \frac{1}{\left(d+ex^3\right)^{\cdots}}}{135d^2e^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\left(17cd^2-2bde-13ae^2\right)x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{2\left(16cd^2+14bde+91ae^2\right)x}{405d^3e^2\sqrt{d+ex^3}} + \frac{(1\cdots}{\cdots}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\left(17cd^2-2bde-13ae^2\right)x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{2\left(16cd^2+14bde+91ae^2\right)x}{405d^3e^2\sqrt{d+ex^3}} + \frac{2\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 0.19, size = 166, normalized size = 0.48

$$\frac{2x\left(e\left(ae\left(157d^2+221dex^3+91e^2x^6\right)+bd\left(-7d^2+34dex^3+14e^2x^6\right)\right)+cd^2\left(-8d^2-19dex^3+16e^2x^6\right)\right)+x\sqrt{\cdots}}{405d^3e^2\left(d+ex^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*x*(c*d^2*(-8*d^2 - 19*d*e*x^3 + 16*e^2*x^6) + e*(b*d*(-7*d^2 + 34*d*e*x^
3 + 14*e^2*x^6) + a*e*(157*d^2 + 221*d*e*x^3 + 91*e^2*x^6))) + (16*c*d^2 +
7*e*(2*b*d + 13*a*e))*x*(d + e*x^3)^2*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1
[1/3, 1/2, 4/3, -((e*x^3)/d)])/(405*d^3*e^2*(d + e*x^3)^(5/2))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)\sqrt{ex^3+d}}{e^4x^{12}+4\,de^3x^9+6\,d^2e^2x^6+4\,d^3ex^3+d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^4*x^12 + 4*d*e^3*x^9 + 6*d^
2*e^2*x^6 + 4*d^3*e*x^3 + d^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

maple [B] time = 0.05, size = 1095, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x)
```

```
[Out] c*(2/15*d*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3-34/135*x/e^4*(e*x^3+d)^(1/2)/(x
^3+d/e)^2+32/405/e^2/d*x/((x^3+d/e)*e)^(1/2)-32/1215*I/e^3/d*3^(1/2)*(-d*e^
2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)
/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I
*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)
*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^
(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3
)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+b*(-2/15*x/e^4*(e*x^3+d)^(1/
2)/(x^3+d/e)^3+4/135/d*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+28/405/e/d^2*x/((x
^3+d/e)*e)^(1/2)-28/1215*I/e^2/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2
)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*(
(x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)
)^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)
/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1
/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)
^(1/3)/e)/e)^(1/2)))+a*(2/15/d*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+26/135/d^2
*x/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+182/405/d^3*x/((x^3+d/e)*e)^(1/2)-182/12
15*I/d^3*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*
(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3
/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e
^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)
/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(
1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^
(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6+b\,x^3+a}{{\left(e\,x^3+d\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)
[Out] Timed out
```

3.42 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{315d^2e^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{2835d^3e^2\left(d+ex^3\right)^{3/2}}$$

```
[Out] 2/21*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(7/2)-2/315*(-19*a*e^2-2*b*d*e+2
3*c*d^2)*x/d^2/e^2/(e*x^3+d)^(5/2)+2/2835*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d
^3/e^2/(e*x^3+d)^(3/2)+2/1215*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d^4/e^2/(e*x^
3+d)^(1/2)+2/3645*(247*a*e^2+26*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*Ellipti
cF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(
e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^4/e^(7/3)/(e*x^3+d)^(1/2)
/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1409, 385, 199, 218}

$$\frac{2x\left(247ae^2+26bde+16cd^2\right)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{2835d^3e^2\left(d+ex^3\right)^{3/2}}-\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{315d^2e^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - (2*(23*c*d^2 -
 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^(5/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]
]*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d
^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e
^(1/3)*x)], -7 - 4*Sqrt[3]])/(1215*3^(1/4)*d^4*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/
3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+19ae)\right)-\frac{21}{2}cdex^3}{\left(d+ex^3\right)^{7/2}}\,dx}{21de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{\left(16cd^2+26bde+247ae^2\right)\int \frac{1}{\left(d+ex \right.}}{315d^2e^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{2\left(16cd^2+26bde+247ae^2\right)x}{2835d^3e^2\left(d+ex^3\right)^{3/2}} + \cdots$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{2\left(16cd^2+26bde+247ae^2\right)x}{2835d^3e^2\left(d+ex^3\right)^{3/2}} + \cdots$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{2\left(16cd^2+26bde+247ae^2\right)x}{2835d^3e^2\left(d+ex^3\right)^{3/2}} + \cdots$$

Mathematica [C] time = 0.24, size = 200, normalized size = 0.51

$$\frac{7x\sqrt{\frac{ex^3}{d}+1}\left(d+ex^3\right)^3 {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\left(13e(19ae+2bd)+16cd^2\right)+2x\left(e\left(ae\left(3388d^3+7182d^2ex^3+5928d\right.\right.\right.}{8505d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*x*(c*d^2*(-56*d^3 - 189*d^2*e*x^3 + 384*d*e^2*x^6 + 112*e^3*x^9) + e*(b*
d*(-91*d^3 + 756*d^2*e*x^3 + 624*d*e^2*x^6 + 182*e^3*x^9) + a*e*(3388*d^3 +
```

```
 7182*d^2*e*x^3 + 5928*d*e^2*x^6 + 1729*e^3*x^9))) + 7*(16*c*d^2 + 13*e*(2*
b*d + 19*a*e))*x*(d + e*x^3)^3*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((e*x^3)/d)])/(8505*d^4*e^2*(d + e*x^3)^(7/2))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)\sqrt{ex^3+d}}{e^5x^{15}+5\,de^4x^{12}+10\,d^2e^3x^9+10\,d^3e^2x^6+5\,d^4ex^3+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^5*x^15 + 5*d*e^4*x^12 + 10*
d^2*e^3*x^9 + 10*d^3*e^2*x^6 + 5*d^4*e*x^3 + d^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6+bx^3+a}{\left(ex^3+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

maple [B] time = 0.06, size = 1182, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x)
```

```
[Out] c*(2/21*d*x/e^6*(e*x^3+d)^(1/2)/(x^3+d/e)^4-46/315*x/e^5*(e*x^3+d)^(1/2)/(x
^3+d/e)^3+32/2835/d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2+32/1215/e^2/d^2*x/((x
^3+d/e)*e)^(1/2)-32/3645*I/e^3/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2
)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*(
(x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)
)^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)
/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1
/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)
^(1/3)/e)/e)^(1/2)))+b*(-2/21*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^4+4/315/d*x/e
^4*(e*x^3+d)^(1/2)/(x^3+d/e)^3+52/2835/d^2*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^
2+52/1215/e/d^3*x/((x^3+d/e)*e)^(1/2)-52/3645*I/e^2/d^3*3^(1/2)*(-d*e^2)^(1
/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*
e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1
/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*
e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)
/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1
/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+a*(2/21/d*x/e^4*(e*x^3+d)^(1/2)/(
x^3+d/e)^4+38/315/d^2*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+494/2835/d^3*x/e^2*
(e*x^3+d)^(1/2)/(x^3+d/e)^2+494/1215/d^4*x/((x^3+d/e)*e)^(1/2)-494/3645*I/d
^4*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^
2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d
*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1
/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^
```

```
3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(
-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2),(I*3^(1/2)*(-d*e^2)^(1/3)/
(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6 + b\,x^3 + a}{\left(e\,x^3 + d\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)
[Out] Timed out
```

3.43 $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=433

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bc\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{2\sqrt[4]{2}\cdots}$$

```
[Out] e*x/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(c*d-b*e+
(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^
(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a
*c+b^2)^(1/2))^(3/4)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(
1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(
-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/
c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 1.13, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1502, 1422, 212, 208, 205}

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bc\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{2\sqrt[4]{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d
- b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
- ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx &= \frac{ex}{c}-\frac{\int \frac{ae-(cd-be)x^4}{a+bx^4+cx^8}\,dx}{c}\\ &= \frac{ex}{c}+\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4a}}\end{aligned}$$

$$= \frac{ex}{c}-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{}{\sqrt{-b}}}{2c\sqrt{-b+\sqrt{b^2-}}}$$

$$= \frac{ex}{c}-\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt[4]{-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)}$$

Mathematica [C] time = 0.08, size = 88, normalized size = 0.20

$$\frac{ex}{c}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4be\log(x-\#1)+\#1^4(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.01, size = 67, normalized size = 0.15

$$\frac{ex}{c}+\frac{\left((-be+cd)\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^4-ae\right)\ln\left(-\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)+x\right)}{4c\left(2\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^7c+\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/c*e*x+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex}{c}-\frac{-\int\frac{(cd-be)x^4-ae}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c
```

mupad [B] time = 9.63, size = 50213, normalized size = 115.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^
2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b
^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*
c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*
```

```
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5
*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5
 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*
b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b
^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3
 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
+ 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*
e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*
c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d
^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^
2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a
^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80
*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e +
61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4
*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*
(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b
^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c
^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3
*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3
*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*
b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c
```

```
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a
^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d
^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*
d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^
2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8
*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b
^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^
2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(3/4) + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b
^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*
c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^
2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 2
0*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*
c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^
5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*
d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*
b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13
*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^
4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) -
 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^
2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c
^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^
4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^
6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d
^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16
*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*
a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5
*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*
(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2
*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*
c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^4*b*c^
7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 2
56*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^
4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e
^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^
5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c
^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2
*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*
b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*
a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-
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(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200
*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d -
 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*
c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240
*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 -
 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) -
 (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2
*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^
5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4
*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*
e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32
*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^
4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3
*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^
4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 +
a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 -
 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e +
48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e -
200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a
*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^
3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*
a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4
*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d
^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*
e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5
*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b
^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6
*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3
*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
- 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*
b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^
2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*
a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*
c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240
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*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 -
4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(
16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^
7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*
b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^
2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*
e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*
c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5
*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*e^5
- 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a
^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6
*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22
*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c
^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^
2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c -
b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2
*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b
^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 +
240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*
b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^
3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^
5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d
*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2
*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*
a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^
3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d
*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^
3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c
^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*
c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^
2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4
*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48
*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20
0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)))*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4
+ 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3
*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^

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3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*
(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^
3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) -
 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288
*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + ata
n((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 -
 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*
a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*
e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*
a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 -
120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5
*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4
 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 1
28*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e
^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3
*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3
*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3
+ 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^
5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 +
13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3
*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*
c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 -
19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b
^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5
*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6
+ a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d
^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^
2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*
b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^
3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^
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4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*
a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^
2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*
(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^
5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c
^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*(40
96*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*
c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e -
1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*
d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*
b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(
4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4
+ 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e
^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*
d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*
c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b
^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4) + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*
c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2
*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 -
 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a
^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4
*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c
^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3
*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3
*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*
b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c
^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*
e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^
3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3
*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e
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- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^
4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*
d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2
- 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 3
20*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^4*b*c^7*d
^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*
a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c
^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d
^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*
d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^
2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8
*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b
^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^
2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12
288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6
*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3
*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*
b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (1
6*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^
7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c
^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^
2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4
 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^
4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e
^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^
5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c
^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2
*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*
b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*
a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200
*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5
*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^
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4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*
e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e +
 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*
d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*
c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^
3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^
3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a
*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*
c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4
*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*
c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4
*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c
^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 -
 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4
*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6
*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3
*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*
b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(163
84*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d
))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c
^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b
^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6
*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2
 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^
2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3
*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^
2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*
a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*e^5 -
4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*
c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^
2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^
4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*
d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 +
 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*
c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240
*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 +
 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
```

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*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) +
 (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^
3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5
*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^
5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4)))*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
 - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 +
80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e
+ 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c
^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66
*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^
3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8
*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + 2*atan
((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 -
2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a
^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4
+ b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*
c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*
a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d +
 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4
 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 1
28*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e
^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3
*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3
*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3
- 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5
*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5
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 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*
b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b
^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3
 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
+ 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*
e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a
^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^
4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4
*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*
c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 -
4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*
c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 +
 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e
 + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*
c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 6
6*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a
^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + ((((4*x*
(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b
^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*
b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
```

)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a
^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*
b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^
3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4
 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*
b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4
+ b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*
c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*
a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6
 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^
2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e
^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^
4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*
c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^
3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*
c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*
b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^
2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d
^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/(((((4*x*(4096*a^4*
b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2
 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^
3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 + b^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c
^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*
c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^
2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4
*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48
*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20
0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d
- 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a
^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4
*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3
+ 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*
a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*
e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2

```
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3
*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3
/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e
^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^
5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22
*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*
b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^
4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c
^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*
e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^
3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3
*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e
- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^
4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*
d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2
- 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 3
20*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - ((((4*x*(4096*a^4*b*c^7*d
^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*
a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c
^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(
4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4
+ 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e
^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*
d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*
c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b
^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288
*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^
5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^
6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^
3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c -
 b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a
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^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c
*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4
*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3
- 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i
- (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^
2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a
^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^
4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d
*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 3
2*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^
3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a
^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 +
 a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4
- 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e +
 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e -
 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*
a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a
^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6
- 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2
*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c
^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*
d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2
*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4
 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3
*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120
*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a
^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*
b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a
^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e
^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^
5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c
^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2
*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*
b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*
a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200
*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) + 2*atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2
+ 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a
^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^
6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*
a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 6
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1*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8
*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*
(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2
*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*
c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a
^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5
*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b
^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b
^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4
*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*
a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6
*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3
*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*
b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i -
 (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^
3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5
*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^
5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4) + ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*
b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5
*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c
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+ ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*
e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*
c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 -
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^
5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^
2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e
^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a
*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a
^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4
 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3
*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120
*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a
^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*
b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a
^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5
 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a
^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3
*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2
*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2
*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^
2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*
a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 6
1*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4
*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^
5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e
^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3
 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*
b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e
^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a
^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4
*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3
+ 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*
a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*
e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3
*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1
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/4))/(((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 2

88*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (
(((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 204
8*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*
c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b
^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6
*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3
*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
+ 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 30
72*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 -
b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b
^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*
a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4
- a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4
- 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e
+ 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e
- 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6
*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^
5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 +
13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3
*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*
c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 -
19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b
^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
- 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*
c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d
^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^
2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a
^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80
*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e +
61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4
*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*

```
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*e
^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a
^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 1
20*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e
^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128
*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^
3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (e*x)/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.44 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=72

$$\frac{e \log \left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c-1/4*(-b*e+2*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e \log \left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx,x,x^4\right)$$
$$= \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c} + \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c}$$
$$= \frac{e\log\left(a+bx^4+cx^8\right)}{8c} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c}$$
$$= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^4+cx^8\right)}{8c}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.99

$$\frac{e\log\left(a+bx^4+cx^8\right) - \frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

fricas [A] time = 1.02, size = 216, normalized size = 3.00

$$\left[\frac{\left(b^2-4\,ac\right)e\log\left(cx^8+bx^4+a\right) - \sqrt{b^2-4\,ac}\,(2\,cd-be)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\left(b^2c-4\,ac^2\right)}, \frac{\left(b^2-4\,ac\right)e\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a
*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c*x^
8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 20.74, size = 70, normalized size = 0.97

$$\frac{e\log\left(cx^8+bx^4+a\right)}{8\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

maple [A] time = 0.00, size = 99, normalized size = 1.38

$$-\frac{be\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\;c} + \frac{d\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}} + \frac{e\ln\left(c\,x^8+b\,x^4+a\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b
^2)^(1/2))*d-1/4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*e*
b/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.21, size = 3704, normalized size = 51.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] - (log(a + b*x^4 + c*x^8)*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) -
 (atan((8*x^4*(((a*c - b^2)*(((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b
^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 1
6*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*
e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16
*b^2*c)) + ((b*e - 2*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^
3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2
*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*
(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) - (((
(b*e - 2*c*d)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4
*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) +
(32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a
*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(b*e - 2*c*d))/
(8*c*(4*a*c - b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4
*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*
(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 1
44*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 +
20*b^3*c*e^3 - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(8*c*(4*a*c - b^2)^(1/2)
) - (b^3*c*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/(2*(64*a*c^2 - 16*b^2*c)*(
4*a*c - b^2)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^3*e^4 + (b^3*(b*e -
 2*c*d)^4)/(8*(4*a*c - b^2)^2) - c^3*d^3*e - ((((b*e - 2*c*d)*(((b*e - 2*c*
d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*
a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a
*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*
a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a
*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^
2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4*d^2 + ((4*b^
2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16
*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^
2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 + 20*b^3*c*e^3
- 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(2*(64*a*c^2 - 16*b^2*c)) + 3*b*c^2*d
^2*e^2 - ((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c
```

```
^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a
*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c
^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + ((b*e - 2
*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d
+ (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 -
 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) - 3*b^2*c*d*e^3))/(8*a^3*c^2*(4*
a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*
e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*e^3) + ((a*c - b^2)*(4*a*c - b^2)^2*(((4*b
^2*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*
a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*
b^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)
) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(768*a*b^2*c^3*e -
 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)
))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e))/(8*c*(
4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - ((b*e - 2*c*d)*(((((b*e -
 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c
*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b
^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + (8*a*b^2*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c -
b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(64*a*c^4*d^2 + ((4*b^2
*e - 16*a*c*e)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e -
 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c
^2*e^2 - 256*a*b*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c*e^3 + 16*
a*c^3*d^2*e - 40*a*b*c^2*d*e^2))/(8*c*(4*a*c - b^2)^(1/2)) - (a*b^2*c*(4*b^
2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2)
)))/(a^3*c^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e -
8*b^3*c*d*e^3)) + ((4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*(a*b^2*e^4 - ((4*b^2
*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*
b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1
/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^
2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + (8*a*
b^2*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c
 - b^2))))/(2*(64*a*c^2 - 16*b^2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 1
6*a*c*e)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(768*a*b^2*c^3*e - 512*a*b*c
^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*
a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e))/(2*(64*a*c^2 - 1
6*b^2*c)) + 24*a*b^2*c*e^3 + 16*a*c^3*d^2*e - 40*a*b*c^2*d*e^2))/(2*(64*a*c
^2 - 16*b^2*c)) + a*c^2*d^2*e^2 - ((((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512
*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(
8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)
)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64
*a*c^2 - 16*b^2*c)) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*
(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64
*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*
b*c^3*d*e))/(8*c*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1
/2)) + (a*b^2*(b*e - 2*c*d)^4)/(4*(4*a*c - b^2)^2) - 2*a*b*c*d*e^3))/(a^3*c
^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*
e^3)))*(b*e - 2*c*d))/(4*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 18.30, size = 287, normalized size = 3.99

$$\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right)+2ae+4b^2\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right)-bd}{be-2cd}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] (e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4
 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2
))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

3.45 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\cdots$$

```
[Out] 1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arcta
nh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*a*c+
b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arctan(2^(1/4
)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/
2))*2^(1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arctanh(2^(1/4)*c^(1/
4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(
1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 0.46, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1510, 298, 205, 208}

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
+ ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}\ldots}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}-\ldots$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4a\ldots}}\right)\ldots}{2\,2^{3/4}c\ldots}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ]/4
```

fricas [B] time = 48.37, size = 13521, normalized size = 36.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a
*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^
2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 -
48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^
2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*
a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*((2*(a*b^4
*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*
b*c^5)*e)*x*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b
*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^
3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)
) + ((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b
^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*x - sqrt
(1/2)*((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a
*b^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 + (2*(a
*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*
```

```
a^3*b*c^5)*e)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))*sqrt((2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a
*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*
c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3
*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)
*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^
2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*
b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 + 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^
3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c + 4*a^4*b*c^2)*e^6 + ((a*b^6*c^4 - 12*a^2
*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4
 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a
*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4
+ 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 +
 (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*
a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^
2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^
3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^
4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(
7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b
^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^5*d^8
- 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a
*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4
+ 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2
 - a^4*c)*e^8)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^
2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^
4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*
b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 +
 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 +
(a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a
^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^4*
d^6 - b*c^3*d^5*e - 5*a*c^3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2
*c^2)*d^2*e^4 + (a*b^3 + 3*a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)) + sqrt(
sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
 - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*(sqrt(1/2)*((b^2
*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c - 4*
a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 - (2*(a*b^4*c^4 -
8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*
e)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*
e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^
2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)
*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(s
qrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
 - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 +
 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*
b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a
^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
+ a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9
)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt((2*(c^5*d^8 - 2*b*c^4*d
^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 +
2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2
```

```
*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e
^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*
d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3
- 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 +
 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c
 + 4*a^4*b*c^2)*e^6 - ((a*b^6*c^4 - 12*a^2*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^
4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e
^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3
*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2
)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(
-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d
*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*s
qrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5
- 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^
6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8
)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3
 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*
e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6
*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^
6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)) + ((2*(a*b^4*c^4
- 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*e)*x*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*
d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3
)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)) - (
(b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c
- 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*x)*sqrt(sqrt(
1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*
a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*
a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^
3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^
4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/
(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5))))/(c^4*d^6 - b*c^3*d^5*e - 5*a*c^
3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2*c^2)*d^2*e^4 + (a*b^3 + 3
*a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3
*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 +
(a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^
6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*
b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(
a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*
b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*
c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 +
 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 +
 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 -
48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)
*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 -
6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a
^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b
^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e
^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3
*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2
)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(
sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
 - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4
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+ 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*
d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*
b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^
5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^
2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3
*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^
3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*
a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*
c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*
c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^
6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*
c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10
)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*
e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*
d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*
a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b
^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2
*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(-1/2*sq
rt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^
2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*
d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*
e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*
c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3
*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6
+ 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48
*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4
*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4
*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2
 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^
4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d
*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7
 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3
*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2
*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c
^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a
^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^
3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a
^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16
*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2
*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^
4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a
^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 -
3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*
c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*
e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*
b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^
4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b
*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 +
a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c
^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3
+ (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((
c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(
a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8
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*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^
2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*
a^2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*
a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3
 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3
 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3
- 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^
3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3
- 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*
(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2
*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)
*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d
^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)
*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c
^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2
*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^
4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a
^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 -
3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*
c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^
3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^
2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*
c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*
a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a
^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*
a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 1
6*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^
3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^
3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*
e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^
2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^
10)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^
2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^
3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^
4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(
7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b
^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(-1/2*
sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*
b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^
3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^
6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a
^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^
6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 +
48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a
^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b
^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e
^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*
d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)
*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c
^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c
^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a
^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a
*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19
*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c -
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a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12
*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 +
16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b
^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*
c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48
*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2
- 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b
^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^
5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^
9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*
a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*
b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3
*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3
+ a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 8.38Unable to divide, perha
ps due to rounding error%%%{-512,[0,10,0,3,5,2,7]%%%}+%%%{1152,[0,10,0,3,4,
4,6]%%%}+%%%{-512,[0,10,0,3,3,6,5]%%%}+%%%{64,[0,10,0,3,2,8,4]%%%}+%%%{1024
,[0,9,1,3,6,1,7]%%%}+%%%{-4352,[0,9,1,3,5,3,6]%%%}+%%%{512,[0,9,1,3,4,5,5]%
%%}+%%%{640,[0,9,1,3,3,7,4]%%%}+%%%{-128,[0,9,1,3,2,9,3]%%%}+%%%{-512,[0,8,
2,3,7,0,7]%%%}+%%%{7296,[0,8,2,3,6,2,6]%%%}+%%%{6144,[0,8,2,3,5,4,5]%%%}+%%
%{-4544,[0,8,2,3,4,6,4]%%%}+%%%{384,[0,8,2,3,3,8,3]%%%}+%%%{64,[0,8,2,3,2,1
0,2]%%%}+%%%{-6144,[0,7,3,3,7,1,6]%%%}+%%%{-22016,[0,7,3,3,6,3,5]%%%}+%%%{9
472,[0,7,3,3,5,5,4]%%%}+%%%{1152,[0,7,3,3,4,7,3]%%%}+%%%{-512,[0,7,3,3,3,9,
2]%%%}+%%%{2048,[0,6,4,3,8,0,6]%%%}+%%%{31232,[0,6,4,3,7,2,5]%%%}+%%%{-4352
,[0,6,4,3,6,4,4]%%%}+%%%{-8064,[0,6,4,3,5,6,3]%%%}+%%%{1792,[0,6,4,3,4,8,2]
%%%}+%%%{1048576,[0,6,0,7,9,1,8]%%%}+%%%{-3670016,[0,6,0,7,8,3,7]%%%}+%%%{2
424832,[0,6,0,7,7,5,6]%%%}+%%%{-589824,[0,6,0,7,6,7,5]%%%}+%%%{49152,[0,6,0
,7,5,9,4]%%%}+%%%{-20480,[0,5,5,3,8,1,5]%%%}+%%%{-11264,[0,5,5,3,7,3,4]%%%}
+%%%{18432,[0,5,5,3,6,5,3]%%%}+%%%{-3584,[0,5,5,3,5,7,2]%%%}+%%%{7340032,[0
,5,1,7,9,2,7]%%%}+%%%{-2097152,[0,5,1,7,8,4,6]%%%}+%%%{-1376256,[0,5,1,7,7,
6,5]%%%}+%%%{622592,[0,5,1,7,6,8,4]%%%}+%%%{-65536,[0,5,1,7,5,10,3]%%%}+%%%
{5120,[0,4,6,3,9,0,5]%%%}+%%%{18176,[0,4,6,3,8,2,4]%%%}+%%%{-23040,[0,4,6,3
,7,4,3]%%%}+%%%{4544,[0,4,6,3,6,6,2]%%%}+%%%{-8388608,[0,4,2,7,10,1,7]%%%}+
%%%{-2359296,[0,4,2,7,9,3,6]%%%}+%%%{3801088,[0,4,2,7,8,5,5]%%%}+%%%{-40960
0,[0,4,2,7,7,7,4]%%%}+%%%{-196608,[0,4,2,7,6,9,3]%%%}+%%%{32768,[0,4,2,7,5,
11,2]%%%}+%%%{-10240,[0,3,7,3,9,1,4]%%%}+%%%{17920,[0,3,7,3,8,3,3]%%%}+%%%{
-3840,[0,3,7,3,7,5,2]%%%}+%%%{4194304,[0,3,3,7,10,2,6]%%%}+%%%{2621440,[0,3
,3,7,9,4,5]%%%}+%%%{-4194304,[0,3,3,7,8,6,4]%%%}+%%%{1343488,[0,3,3,7,7,8,3
]%%%}+%%%{-131072,[0,3,3,7,6,10,2]%%%}+%%%{2048,[0,2,8,3,10,0,4]%%%}+%%%{-9
216,[0,2,8,3,9,2,3]%%%}+%%%{2176,[0,2,8,3,8,4,2]%%%}+%%%{5242880,[0,2,4,7,1
1,1,6]%%%}+%%%{-12582912,[0,2,4,7,10,3,5]%%%}+%%%{8454144,[0,2,4,7,9,5,4]%%
%}+%%%{-2195456,[0,2,4,7,8,7,3]%%%}+%%%{196608,[0,2,4,7,7,9,2]%%%}+%%%{2147
483648,[0,2,0,11,12,2,8]%%%}+%%%{-2147483648,[0,2,0,11,11,4,7]%%%}+%%%{8053
06368,[0,2,0,11,10,6,6]%%%}+%%%{-134217728,[0,2,0,11,9,8,5]%%%}+%%%{8388608
,[0,2,0,11,8,10,4]%%%}+%%%{3072,[0,1,9,3,10,1,3]%%%}+%%%{-768,[0,1,9,3,9,3,
2]%%%}+%%%{5242880,[0,1,5,7,11,2,5]%%%}+%%%{-4718592,[0,1,5,7,10,4,4]%%%}+%
%%{1376256,[0,1,5,7,9,6,3]%%%}+%%%{-131072,[0,1,5,7,8,8,2]%%%}+%%%{-2147483
648,[0,1,1,11,12,3,7]%%%}+%%%{2147483648,[0,1,1,11,11,5,6]%%%}+%%%{-8053063
68,[0,1,1,11,10,7,5]%%%}+%%%{134217728,[0,1,1,11,9,9,4]%%%}+%%%{-8388608,[0
,1,1,11,8,11,3]%%%}+%%%{-512,[0,0,10,3,11,0,3]%%%}+%%%{128,[0,0,10,3,10,2,2
]%%%}+%%%{-2097152,[0,0,6,7,12,1,5]%%%}+%%%{1835008,[0,0,6,7,11,3,4]%%%}+%%
```

```
%{-524288,[0,0,6,7,10,5,3]%%%}+%%%{49152,[0,0,6,7,9,7,2]%%%}+%%%{-214748364
8,[0,0,2,11,13,2,7]%%%}+%%%{3221225472,[0,0,2,11,12,4,6]%%%}+%%%{-187904819
2,[0,0,2,11,11,6,5]%%%}+%%%{536870912,[0,0,2,11,10,8,4]%%%}+%%%{-75497472,[
0,0,2,11,9,10,3]%%%}+%%%{4194304,[0,0,2,11,8,12,2]%%%} / %%%{1,[0,6,0,0,1,2
,2]%%%}+%%%{-2,[0,5,1,0,2,1,2]%%%}+%%%{-2,[0,5,1,0,1,3,1]%%%}+%%%{1,[0,4,2,
0,3,0,2]%%%}+%%%{6,[0,4,2,0,2,2,1]%%%}+%%%{1,[0,4,2,0,1,4,0]%%%}+%%%{-6,[0,
3,3,0,3,1,1]%%%}+%%%{-4,[0,3,3,0,2,3,0]%%%}+%%%{2,[0,2,4,0,4,0,1]%%%}+%%%{6
,[0,2,4,0,3,2,0]%%%}+%%%{-2048,[0,2,0,4,5,1,3]%%%}+%%%{512,[0,2,0,4,4,3,2]%
%%}+%%%{-4,[0,1,5,0,4,1,0]%%%}+%%%{2048,[0,1,1,4,5,2,2]%%%}+%%%{-512,[0,1,1
,4,4,4,1]%%%}+%%%{1,[0,0,6,0,5,0,0]%%%}+%%%{2048,[0,0,2,4,6,1,2]%%%}+%%%{-1
536,[0,0,2,4,5,3,1]%%%}+%%%{256,[0,0,2,4,4,5,0]%%%} Error: Bad Argument Val
ue
```

maple [C] time = 0.00, size = 51, normalized size = 0.14

$$\frac{\left(\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^6 e+\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^2 d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^7 c+4\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^4+d\right)x^2}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 9.57, size = 29445, normalized size = 78.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] 2*atan(((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*
c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 1
2*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*
c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^
2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d
^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4
- a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4
+ a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^
3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4
*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*
e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*
c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4
)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^
3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*
b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3
*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*
```

```
a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*
c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d
*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*
c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 102
4*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*
b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^
4*d*e - 16384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 -
4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b
^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^
5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e
^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*
a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b
^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*
a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2
*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5
*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e
 + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*
c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*
d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*
a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128
*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 -
 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*
b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c
^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*
b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^
3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e +
6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*
b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c
^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*
c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 -
32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 3276
8*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a
^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e -
 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^
2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 +
c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 4
0*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*
d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 +
 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 +
 a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4))/((
x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5
 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3
*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 +
 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^
3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*
```

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d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^
3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*
e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*
a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b
^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a
*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4
 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4
 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*
e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c
^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e
 + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*
a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256
*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c
^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e
^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 1
6384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*
b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^
3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*
1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3
*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b
^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*
c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a
^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c
^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*
e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c
^5 - 256*a^4*b^2*c^6)))^(1/4)*1i - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 +
 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 +
 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a
*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^
3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5)
 + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*
c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^
5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c
^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^
2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^
4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e
^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^
5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*
e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^
5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*
d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 1
6*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^
2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*
a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*
b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5
*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*
a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*
c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4
```

*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*
b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e
+ 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^
3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8
*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + 2*a*
c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b
*c*e^7 - 8*a*b*c^4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a
^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*
d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c
^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*
d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*
e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*
d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a
*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^
2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^
(1/4) - atan(-((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e +
16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*
e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a
^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4
*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^
5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c
^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c
^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4
*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^
2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 20
48*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2
*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^
3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a
^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^
2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*
e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64
*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*
b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96
*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b
*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b
*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d
^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*
b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*

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c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e
^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*
a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b
^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*
a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^
3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*
e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*
a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b
^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*
a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^
2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 +
32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*
a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e
- 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a
^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^
3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*
a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^
3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 9
6*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a
*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i)/(
(x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^
5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^
3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5
+ 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a
^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3
*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a
^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3
*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96
*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*
b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(
a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^
4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^
4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3
*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*
c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*
e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6
*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 25
6*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*
c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*
e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e -
16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*
c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3
+ 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(
-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*
```

d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*
e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d
^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^
3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^
3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 +
6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 -
256*a^4*b^2*c^6)))^(1/4) - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*
c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*
b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d
^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 -
36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*
b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4
+ 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4
- 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e
+ 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^
3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e
+ 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a
*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*
a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*
a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^
2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2
- 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^
6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 3276
8*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*
c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*
e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*
a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5
*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e +
6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^
2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^
2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 -
4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*
d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^
2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*
b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*
b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + 2*a*c^5*d^7 + 2*a^4*c
^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^
4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 +
12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b
^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^
3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e +
6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*
b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c
^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*2i - atan(-
((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e
^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b
^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5
+ 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*

a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c

- b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i)/((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 12

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8*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3
- 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2
*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*
c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a
*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3
*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2
- 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048
*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b
^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3
+ 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3
*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*
c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + 2*a*c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a
^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d^6*e + 18*a
^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d
^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 -
 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*
a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^
2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2
- 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^
6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*2i + 2*atan(((x*(4*a^3*b^
3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c
^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^
4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c
^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2
*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e
^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5
*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d
^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16
*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 +
b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b
*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b
*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4
*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2
- 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b
^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^
6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10
240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*
a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^
3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 +
 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^
4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^
```

7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4))/((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4

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- a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^
3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4
*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*
e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*
c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4
)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b
^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c
^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e
)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^
4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4
*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b
^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*
c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*
c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*
c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^
4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6
)))^(1/4)*1i - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e +
16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*
e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a
^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4
*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^
5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c
^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c
^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4
*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^
2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 20
48*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2
*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4
*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 25
6*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3
*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^
6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^
2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 12
8*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*
e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^
4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + 2*a*c^5*d^7 + 2*a^4*c^2*
d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d
^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*
a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*
```

```
c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c
^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a
*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c
^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3
- 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.46 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=184

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d
)/(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arct
an(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1490, 1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx,x,x^2\right)$$
$$=\frac{1}{4}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)+\frac{1}{4}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}$$
$$=\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.15, size = 179, normalized size = 0.97

$$\frac{\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 1.47, size = 1535, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 + 1/2
*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4
*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 -
 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4
 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*
d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*
c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2
*e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2
+ ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*
c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2
```

```
*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*s
qrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^
2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d -
 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4
*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

giac [B] time = 20.31, size = 1406, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^
2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^
2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/8*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*
b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*
(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arc
tan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c
- 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [B] time = 0.02, size = 340, normalized size = 1.85

$$\frac{\sqrt{2}\, be \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{\sqrt{2}\, be \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\, cd \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+\cdots}\right.}}\right.}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x^2)*e+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^
2)*b*e-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*d+1/4*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x^2)*e+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b*e-1/2*c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x^2)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^4 + d\right)x}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 7.05, size = 4501, normalized size = 24.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i - a^2*e^3*
x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3
*x^2*4i - a*b^4*d*e^2*x^2*1i - b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i + a
*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a*
c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2
*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2
*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)
/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(
a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^
3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^
3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*
d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2
- 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^
2*c^2 + 32*a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b
^4*c))^(1/2) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*
d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) +
2048*a^4*b*c^3*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 48*a^3*b^2*c*e
```

```
^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*
a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 16*a^2*b^2*c^2*d^2*(-(a*b^
3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*
b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 -
256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*
c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 -
4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*
c^2 + 32*a*b^4*c))^(1/2) + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b
^4*c))^(1/2)))*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)*2i + atan((b^4*c
*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i + a^2*e^3*x^2*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3*x^2*4i - a
*b^4*d*e^2*x^2*1i + b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i - a*b*d*e^2*x^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a*c*d^2*e*x^2
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*b*c^2*d^2*
e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2*b^4*e^2*(-
(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c
^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(a*b^3*e^2 +
 b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) +
 c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d
^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2
*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^3*c*d^2 - a
*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c
*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*
a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*
c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2
) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b
^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) + 2048*a^4*b*
c^3*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512
*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 48*a^3*b^2*c*e^2*(-(a*b^3
*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b
*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 2
56*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 16*a^2*b^2*c^2*d^2*(-(a*b^3*e^2 + b^3
*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d
^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 -
 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2
*c^2 + 32*a*b^4*c))^(1/2) - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e
^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
```

```
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*
b^4*c))^(1/2) + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*
c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2
)))*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512
*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)*2i
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.47 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arct
anh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctan(2^(1/
4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1
/2))*2^(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1
/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^
(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.35, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1422, 212, 208, 205}

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4
)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}} - \cdots$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\cdots\right.}{2\sqrt[4]{2}\sqrt[4]{c}\cdots}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1^
3 + 2*c*#1^7) & ]/4
```

fricas [B] time = 9.67, size = 13304, normalized size = 35.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*
c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2
*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5
*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2
 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c
- 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*
a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/4*(2*sqrt(1/
2)*(((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128
*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b
*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5
)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)
*x*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6
*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^
7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^
```

```
4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)) + ((b^
7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 5
9*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a
^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*
b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*e^7)*x)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^
3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (
a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4
*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^
4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^
6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) -
 ((b^7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^
2 - 59*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2
- 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168
*a^4*b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5
*b*c^3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^7 + ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*
a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*
b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^
4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 +
48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c
*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d
^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^
2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*
a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*
b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*
c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3
*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 +
 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 +
2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*
b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*sqrt((2*(
14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b
^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*
b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b
^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7*a*b^4*c^2 + 14*a^2*b^2*c
^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^2 + 20*a^3*b*c^3)*d^5*e +
 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4*
a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4
*b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 + ((a^3*b^7*c - 12*a^4*
b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4*c^
2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b
*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)
*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*
e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4
8*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^
3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^
4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d
^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8
 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
+ 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^
8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(14*a^
3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^
2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c
 + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 -
4*a^4*c)*d^2*e^6)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3
 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a
^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*
```

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b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4
)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6
*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(
3*a^5*b*d*e^9 - a^6*e^10 + (b^2*c^4 - a*c^5)*d^10 - (3*b^3*c^3 + a*b*c^4)*d
^9*e + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^8*e^2 - (b^5*c + 17*a*b^3*c^2
+ 24*a^2*b*c^3)*d^7*e^3 + 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^6*e^4 -
 21*(a^2*b^3*c + 2*a^3*b*c^2)*d^5*e^5 + 14*(2*a^3*b^2*c + a^4*c^2)*d^4*e^6
+ (a^3*b^3 - 16*a^4*b*c)*d^3*e^7 - 3*(a^4*b^2 - a^5*c)*d^2*e^8)) - sqrt(sqr
t(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*
b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e
^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b
*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4
*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/4*(2*sqrt(1/2)*(((a^
3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6
)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^
2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2
- (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*x*sqrt(
-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (
b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*
(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(
a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)) - ((b^7*c^2 -
9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59*a^2*b^
4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c
^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3
- 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e
^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*e^7)*x)*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3
*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^
3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3
- 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 +
 a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*
c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^
4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*
b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e
^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b
*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4
*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) + ((b^7*c^2 - 9*a*b^5*c^3 + 24
*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59*a^2*b^4*c^3 + 136*a^3*
b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^
4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3 - 304*a^5*c^4)*d
^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^4 + 9*(a^4*b^4*
c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*e^7 - ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5
 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64
*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a
^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4
)*e^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 -
 a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3
)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2
)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*s
qrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c
 - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*
c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*
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c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2
- a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c -
 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a
^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*sqrt(-(6*a^2*b*c*d^2*e
^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a
^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^
2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a
*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^
2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 -
 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2
 + 16*a^5*c^3))*sqrt((2*(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^
2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4
*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*
a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7
*a*b^4*c^2 + 14*a^2*b^2*c^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^
2 + 20*a^3*b*c^3)*d^5*e + 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4
*e^2 - 20*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*
a^5*c^2)*d^2*e^4 + 2*(a^4*b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^
6 - ((a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(
a^4*b^6*c - 12*a^5*b^4*c^2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*sqrt(-(48*a^
3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2
 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*
b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6
*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2
*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2
*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*
c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2
*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*
c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2
 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)))/(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3
 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*
c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c
^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)))/(3*a^5*b*d*e^9 - a^6*e^10 + (b
^2*c^4 - a*c^5)*d^10 - (3*b^3*c^3 + a*b*c^4)*d^9*e + 3*(b^4*c^2 + 4*a*b^2*c
^3 + a^2*c^4)*d^8*e^2 - (b^5*c + 17*a*b^3*c^2 + 24*a^2*b*c^3)*d^7*e^3 + 7*(
a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^6*e^4 - 21*(a^2*b^3*c + 2*a^3*b*c^2)
*d^5*e^5 + 14*(2*a^3*b^2*c + a^4*c^2)*d^4*e^6 + (a^3*b^3 - 16*a^4*b*c)*d^3*
e^7 - 3*(a^4*b^2 - a^5*c)*d^2*e^8)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d
^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c -
 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*
b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 -
 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^
2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c
^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2
*c^2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^
5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*
c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a
*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2
- 4*a^4*c)*d*e^4 - ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b
^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b
*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)
*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*
e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4
8*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a
^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*
d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^
3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3
 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^
```

3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^
5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a
^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*
d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((
10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a
*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x -
1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*
e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 - ((a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*
a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*
e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*
b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^
4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5
)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (
b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4
*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12
*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3
*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^
2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 -
 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))) + 1/4*sqrt(sqrt(
1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*
c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*
a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6
 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c
^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c
^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))
/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3
*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b
*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2
+ 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c
^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*
a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*
b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 -
 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^
2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c
^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(
6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4
*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqr
t(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 -
 (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e -
4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)
/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c
- 8*a^4*b^2*c^2 + 16*a^5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*
e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*
a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c
^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*
a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c
^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2
- 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^
2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 +
 a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c +
 a^2*c^2)*d^4*e^2)*x - 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^
3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4
*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*

```
c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*
d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*
c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3
*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 -
 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 +
a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4
*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c
^3))))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.00, size = 47, normalized size = 0.13

$$\frac{\left(\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^4 e+d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^7 c+4\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(c*x^8+b*x^4+a),x)
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4+d}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 8.75, size = 36707, normalized size = 97.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(a + b*x^4 + c*x^8),x)
[Out] - atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128
*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*
b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c
^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*
b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(
```

-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3
*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 4
0*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*
b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*
c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*
c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 4
9152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b
*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 +
16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*
a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^
5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a
*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*
c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*
d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2
*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3
*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 1
6*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 -
16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e -
16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*
d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3
- 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 +
4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^
3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c
^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^
5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*
d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e
^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d
*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^
3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^
3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a
^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(
1/4)*1i - ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 1
28*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e
- 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^
2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2
*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^
5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b
^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 +
40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^
4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^
8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^
5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e +
49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4
*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2
+ 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 819
2*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*

c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^
3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a
^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c
^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c -
16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5
- 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e
- 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^
3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e
^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2
+ 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*
d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3
*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*
b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^
4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2
*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3
*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*
a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*
c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))
^(1/4)*1i)/(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*
a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b
^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*
a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*
a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
+ 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a
^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d
^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8
192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^
4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-
(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*
c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40
*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b
*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c
- 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^
5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*
e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*
c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2
*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e

^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^
4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b
^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^
3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*
c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c
^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c
^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 -
8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^
2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)
))^(1/4) + ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*
e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a
^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^
2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a
^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*
b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a
^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b
^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a
^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
+ 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^
4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^
2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 81
92*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4
*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(
4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c
^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*
a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*
c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c
- 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5
- 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e
- 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c
^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*
e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^
2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4
*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^
3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3
*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c
^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^
2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^
3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8
*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2
*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))
)^(1/4)))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 1
1*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8

```
*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 12
8*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e
- 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2
*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*
c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a
*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5
*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*2i - atan((((-(b^7*c*d^4 + a^3*b^5*e^4
- a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b
^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*
c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*
e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(
((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c
^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*
c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4
*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*
b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^
2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3
- 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608
*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*
c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 1126
4*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3
*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e
- 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b
^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^
4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4
 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*
e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c
^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2
 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*
a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256
*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 -
192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3
*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*
b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*
d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e
^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6
*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*
b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(
1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*
c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e
 + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*
a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*
e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2
+ 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i - ((-(b^7*c*d^4 + a^3*b^5*e^
4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
- a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
 b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^
3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*
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c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c
^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)
*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5
*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^
3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c
^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^
3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*
d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e
^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^
3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 1966
08*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^
2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11
264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b
^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*
e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-
(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*
d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e
^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^
3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3
*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e
^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 +
6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 2
56*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5
- 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d
^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*
a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^
7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2
*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c
^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*
a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)
^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^
6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3
*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 6
4*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^
3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^
2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i)/(((-(b^7*c*d^4 + a^3*b^5*
e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^
4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e
^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*
b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*
d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7
*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/
4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b
^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*
b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4
*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*
a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*
c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d
*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*
c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 19
6608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*
b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 -
```

11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2
*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*
d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*
(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^
2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c
*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*
d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b
^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2
*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
+ 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^
5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6
*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 6
4*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*
c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d
^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b
*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 1
6*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^
5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*
b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d
^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*
c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4) + ((-(b^7*c*d^4 + a^3*b^5*e
^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
- a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
+ b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^
3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b
^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*
c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4
)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^
5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b
^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*
c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a
^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c
*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*
e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c
^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196
608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b
^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 1
1264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*
b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d
*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(
-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2
*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*
e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d
^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^
3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*
e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 +
6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5
- 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*

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d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64
*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c
^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^
2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*
c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16
*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b
^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^
3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c
^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)))*(-(b^7*c*d^4 + a^3*b^5*e^4
 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
 a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
+ 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4
*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3
*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^
5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*
2i - 2*atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
 - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
 - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^
3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6
*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*
b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96
*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*
e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d
^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^
2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^
3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96
*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3
*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144
*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*
e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*
a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c
^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2
 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^
2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*
b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a
^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b
^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a
*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c
^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16
*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*
c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a
*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 -
 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e
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^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*
c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +
16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*
d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^
6*b^2*c^4)))^(1/4) - ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^
5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*
b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d
^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*
c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^
4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
+ a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 -
 b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^
3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c
^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)
*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*
b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d -
 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152
*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5
*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e +
20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4
+ a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 +
a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b
^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*
c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*
e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1
i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 -
16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^
4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 25
6*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7*d^6 - 8*a^3*c^4*e
^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d
^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c
^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))
*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c
^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*
c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4
*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*
b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^
2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3
- 256*a^6*b^2*c^4)))^(1/4))/(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^
```

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4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b
^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d
^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*
a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^
3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*
c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c
^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c
^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 -
8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^
2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)
))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4
096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7
*c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2
 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024
*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^
4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*
b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^
4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2
*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3
*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*
a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*
c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))
^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*
d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*
b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*
e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a
^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b
^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e -
16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d
^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128
*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*
b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c
^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*
b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i + ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7
*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4
- 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +
 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e
- 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2
*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^
2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a
```

^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^
3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e +
 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*
b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6
*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 20
48*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c
*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 1
6*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d
^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 1
28*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*
c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6
*b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 1
92*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*
e^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b
^3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^
7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2
*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c
^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*
a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)
^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^
6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3
*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 6
4*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^
3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^
2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i))*(-(b^7*c*d^4 + a^3*b^5*e
^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
- b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^
3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b
^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*
c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4
) - 2*atan((((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*
a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b
^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*
a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*
a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
 + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a
^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^
7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2

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- 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2
*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b
^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 +
 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^
4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^
8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a*
c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^
5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16*
a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c
^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*
c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 -
16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^
3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c
*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 1
6*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d
^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 1
28*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*
c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6
*b^2*c^4)))^(1/4) - ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b
^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^
3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c
^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4
 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
 a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
+ 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4
*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3
*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^
5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*
(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b
^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d -
262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*
a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*
c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 2
0480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 -
 a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^
2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c
*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e
 + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
+ a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i
 - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 1
6*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4
*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256
```

*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7*d^6 - 8*a^3*c^4*e^
6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^
4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^
5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*
(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^
2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c
*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*
d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b
^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2
*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
+ 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
256*a^6*b^2*c^4)))^(1/4))/(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4
- 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^
4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^
2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a
^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3
*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c
^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^
2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^
3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8
*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2
*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))
)^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 40
96*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*
c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2
- 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*
a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4
*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b
^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4
*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*
e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*
d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a
^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c
^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^
(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d
*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b
*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e
^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^
3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^
2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 1
6*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^
2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*
a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a
^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*
a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b
^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^
2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b
^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i + ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^

```
4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*
b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a
^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b
^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*
c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +
16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*
d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^
6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3
*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e +
4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b
^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*
d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 204
8*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*
d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 4
8*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16
*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 1
28*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^
2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 12
8*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*
b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 19
2*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e
^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^
3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7
*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*
e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^
6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a
*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6
*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*
e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64
*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2
 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i))*(-(b^7*c*d^4 + a^3*b^5*e^
4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
- a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
 b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^
3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c
^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.48 $\int \frac{d+ex^4}{x(a+bx^4+cx^8)} \, dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

```
[Out] d*ln(x)/a-1/8*d*ln(c*x^8+b*x^4+a)/a+1/4*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^4\right)\\
&= \frac{d\log(x)}{a}+\frac{\operatorname{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx,x,x^4\right)}{4a}\\
&= \frac{d\log(x)}{a}-\frac{d\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8a}+\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}-\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\operatorname{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

fricas [A] time = 2.40, size = 240, normalized size = 3.08

$$\left[-\frac{\left(b^2-4\,ac\right)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4\,ac\right)d\log(x)+\sqrt{b^2-4\,ac}\,(bd-2\,ae)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-\left(2\,cx^4\right.}{cx^8+bx^4+a}\right.}{8\left(ab^2-4\,a^2c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c -
(2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c), -
1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 20.63, size = 78, normalized size = 1.00

$$-\frac{d\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{d\log\left(x^4\right)}{4\,a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arct
an((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

maple [A] time = 0.01, size = 106, normalized size = 1.36

$$-\frac{bd\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\;a}+\frac{e\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}+\frac{d\ln(x)}{a}-\frac{d\ln\left(c\,x^8+b\,x^4+a\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/a*d*ln(x)-1/8*d*ln(c*x^8+b*x^4+a)/a+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4
+b)/(4*a*c-b^2)^(1/2))*e-1/4/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-
b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.27, size = 8454, normalized size = 108.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^4 + c*x^8)*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 -
 64*a^2*c)) + (atan((128*a^5*x^4*(((c^4*e^5 - ((4*b^2*d - 16*a*c*d)*(11*b*c
^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^
2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*
(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e)
)/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c
^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b*
c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(2*(16*a*b^2 - 64*a^2*c)) - ((4*b^2
*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c)
) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 -
 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*
e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1
280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4
*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a
*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 460
```

```
8*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^
5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2)))))/(8*a*(4*a*c - b^2)^(1
/2))))/(2*(16*a*b^2 - 64*a^2*c)) + (((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*
c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*
a*e - b*d)^3)/(1024*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)))*(2*a*e
- b*d))/(8*a*(4*a*c - b^2)^(1/2)) - ((((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*
d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 6
4*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c
 - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a
*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*
a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^
2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 -
64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*
a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d -
 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 46
08*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c
^5*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^
2*c^4*e^3 + 108*b*c^5*d*e^2))/(8*a*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*
a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c
^5)*(2*a*e - b*d)^4)/(8192*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2))*(5*b
^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d + 3*a^2*b^2*c*e)
)/(32*a^5*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e)) - ((((4*b^2*d
- 16*a*c*d)*(((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608
*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3
456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b
^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c
- b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*
c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 -
64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^
2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^3)/(1024*a^3*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((
4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 1
6*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b
^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) +
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*
b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*
c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c
^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*
b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))
))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b
^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2
*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e
))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*
c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b
*c^5*d*e^2))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*
e - b*d)*(11*b*c^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 1
6*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608
*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3
456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5
```

```
*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*
c^4*e^3 + 108*b*c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(8*a*(4*a*c - b^2)^
(1/2)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d
 - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 5
76*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)
) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(1
6*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c
 - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*
e - b*d)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 -
64*a^2*c)) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*
b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024
*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*
a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*
a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)
) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*
c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4
*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 8
64*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c
 - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) - ((1280*b^5*c^4 - 4608*a*b^3*c^5
)*(2*a*e - b*d)^5)/(32768*a^5*(4*a*c - b^2)^(5/2)))*(144*a^3*c^3*d - 40*b^6
*d + 8*a*b^5*e - 488*a^2*b^2*c^2*d + 272*a*b^4*c*d - 40*a^2*b^3*c*e + 40*a^
3*b*c^2*e))/(256*a^5*c^4*(4*a*c - b^2)^(1/2)*(a^2*e^2 - 20*b^2*d^2 + 81*a*c
*d^2 - a*b*d*e)))*(4*a*c - b^2)^(5/2))/(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a
^2*b^2*c^4*d^2*e^2 - 8*a*b^3*c^4*d^3*e - 32*a^3*b*c^4*d*e^3) + (4*(4*a*c -
b^2)^(5/2)*(5*b^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d +
 3*a^2*b^2*c*e)*(c^4*d*e^4 + ((4*b^2*d - 16*a*c*d)*(a*c^4*e^4 + ((4*b^2*d -
 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 2
56*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)
))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*
a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(2*(16*a*b^2 - 64*
a^2*c)) - 16*b*c^4*d*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c
*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128
*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)
^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a
^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^2*
d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2
*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e -
b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d)
)/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d
 - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2
*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^
4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64
*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/
(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d)
)/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d
)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*
d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*
c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2
)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d
)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(
16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b
^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*
e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256
*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))
/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*
b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(8*a*(4*a*c - b^2)^(
1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((2*a*e - b*d)*(((2*a*e -
 b*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (1
28*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^
```

```
2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64
*a^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^
2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/
(8*a*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^3)/
(4*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(8*a*(4*a*c - b^2)^(1/2
)) + (b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^4)/(32*a^3*(16*a*b^2 - 64*
a^2*c)*(4*a*c - b^2)^2)))/(c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e
)*(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*a*b^3*c^4*d^3*
e - 32*a^3*b*c^4*d*e^3)) + ((4*a*c - b^2)^2*(((4*b^2*d - 16*a*c*d)*(((4*b^2
*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*
a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/
(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))
/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) +
((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (1
28*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 6
4*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1/2)))
)/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^
2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d
- 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c
^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 1
6*a*b*c^4*e^3))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((4
*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4
*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 -
 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(
2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b
^2)^(1/2)) + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2
- 64*a^2*c)*(4*a*c - b^2))))/(8*a*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4*b^2*d
- 16*a*c*d)*(2*a*e - b*d)^3)/(4*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/
2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(
((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d -
 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4
*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(
1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e
- b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c
)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(256*b^4*c^4*d -
 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*
c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e -
b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c
)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e
 + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^
2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1
/2))))/(8*a*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2))
+ ((2*a*e - b*d)*(a*c^4*e^4 + ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*
(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4
*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*
a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*
e^2 - 16*a*b*c^4*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - 16*b*c^4*d*e^3))/(8*a*(4
*a*c - b^2)^(1/2)) + (b^4*c^4*(2*a*e - b*d)^5)/(128*a^4*(4*a*c - b^2)^(5/2)
))*(144*a^3*c^3*d - 40*b^6*d + 8*a*b^5*e - 488*a^2*b^2*c^2*d + 272*a*b^4*c*
d - 40*a^2*b^3*c*e + 40*a^3*b*c^2*e))/(2*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c
*d^2 - a*b*d*e)*(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*
a*b^3*c^4*d^3*e - 32*a^3*b*c^4*d*e^3)))*(2*a*e - b*d))/(4*a*(4*a*c - b^2)^(
1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.49 $\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4a\cdots}}}$$

```
[Out] -d/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+
1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(d+(2*
a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^
(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b*
d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*
arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b*d)/(-
4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] time = 0.68, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1504, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4a\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b
 + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{ax}-\frac{\int \frac{x^2\left(bd-ae+cdx^4\right)}{a+bx^4+cx^8}\,dx}{a}$$

$$= -\frac{d}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}$$

$$= -\frac{d}{ax}+\frac{\left(\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a}-\frac{\left(\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{-b-\sqrt{b}}}}{2\sqrt{2}a}$$

$$= -\frac{d}{ax}-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4a}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.06, size = 85, normalized size = 0.22

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.01, size = 72, normalized size = 0.18

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{6}cd+\left(-ae+bd\right)\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{2}\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)\right.}{4a\left(2\,\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{7}c+\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x)
[Out] -1/a*d/x-1/4/a*sum((_R^6*c*d+(-a*e+b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_
R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 9.46, size = 39028, normalized size = 99.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x)
[Out] atan((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*
c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4
*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d
^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*
b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/
2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^
3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^
3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a
*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 2
40*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48
*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*
c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2
```

```
*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*
c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1
024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81
920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 327
68*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*
a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*
b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5
*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e
^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*
e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d
^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^
8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*
c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3
*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6
*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a
*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 -
8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e +
128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3
*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 -
 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^
4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*
c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))
^(1/4)*1i + ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*
a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 6
1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a
^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b
*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8
*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*
b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^
2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(
4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b
^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3
*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^
2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 5
1200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10
240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048
*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096
*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b
^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5
*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^
4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3
```

*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^1
1*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32
*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^
12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13
*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^
3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^
2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b
^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2
*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e +
 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^
4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*
c^3)))^(1/4)*1i)/(((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e
^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b
*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288
*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*
a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^
9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*
a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
- 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^
2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e -
 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128
*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17
*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*
c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c
^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*
d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3
+ 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*
a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^
14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*
b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^
14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*
(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^
5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
+ 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 4
4*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^
4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*
c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 -
 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c
^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*

c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*

```
e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*
b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3
- 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 28
8*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256
*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i + at
an((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^
2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^
5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*
a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*
c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c
^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2
*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8
*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a
^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^
5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3
- 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*
c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b
^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240
*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a
^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*
d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d
*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^
2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 102
4*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 8192
0*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768
*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^
14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*b*
c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d
^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3
 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e
- 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2
*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^8*
d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^
7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c
^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d
^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*
a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 12
8*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e
 - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3
*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*
b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^
3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i + ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^
```

6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*
a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3
*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c
^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8*d
^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*
c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4
*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d
^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2
- 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 512
00*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 1024
0*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a
^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a
^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6
*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c
^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*
d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c
^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*
b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a
^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12
*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b
*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^
4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*
e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8
*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e
^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 4
0*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*
b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(
512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^
3)))^(1/4)*1i)/(((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 +
 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3
 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d
e^3(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 2
00*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a
^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^
9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*
d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b

```
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^
3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c
^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^
6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5
*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*
e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 +
4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^
12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14
*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^
7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14
*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4
*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5
- 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 +
4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*
a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^
4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2
*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3
*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 32
0*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
b^2*c^3)))^(1/4) - ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(
1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^
4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*
e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*
b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3
- 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 28
8*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256
*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a
^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 +
16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3
+ 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 20
0*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^
5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9
*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16
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*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*
d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e
^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e
 - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)
 - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816
*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a
^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12
*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a
^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x
*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e
^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 +
44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5
*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 -
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4
- 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*
c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*
d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*
a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4) + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^
3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 +
6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^1
2*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/
2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
+ 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^
3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*
a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a
^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - 2*at
an((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^
2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^
5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*
a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*
c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c
^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2
*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8
*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a
^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^
5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3
- 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*
c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b
^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240
*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a
^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*
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d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d
*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^
2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 102
4*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 8192
0*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768
*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^
14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16
*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^
5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*
e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2
*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*
d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*
b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a
^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12
*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b
*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^
4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*
e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8
*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e
^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 4
0*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*
b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(
512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^
3)))^(1/4) + ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16
*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*
a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*
b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^
8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4
d^4(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6
*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a
^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*
b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^
3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d
^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 +
51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 1
0240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 204
8*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i -
4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^
12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14
*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^

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7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14
*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i - x
*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e
^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 +
44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5
*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4
- 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*
c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*
d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*
a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4))/(((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c
*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3
*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*
b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a
^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e
^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e -
 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 +
256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(409
6*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e
^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b
*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288
*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*
a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a
^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c
^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^
4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*
d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d
*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3
- 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 -
 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 76
8*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 2
4576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^
2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^1
4*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^
6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d
^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^
5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3
- 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*
c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b
^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240
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*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a
^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*
d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d
*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^
2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*
a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 -
 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e +
 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^
3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2
- 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a
^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2
*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))
)^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^
6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*
a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3
*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c
^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*
d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 +
 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 -
10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 20
48*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i -
 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*
b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b
^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^
7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c
^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*
b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^
5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32
*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*
a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*
d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^
3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + 2*a^14*c^5*e^7 + 2*a^11*c^8*d
^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2
*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^1
1*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^
4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
```

4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*
d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*
c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a
*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d
^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e
 + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*
a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^
2*c^3)))^(1/4) - 2*atan((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^
3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a
^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*
d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*
e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8
 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(
x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e
^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c
^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2
*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*
d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^
2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^
2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*
b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 327
68*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^
14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^1
5*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b
^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^1
5*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d
^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^
3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e +
 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2
+ 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d
*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*
a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2
*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^
7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4
*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e
^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b
^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*
a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4
8*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5
*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^
2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a
c - b^2)^5)^(1/2))/(512(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4
*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*

```
a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 -
 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e +
 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^
3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2
+ 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a
^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2
*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))
)^(3/4)*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a
^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128
*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e
- 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*
a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b
^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3
*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1
/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 1228
8*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024
*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304
*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^
15*b^3*c^6*d*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*
b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b
^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^
7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c
^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*
b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^
5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32
*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*
a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*
d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^
3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9*d^4 + a^4*b^5*e^4 - a^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^
4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2
*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3
*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 32
0*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
```

b^2*c^3)))^(3/4)*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a

```
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 122
88*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 102
4*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 9830
4*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a
^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*
c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c
^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*
e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d
^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^
5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5
*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11
*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^1
1*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^
2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16
*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*
a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*
b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + 2*a^14*c
^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^1
1*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^1
3*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 +
 a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6
*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a
^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 -
 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*
b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3
*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^
2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7
*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - d/(a*x)
```

sympy [**F(-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.50 $\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

```
[Out] -1/2*d/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2)-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)
*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1490, 1281, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(2*a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^
2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
```

```
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex^2}{x^2\left(a+bx^2+cx^4\right)}\,dx, x, x^2\right)$$

$$= -\frac{d}{2ax^2} - \frac{\text{Subst}\left(\int \frac{bd-ae+cdx^2}{a+bx^2+cx^4}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \cdots}{\cdots}$$

$$= -\frac{d}{2ax^2} - \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b+\sqrt{b^2\cdots}}}\right)}{2\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.45

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^6c+\#1^2b}\&\right]}{4a} - \frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

fricas [B] time = 2.63, size = 2772, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2
- 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d
^3*e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 -
 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2
 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5
*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 +
 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (
a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*
c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 +
 (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-
(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2
*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
```

```
 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^
2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a
^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2
*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b
^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*
c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3
 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
+ sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3
*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*
e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13
*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 -
4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*
c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*
(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c
)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3
*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b
^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*
a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*
c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 -
 a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*
c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*
b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c
+ 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 -
(b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2
- a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*
d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - 2
*d)/(a*x^2)
```

giac [B] time = 22.52, size = 3006, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*
c^3)*d*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqr
```

```
t(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*
a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
- 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2
*c^2)*abs(a)*e + (2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2)*d - (2*a^2*b^3*c^2 - 8
*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2
- 4*a*c)*a^2*b*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 - 4*
a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*
b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) - 1/8*((sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3
- 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c
^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(a) - (2*a*b
^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c
)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 16*
a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 32*a^3*c^
3 - 2*(b^2 - 4*a*c)*a*b^2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(a)*e - (2*a*b^4*
c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
2*(b^2 - 4*a*c)*a*b^2*c^2)*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*e)*arc
tan(2*sqrt(1/2)*x^2/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^2*b^4
- 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^
3*c^3)*abs(a)*abs(c)) - 1/2*d/(a*x^2)
```

maple [B] time = 0.02, size = 365, normalized size = 1.83

$$\frac{\sqrt{2}\, bcd \arctan\text{h}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\, \sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, bcd \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\, \sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{\sqrt{2}\, ce \arctan\text{h}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+\cdots}\right.}}\right)}{2\sqrt{-4ac+b^2}\, \sqrt{\left(-b+\sqrt{-4ac\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*d-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x^2)*e+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b*d-1/4/a*c*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x^2)*d-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*e+1/4/a*
c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b*d-1/2/a*d/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^2
)
```

mupad [B] time = 7.62, size = 15013, normalized size = 75.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b
^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(3
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 +
 a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^
12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*
c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e -
8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^
2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2
```

```
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2
 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e +
4096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a
^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7
*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e
- 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2
))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d
^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 +
 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*
(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*
d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*
e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i
- ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
 - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^
3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4
*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6
*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d
- 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a
^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^
2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d
*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 307
2*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a
^12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^
7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 +
 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*
a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(
b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d
*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 6
4*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a
^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^1
1*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^
3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 +
a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/(((-(
```

b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*

```
b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*
d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e
) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^
8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c
^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^
2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b
^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
+ 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d
^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e
 - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11*c^6*e^5 - 8*a
^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c
^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e
^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5*d^2 + a^2*b^3*e
^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i -
 atan((((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^
2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*
a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^
4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32
*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 -
a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^1
2*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c
^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8
192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2
 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*
a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2
- 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4
096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^
12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*
d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e -
 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2)
)*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^
2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^
4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 +
128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(
8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d
^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
```

12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i -
((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^
2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
- 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3
*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*
c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d -
1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^
12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2
*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2
) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072
*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^
12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7
*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 +
192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a
^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b
^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64
*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^
10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11
*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3
- 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/(((-(b
^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*
b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d
*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
- 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*
c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1
/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 -
32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c^5*d - 1024*
a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4
*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +

12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 40
96*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*
b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2
*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2
 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^
11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^
3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2
 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*
a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b
^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c
^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(8*a^11*c^6*e
^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*
a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2
*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^
3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + ((-(b^5*d^2 +
a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(
1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2
 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a
^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4
*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 1
6*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^1
3*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*
c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e +
16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*
e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b
*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*d
^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e)
 - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^8
*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c^
5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^2
*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b^
3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(
-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) +
 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d^
4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e
- 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11*c^6*e^5 - 8*a^
9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c^
6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*
a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^

```
2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5*d^2 + a^2*b^3*e^
2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i -
d/(2*a*x^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=394

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\ldots\right.}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-\ldots\right.}$$

```
[Out] -1/3*d/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(
1/4))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^
(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+
1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*
a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b
*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] time = 0.63, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1504, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\ldots\right.}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(3*a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/
4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[
b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(
-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{3ax^3}-\frac{\int \frac{3(bd-ae)+3cdx^4}{a+bx^4+cx^8}\,dx}{3a}$$

$$= -\frac{d}{3ax^3}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}$$

$$= -\frac{d}{3ax^3}+\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{-b-\sqrt{b^2-}}}}{2a\sqrt{-b-\sqrt{b^2-4}}}$$

$$= -\frac{d}{3ax^3}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}}{\sqrt[4]{-b+\sqrt{b^2}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.07, size = 86, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]+\frac{4d}{x^3}}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/12*((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e
*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.01, size = 68, normalized size = 0.17

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)^{4} c d+a e-b d\right) \ln \left(-\operatorname{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)+x\right)}{4 a\left(2 \operatorname{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)^{7} c+\operatorname{RootOf}\left(\_Z^{8} c+\_Z^{4} b+a\right)^{3} b\right)}-\frac{d}{3 a x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x)
[Out] -1/3/a*d/x^3+1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=R
ootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 10.22, size = 65350, normalized size = 165.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x)
[Out] atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b
^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86
*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^
3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^
5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3
- 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3
*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^
4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2
) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4
*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*
e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^
3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^
4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c
```

```
- b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
- 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3
*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c
*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 29
2*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^
5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b
^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4
 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c
^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d -
 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 19660
8*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a
^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13
*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14
*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*
c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^
2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3
*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^
2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d
*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 7
8*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a
^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b
^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^
5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13
*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^
3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^
13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^
2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c
^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*
c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3
*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7
*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^
7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) +
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-
```

(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(1/4)*1i - ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3
d^4(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^
2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d
*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 7
8*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a
^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b
^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b
*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c
^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2
80*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c
*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*
b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^
3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*
b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6
40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*
a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a
^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^
6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e -
196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49
152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464
*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264
*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12
*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^
15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-
(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*
d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^
3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e +
 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^
6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*
e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e -
200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*
a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c

^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b

^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5
*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2
- 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 +
62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 -
11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048
*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 1638
40*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7
*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
- a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^
3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*
a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a
^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2
*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3
*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
+ 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^1
1*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5
+ 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 2
88*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^
3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 1
12*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x
*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2
+ 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 1
6*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*
a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^
4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11
*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*
a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
- 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5
)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2
*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5
*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480
*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b
^2*c^3)))^(1/4) + ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^
4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 -
a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e
+ 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4
*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1
/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*
b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^

3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)

^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i + atan((((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^

(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^1
1*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^
14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^1
5*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6
*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4
+ a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a
^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^
3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^
(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b
^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c
^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 68
0*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a
^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e -
 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*
c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*
b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*
b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b
*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6
 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*
c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b
^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d
^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4
+ a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e
- 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^
3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*
a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2
*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3
*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^
3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^
2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c
^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 1
6*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11*d^4
+ a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a
^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^
3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^
(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b
^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c

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^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 68
0*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a
^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d
*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3
*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e -
6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^
2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6
*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e
 - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 2
00*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a
^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^
8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^
7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e +
262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 102
4*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 1228
80*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 4096
0*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^1
3*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11
*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 -
 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d
^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*
a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*
b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2
 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1
0*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^
4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*
a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*
a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16
*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a
^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a^10*c^1
0*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9
*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a
^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*
c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(
```

4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7* c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^

4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d
^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6
*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*
a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^
2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^
3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^
6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*
a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16
*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c
^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^
2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2
 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2
 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24
576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*
(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12
*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5
 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^
2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e
^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e
- 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a
^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4
+ 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3
 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a
^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^
7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^
6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4
+ 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*
(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^
9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a
^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5
*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d
^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a
^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1
/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e
- 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 3
20*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3

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*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*
a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b
^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^
4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^
2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c -
b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 +
 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a
^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(
4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3
*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*
e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3
 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*
b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*
a^10*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3
*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d
^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a
^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^
3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*
c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 2
92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a
^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*
b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4
 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*
a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a
^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)
^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*
b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*
c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c -
 b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*
(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 6
80*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*
a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^
2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^
5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e +
 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i + x
*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 133
12*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 10
24*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 655
36*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a
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^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6
*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a
^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a
^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4
 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4
*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b
^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*
a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^
5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*
a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(
1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e
 - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 +
320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^
3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256
*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i +
64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*
d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64
*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3
+ 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e +
48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 6
4*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*
c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 +
 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^
4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^1
1*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1
/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4
*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a
^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e
+ 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b
^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3
*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e -
 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16
*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*
e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11*d^4 + a^4*b^7*e^4 +
b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 4
8*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 12
8*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*
d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*
a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720
*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((
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(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*
a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*
b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^
7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^
4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c
^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^
6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e
 - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^
7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) +
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a
^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d
^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c
^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c
^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*
d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 +
 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^
7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2
*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5
*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 -
a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e -
128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*
c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2
)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*
b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e
^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c -
b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e
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 + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*
b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*
c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3
*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8
*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/(((-(b^11*d^4 + a^4
*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^
5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8
*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a
^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^
2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^
2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a
*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4
*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*
c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)
))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
+ 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*
d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^
6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2
*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*
a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*
e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78
*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^
5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*
a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880
*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*
a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*
b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d
^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 1
1*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^
5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^
2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*
c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^
5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 48
```

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0*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c -
 b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d
^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128
*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96
*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 1
6*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*
a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10
*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4
*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 +
28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*
b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b
*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c
^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2
80*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c
*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*
b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^
3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*
b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6
40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*
a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(
b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d
^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/
2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c
^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4
+ 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(
-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*
a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^
3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2
*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^
3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3
*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256
*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*
c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*
a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*
d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)
^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10
*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 +
 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 4
8*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*
c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*
d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e
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^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4
*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*
b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 409
6*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^
16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3
*c^6*e + 262144*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c
^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^
6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^
5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e
 + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d
*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2
)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a
^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-
(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*
c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 -
4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b
^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*
c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^
3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
- 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c
^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 -
192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2
*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^
7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b
*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^
8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^
7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c
^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*
c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*
d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7
*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*
e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e
^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b
^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a
^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^
2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^
2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(
1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4
*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*
d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7
*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(
1/4)*1i))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 11
2*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 +
 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d
^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6
*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*
```

b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a
*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e
^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*
a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5
*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(
1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + 2*atan((((-(b^11*d^4 + a^4*b^7*e^4 -
b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 4
8*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 12
8*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*
d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*
a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720
*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
+ 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((
(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*
a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*
b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^
7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^
4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c
^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^
6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e
- 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^
7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +

5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*

a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*
e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78
*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^
5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*
a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880
*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*
a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*
b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d
^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 1
1*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
- 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^
5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^
2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*
c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^
5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 48
0*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d
^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128
*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96
*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 1
6*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*
a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^7*e^6 - 8*a^10
*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4
*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 +
28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*
b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b
*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c
^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2
80*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c
*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*
b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^
3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*
b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
- 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6
40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*
a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/(((-(b^1
1*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4
- 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2)

- 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*
d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^
2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6
*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4
*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d
e^3(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^
2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^
2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^
10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7
*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
+ a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^
3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*
a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a
^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2
*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3
*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
- 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a
^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*
b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^
6*e + 262144*a^17*b*c^7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*
e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d
^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e
^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e +
24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e)
)*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*
c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*
b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*
a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2
*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3
e(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a
^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*
c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2
*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 1
8*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2
- 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192
*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^
6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d
^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^
7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d

^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e ^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8* d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7 *d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e ^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^ 4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5* c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4* b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/ 2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a *b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^ 2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^ 3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2 *e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a *c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^ 8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112* a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 8 6*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4 *(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b ^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^ 2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^ 2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b ^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^ 3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b ^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c *d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/ 2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a *c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^ 4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3 *e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d ^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a ^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a* c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4* d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^ 3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4 *a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8* c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 2 92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a ^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6* b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2* d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1 /2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^ 4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17* c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 1966 08*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 491 52*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464* a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*

a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*
b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^1
5*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d
*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3
*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e -
6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^
2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6
*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e
 - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 2
00*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a
^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b
*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 -
16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*
a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 -
 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*
a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x
*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2
 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 1
6*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*
a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^
4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11
*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*
a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
- 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5
)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2
*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5
*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480
*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b
^2*c^3)))^(1/4)*1i))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*
c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
+ a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3
*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a
^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^
4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*
d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*
e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 -
 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c

```
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*
c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - d/(3*a*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{x^4(1-x^4)}{1-x^4+x^8} dx$

Optimal. Leaf size=278

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

```
[Out] -x-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x
^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^
(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1502, 1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcT
an[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[
2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[
3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 -
Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^4\left(1-x^4\right)}{1-x^4+x^8}\,dx = -x+\int \frac{1}{1-x^4+x^8}\,dx$$

$$= -x+\frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -x+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+$$

$$= -x-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}$$

$$= -x-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}$$

$$= -x-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

fricas [A] time = 0.96, size = 218, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) -
 x
```

giac [A] time = 0.45, size = 208, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

maple [C] time = 0.01, size = 34, normalized size = 0.12

$$-x+\frac{\text{RootOf}\left(9\_Z^4+1\right)\ln\left(3\,\text{RootOf}\left(9\_Z^4+1\right)^2+3\,\text{RootOf}\left(9\_Z^4+1\right)x+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x+\int\frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.92, size = 56, normalized size = 0.20

$$-x+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] - x - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/
12) - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/
12)
```

sympy [A] time = 0.23, size = 170, normalized size = 0.61

$$-x-\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}\,x^3-4x^2+2\sqrt{6}\,x-3\right)\right)}{24}-\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}\,x^3+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2
*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*
x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*
x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(
6)*x + 1)/24
```

3.53 $\int \frac{x^3\left(1-x^4\right)}{1-x^4+x^8} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

[Out] -1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) - \frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)$$
$$= -\frac{1}{8}\log\left(1-x^4+x^8\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

fricas [A] time = 0.89, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

giac [A] time = 0.63, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

maple [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^4+1)/(x^8-x^4+1),x)
[Out] -1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.51, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

sympy [A] time = 0.15, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

3.54 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(\right.$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqr
t[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[
3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\int \frac{x^2(\sqrt{3}-2x^2)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2(\sqrt{3}+2x^2)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{2\sqrt{2-\sqrt{3}}-(2-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3(2-\sqrt{3})}} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+(2-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3(2-\sqrt{3})}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-(-2-\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3(2+\sqrt{3})}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}\ \cdots}{1+\sqrt{2+\cdots}}}{4\sqrt{3(2\cdots}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\ \cdots$$

$$= \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\cdots$$

$$= \frac{1}{4}\sqrt{\frac{1}{3}(2+\sqrt{3})}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}(2-\sqrt{3})}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\cdots$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2
*#1^5) & ]
```

fricas [B] time = 1.19, size = 715, normalized size = 2.01

$$\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(12x^2+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12\right)-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/12*sqr
t(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
 - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
```

```
 + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

giac [A] time = 0.48, size = 253, normalized size = 0.71

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1)
```

maple [C] time = 0.01, size = 46, normalized size = 0.13

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{6}-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{2}\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{7}-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(x^{4}-1\right)x^{2}}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.99, size = 248, normalized size = 0.70

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) - (3
^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/
4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4)) - (2^
(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/
12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)) + (2^(
3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [**A**] time = 3.16, size = 27, normalized size = 0.08

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(442368t^7 - 384t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 - 3
84*_t**3 + x)))
```

3.55 $\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx$

Optimal. Leaf size=50

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[Out] -1/12*ln(1+x^4-3^(1/2)*x^2)*3^(1/2)+1/12*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1490, 1164, 628}

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqr
t[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(1-x^4\right)}{1-x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}\\
&= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.88

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)-\log\left(-x^4+\sqrt{3}x^2-1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] (-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])
```

fricas [A] time = 1.06, size = 41, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5x^4+2\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1))
```

giac [A] time = 0.44, size = 31, normalized size = 0.62

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

maple [A] time = 0.02, size = 39, normalized size = 0.78

$$-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+1)/(x^8-x^4+1),x)
[Out] -1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(x^4-1\right)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.89, size = 20, normalized size = 0.40

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x^2}{x^4+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atanh((3^(1/2)*x^2)/(x^4 + 1)))/6
```

sympy [A] time = 0.13, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12
```

3.56 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(\right.$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.22, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3]
)]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}}{1+\sqrt{2+}}}{4\sqrt{3\left(2\right.}}$$

$$= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{}$$

$$= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]
```

fricas [B] time = 0.92, size = 715, normalized size = 2.01

$$\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(12x^{2}+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12\right)-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/12*sqr
t(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
 - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
 + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

giac [A] time = 0.54, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.12

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{4}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{7}-4\,\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.00, size = 208, normalized size = 0.59

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}+\frac{2^{3/4}\,\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4
))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3
^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/
4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2
)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.23, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(9216t^5-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

3.57 $\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^4\right)\\
&= \log(x)-\frac{1}{4}\,\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) & ]/4
```

fricas [A] time = 0.79, size = 34, normalized size = 0.83

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + lo
g(x)
```

giac [A] time = 0.45, size = 38, normalized size = 0.93

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 0.97, size = 38, normalized size = 0.93

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

mupad [B] time = 1.89, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

sympy [A] time = 0.16, size = 41, normalized size = 1.00

$$\log(x)-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

3.58 $\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=280

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

```
[Out] -1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2))
)*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1
+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*
2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1504, 1372, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) +
 ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 +
Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*
x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x +
 x^2]/(4*Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{}}}{1+\sqrt{}}}{4\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x}}{4\sqrt{6}}$$

$$= -\frac{1}{x}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x\right.}{4\sqrt{6}}$$

$$= -\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{}}}\right.}{2\sqrt{6}}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/
4
```

fricas [A] time = 0.97, size = 224, normalized size = 0.80

$$4\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3\,x^2+1}\left(\sqrt{3}\sqrt{2}\,x-2\right)}{3\,x^2-2}\right)+4\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x
^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2
 - 2)) + 4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqr
t(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*
x^2 - 2)) + sqrt(3)*sqrt(2)*x*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 +
 1) - sqrt(3)*sqrt(2)*x*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) -
24)/x
```

giac [A] time = 0.58, size = 210, normalized size = 0.75

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

maple [C] time = 0.01, size = 38, normalized size = 0.14

$$-\frac{\text{RootOf}\left(9\_Z^4+1\right)\ln\left(9\,\text{RootOf}\left(9\_Z^4+1\right)^3x-3\,\text{RootOf}\left(9\_Z^4+1\right)^2+x^2\right)}{4}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.86, size = 58, normalized size = 0.21

$$-\frac{1}{x}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
1/x
```

sympy [A] time = 0.23, size = 168, normalized size = 0.60

$$-\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3-4x^2+2\sqrt{6}\,x-3\right)\right)}{24}-\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3+4x^2+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

```
[Out] -sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
 sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
 1)/24 - 1/x
```

3.59 $\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-
3^(1/2)*x^2)*3^(1/2)+1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1490, 1281, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1
- Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x^2}{x^2\left(1-x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx,x,x^2\right)-\\
&= -\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\right.\\
&= -\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1-\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.55

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4) &
 ]/4
```

fricas [B] time = 0.84, size = 188, normalized size = 2.11

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^2\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^2\arctan\left(-\frac{1}{3}\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
sqrt(2)*x^2*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(6)*sqrt(2)*x^2*log(
2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) - 24)/x^2
```

giac [A] time = 0.53, size = 81, normalized size = 0.91

$$-\frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right)-\frac{1}{4}x^4\arctan\left(2x^2+\sqrt{3}\right)-\frac{1}{4}x^4\arctan\left(2x^2-\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/4*x^4*arctan(2*x^2 + sqrt(3)) - 1/4*x^4*arctan(2*x^2 -
sqrt(3)) - 1/2/x^2
```

maple [A] time = 0.01, size = 70, normalized size = 0.79

$$-\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{24}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^3/(x^8-x^4+1),x)
```

```
[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*3^(1/2)*l
n(x^4-3^(1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2x^2}-\int\frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.10, size = 56, normalized size = 0.63

$$\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^3*(x^8 - x^4 + 1)),x)
```

```
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - 1/(2*x^2)
```

sympy [A] time = 0.23, size = 76, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - 1/(2*x**2)
```

3.60 $\int \frac{1-x^4}{x^4(1-x^4+x^8)} \, dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}$$

```
[Out] -1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)
))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*
6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)
+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^
(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))
/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((2*x+1/2*6
^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/
8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/8*ln(1+
x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A] time = 0.27, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3}-\frac{1}{3}\int \frac{3x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$
$$= -\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}$$
$$= -\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}}{8\sqrt{3}}$$
$$= -\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x\right.}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$
$$= -\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}\,\cdot}{\sqrt{2-\sqrt{}}}\right.}{4\sqrt{3\left(2-\sqrt{}\right.}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.13

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]
/4
```

fricas [B] time = 1.13, size = 608, normalized size = 1.64

$$\frac{8\sqrt{6}\sqrt{2}x^3\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\frac{1}{6}\sqrt{6}\sqrt{2}\sqrt{2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12x^2+12}\sqrt{\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/96*(8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*s
qrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqr
t(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) +
8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)
*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*
x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 4*sqr
t(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x
*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x
*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) -
4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt
```

```
(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqr
t(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) +
 2) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*log
(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) + 2*sqrt(6)*(
sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(
3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - sqrt(6)*(sqrt(3)*sqrt(2)*x^
3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(
-4*sqrt(3) + 8) + 12*x^2 + 12) + sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x
^3)*sqrt(-4*sqrt(3) + 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8
) + 12*x^2 + 12) - 32)/x^3
```

giac [A] time = 0.44, size = 258, normalized size = 0.70

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/3/x^3
```

maple [C] time = 0.01, size = 46, normalized size = 0.12

$$-\frac{\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3\right)}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^4/(x^8-x^4+1),x)
```

```
[Out] -1/3/x^3-1/4*sum(1/(2*_R^7-_R^3)*_R^4*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3\,x^3}-\int\frac{x^4}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.07, size = 479, normalized size = 1.29

$$-\frac{1}{3\,x^3}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{\right.}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}}}{4}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - 1/(3*x^3) + (3^(1/2)*atan((x*(8 - 3^(1/2)*8i
)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1
/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^
(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (2^(
3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*
1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2^(3/
4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2)
)/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/
4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*
((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/
2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/
2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(
1/2)*1i + 1)^(1/4))/12
```

sympy [**A**] time = 3.17, size = 32, normalized size = 0.09

$$-\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4
*_t + x))) - 1/(3*x**3)
```

3.61 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=280

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2\right.}{a^4\sqrt{b^2-4ac}\,\left(a\right.}$$

```
[Out] (a^2*d^2+b^2*e^2+a*e*(b*d-c*e))*x/a^3/e^3-1/2*(a*d+b*e)*x^2/a^2/e^2+1/3*x^3
/a/e-d^5*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a*b
*c^2*e+b^4*d-b^3*c*e)*ln(a*x^2+b*x+c)/a^4/(a*d^2-e*(b*d-c*e))+(5*a^2*b*c^2*
d-2*a^2*c^3*e-5*a*b^3*c*d+4*a*b^2*c^2*e+b^5*d-b^4*c*e)*arctanh((2*a*x+b)/(-
4*a*c+b^2)^(1/2))/a^4/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(a^2c^2d-3ab^2cd+2abc^2e-b^3ce+b^4d\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e+4ab^2c^2e-5ab^3cd-b^4ce+b^5\right.}{a^4\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*
a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e +
4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2
- e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2
*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx=\int \frac{x^5}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\int \left(\frac{a^2d^2+b^2e^2+ae(bd-ce)}{a^3e^3}-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{ae}+\frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)(d+ex)}\right.$$

$$=\frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\int \frac{c\left(b^3 a\right.}{}$$

$$=\frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^4d-\right.}{}$$

$$=\frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^4d-\right.}{}$$

$$=\frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}+\frac{\left(b^5d-5ab^3cd+5a^2bc^2d-b^4\right.}{a^4\sqrt{b^2-4}}$$

Mathematica [A] time = 0.23, size = 283, normalized size = 1.01

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-bde+ce^2\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e\right.}{a^4\sqrt{4ac-b^2}\left(-ad^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(
2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e
+ 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^4
*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^5*Log[d + e*x])/(e^4*(
a*d^2 - b*d*e + c*e^2)) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a
*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))
```

fricas [A] time = 80.22, size = 1027, normalized size = 3.67

$$\left[-\frac{6\left(a^4b^2-4a^5c\right)d^5\log(ex+d)-2\left(\left(a^4b^2-4a^5c\right)d^2e^3-\left(a^3b^3-4a^4bc\right)de^4+\left(a^3b^2c-4a^4c^2\right)e^5\right)x^3+3\left(\left(a^4b^2-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*
e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((
a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a
^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 + 3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4
 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^5)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2
+ 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6), -1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*
x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^
3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 -
 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 - 6*((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^
5)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6)]
```

giac [A] time = 0.38, size = 295, normalized size = 1.05

$$-\frac{d^5\log\left(|xe+d|\right)}{ad^2e^4-bde^5+ce^6}+\frac{\left(b^4d-3\,ab^2cd+a^2c^2d-b^3ce+2\,abc^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^5d^2-a^4bde+a^4ce^2\right)}-\frac{\left(b^5d-5\,ab^3cd+5\,a^2bc^2d-\cdots\right.}{\left(a^5d^2-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^5*log(abs(x*e + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d - 3*a*b^2
*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*log(a*x^2 + b*x + c)/(a^5*d^2 - a
^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4*
a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*d^2
 - a^4*b*d*e + a^4*c*e^2)*sqrt(-b^2 + 4*a*c)) + 1/6*(2*a^2*x^3*e^2 - 3*a^2*
d*x^2*e + 6*a^2*d^2*x - 3*a*b*x^2*e^2 + 6*a*b*d*x*e + 6*b^2*x*e^2 - 6*a*c*x
*e^2)*e^(-3)/a^3
```

maple [B] time = 0.01, size = 662, normalized size = 2.36

$$-\frac{5b\,c^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{2c^3e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{5b^3cd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^3}-\frac{4b^2\cdots}{\left(a\,d^2-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/3*x^3/a/e-1/2/a/e^2*x^2*d-1/2/a^2/e*x^2*b+1/a/e^3*d^2*x+1/a^2/e^2*b*d*x-1
/a^2/e*c*x+1/a^3/e*b^2*x+1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*d-
3/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^2*c*d+1/(a*d^2-b*d*e+c*e^2)/a
^3*ln(a*x^2+b*x+c)*b*c^2*e+1/2/(a*d^2-b*d*e+c*e^2)/a^4*ln(a*x^2+b*x+c)*b^4*
d-1/2/(a*d^2-b*d*e+c*e^2)/a^4*ln(a*x^2+b*x+c)*b^3*c*e-5/(a*d^2-b*d*e+c*e^2)
/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d+2/(a*d^2
-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3
```

```
*e+5/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)
^(1/2))*b^3*c*d-4/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*b^2*c^2*e-1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*d+1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-
b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*c*e-1/e^4*d^5/(a*d^2-b*d
*e+c*e^2)*ln(e*x+d)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 6.21, size = 2490, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(4*a^5*c*d^7 - a^4*b^2*d^7 + b^3*c^3*e^7 - b^6*d^3*e^4 - 6*a^2*c^4*d*e^
6 - 3*b^4*c^2*d*e^6 + 3*b^5*c*d^2*e^5 - 2*a^2*c^4*e^7*x - b^2*c^3*e^7*(b^2
- 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 2*a^3*c^3*d^3*e^4 - 4*a^
4*c^2*d^5*e^2 - 3*a*b*c^4*e^7 + a^4*b*d^7*(b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(
b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b
^2 - 4*a*c)^(1/2) + 8*a^5*c*d^6*e*x - 9*a^2*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e
*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^3*d*e^6 + 6*a*b^4*c*d^3*e^4 + a*b^2*c^3*e
^7*x - a*b^5*d^3*e^4*x - 2*a^4*b^2*d^6*e*x + 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^
(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 15*a*b^3*c^2*d^2*e^5 + 15*a^2
*b*c^3*d^2*e^5 + a^3*b^2*c*d^5*e^2 + a^3*b^3*d^5*e^2*x + 6*a^3*c^3*d^2*e^5*
x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(
1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c
)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^3*c*d^3*e^4*x - 5
*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^
2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3
*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*a^2*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d
*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*
e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c^2*d*e^6*x + 3*a*b^4*c*d^2*e^5*x + 9*a^2
*b*c^3*d*e^6*x - 4*a^4*b*c*d^5*e^2*x + 3*a*b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1
/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2
- 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^5*d*(b^2 - 4
*a*c)^(1/2) - b^6*d + 4*a^3*c^3*d + b^5*c*e - 13*a^2*b^2*c^2*d + 7*a*b^4*c*
d - b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*e + 8*a^2*b*c^3*e - 2*a^2*c^3
*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*d*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^2*e*
(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a
^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2 + a^4*b^3*d*e - 4*a^5*b*c*d*e))
- (d^5*log(d + e*x))/(c*e^6 + a*d^2*e^4 - b*d*e^5) - x*((b*d + c*e)/(a^2*e^
2) - (a*d + b*e)^2/(a^3*e^3)) + (log(a^4*b^2*d^7 - 4*a^5*c*d^7 - b^3*c^3*e^
7 + b^6*d^3*e^4 + 6*a^2*c^4*d*e^6 + 3*b^4*c^2*d*e^6 - 3*b^5*c*d^2*e^5 + 2*a
^2*c^4*e^7*x - b^2*c^3*e^7*(b^2 - 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^
(1/2) - 2*a^3*c^3*d^3*e^4 + 4*a^4*c^2*d^5*e^2 + 3*a*b*c^4*e^7 + a^4*b*d^7*(
b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a
*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 8*a^5*c*d^6*e*x + 9*a^2
*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 12*a*b^2*c^3*d*e^6 -
```

```
 6*a*b^4*c*d^3*e^4 - a*b^2*c^3*e^7*x + a*b^5*d^3*e^4*x + 2*a^4*b^2*d^6*e*x
+ 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2)
 + 15*a*b^3*c^2*d^2*e^5 - 15*a^2*b*c^3*d^2*e^5 - a^3*b^2*c*d^5*e^2 - a^3*b^
3*d^5*e^2*x - 6*a^3*c^3*d^2*e^5*x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) +
 a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2)
- 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(
1/2) - 5*a^2*b^3*c*d^3*e^4*x + 5*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*
(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5
*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 12*a^2
*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b
^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^3*c^2*d*e^6
*x - 3*a*b^4*c*d^2*e^5*x - 9*a^2*b*c^3*d*e^6*x + 4*a^4*b*c*d^5*e^2*x + 3*a*
b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/
2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2
 - 4*a*c)^(1/2))*(4*a^3*c^3*d - b^5*d*(b^2 - 4*a*c)^(1/2) - b^6*d + b^5*c*e
 - 13*a^2*b^2*c^2*d + 7*a*b^4*c*d + b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c
^2*e + 8*a^2*b*c^3*e + 2*a^2*c^3*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*d*(b^2
 - 4*a*c)^(1/2) - 4*a*b^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*d*(b^2 - 4*
a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2
+ a^4*b^3*d*e - 4*a^5*b*c*d*e)) + x^3/(3*a*e) - (x^2*(a*d + b*e))/(2*a^2*e^
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.62 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(-2abcd+ac^2e+b^3d-b^2ce\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{x(ad+be)}{a^2e^2}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)\tanh^{-1}\left(\right.}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}$$

```
[Out] -(a*d+b*e)*x/a^2/e^2+1/2*x^2/a/e+d^4*ln(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))-1/2*
(-2*a*b*c*d+a*c^2*e+b^3*d-b^2*c*e)*ln(a*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))-
(2*a^2*c^2*d-4*a*b^2*c*d+3*a*b*c^2*e+b^4*d-b^3*c*e)*arctanh((2*a*x+b)/(-4*a
*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2abcd+ac^2e-b^2ce+b^3d\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e-b^3ce+b^4d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-x$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*Log[d + e*x])/(e^3*(a*d^
2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x
+ a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^4}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{-ad-be}{a^2e^2}+\frac{x}{ae}+\frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-c\left(b^2d-acd-bce\right)-\left(b^3\right.}{a^2\left(ad^2-e(bd-c\right.}\right.$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-c\left(b^2d-acd-bce\right)-\left(b^3d-2abcd-b^2ce+ac^2e\right)}{c+bx+ax^2}}{a^2\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)\int \frac{b+2}{c+bx}}{2a^3\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)\log\left(c\right.}{2a^3\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}-\frac{\left(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A] time = 0.17, size = 218, normalized size = 1.00

$$\frac{\left(2abcd-ac^2e+b^3(-d)+b^2ce\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)}-\frac{x(ad+be)}{a^2e^2}+\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)\tan^{-}}{a^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3
*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))) + (d^4*Log[d + e*x])/(e^3*(
a*d^2 + e*(-(b*d) + c*e))) + ((-(b^3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e)*Lo
g[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e)))
```

fricas [A] time = 35.52, size = 798, normalized size = 3.66

$$\left[\frac{2\left(a^3b^2-4\,a^4c\right)d^4\log\left(ex+d\right)+\left(\left(a^3b^2-4\,a^4c\right)d^2e^2-\left(a^2b^3-4\,a^3bc\right)de^3+\left(a^2b^2c-4\,a^3c^2\right)e^4\right)x^2+\left(\left(b^4-4\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 - 4*a^4*c)*d^2*e^2
 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^4 -
 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*sqrt(b^2 - 4*a*c)*
log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*
x^2 + b*x + c)) - 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d*e^3 + (a*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*
b*c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))
/((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c -
4*a^4*c^2)*e^5), 1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 -
4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
4)*x^2 - 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) -
2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*e^3 + (a
*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d*e^3 - (b^
4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))/((a^4*b^2 - 4*a^5
*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)]
```

giac [A] time = 0.37, size = 224, normalized size = 1.03

$$\frac{d^4\log\left(|xe+d|\right)}{ad^2e^3-bde^4+ce^5}-\frac{\left(b^3d-2\,abcd-b^2ce+ac^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^4d^2-a^3bde+a^3ce^2\right)}+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-b^3ce+3\,abc^2e\right)\mathrm{a}}{\left(a^4d^2-a^3bde+a^3ce^2\right)\sqrt{-b^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^4*log(abs(x*e + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d - 2*a*b*c*
d - b^2*c*e + a*c^2*e)*log(a*x^2 + b*x + c)/(a^4*d^2 - a^3*b*d*e + a^3*c*e^
2) + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*arctan((2*
a*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*d^2 - a^3*b*d*e + a^3*c*e^2)*sqrt(-b^2 +
 4*a*c)) + 1/2*(a*x^2*e - 2*a*d*x - 2*b*x*e)*e^(-2)/a^2
```

maple [B] time = 0.01, size = 512, normalized size = 2.35

$$\frac{2c^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a}-\frac{4b^2cd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{3b\,c^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{b^4d\,\mathrm{arc}}{\left(a\,d^2-deb+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/2*x^2/a/e-1/a/e^2*x*d-1/a^2/e*b*x+1/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+
c)*b*c*d-1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*e-1/2/(a*d^2-b*d*e
+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^3*d+1/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+
c)*b^2*c*e+2/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*c^2*d-4/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d+3/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2
)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*e+1/(a*d^2-b*d*e+c*e^2)/a^3/(4*
a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d-1/(a*d^2-b*d*e+c*e
^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*e+1/e^3
*d^4/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.24, size = 2051, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^4*log(d + e*x))/(c*e^5 + a*d^2*e^3 - b*d*e^4) - (log(4*a^4*c*d^6 - 2*a*c
^4*e^6 - a^3*b^2*d^6 + b^2*c^3*e^6 - b^5*d^3*e^3 - 3*b^3*c^2*d*e^5 + 3*b^4*
c*d^2*e^4 + b^4*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^3*d^2*e^4 - 4*a^3*c^2
*d^4*e^2 + a^3*b*d^6*(b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) +
2*a^4*d^6*x*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 -
4*a*c)^(1/2) + a*b*c^3*e^6*x + 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c
)^(1/2) - 4*a^3*c*d^5*e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/
2) + 5*a*b^3*c*d^3*e^3 - a*b^4*d^3*e^3*x - 2*a^3*b^2*d^5*e*x + 6*a^2*c^3*d*
e^5*x + 3*b^2*c^2*d*e^5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)
^(1/2) - 12*a*b^2*c^2*d^2*e^4 - 5*a^2*b*c^2*d^3*e^3 + a^2*b^2*c*d^4*e^2 + a
^2*b^3*d^4*e^2*x - 2*a^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1
/2) - 3*a*b^2*c*d^3*e^3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)
^(1/2) + a*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a
*c)^(1/2) - 9*a^2*b*c^2*d^2*e^4*x + 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2
*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*
d^5*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c^2*d*e^5*x + 3*a*b^3*c*d^2*e^4*x - 4
*a^3*b*c*d^4*e^2*x + 3*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*
e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d
*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^2*c^3*e + b^4*c*e + 6*a*b^3*c*d - b^3*c*
e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 -
4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^5*c*d^2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3
*b^3*d*e - 4*a^4*b*c*d*e)) + (log(2*a*c^4*e^6 - 4*a^4*c*d^6 + a^3*b^2*d^6 -
 b^2*c^3*e^6 + b^5*d^3*e^3 + 3*b^3*c^2*d*e^5 - 3*b^4*c*d^2*e^4 + b^4*d^3*e^
3*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^4 + 4*a^3*c^2*d^4*e^2 + a^3*b*d^6*(
b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) + 2*a^4*d^6*x*(b^2 - 4*a
*c)^(1/2) - 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) - a*b*c^3
*e^6*x - 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*
e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d^3*e^3
 + a*b^4*d^3*e^3*x + 2*a^3*b^2*d^5*e*x - 6*a^2*c^3*d*e^5*x + 3*b^2*c^2*d*e^
5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^2*
d^2*e^4 + 5*a^2*b*c^2*d^3*e^3 - a^2*b^2*c*d^4*e^2 - a^2*b^3*d^4*e^2*x + 2*a
^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^3*e^
3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) + a*b^3*d^3*e^3
*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 9*a^2*b*c^
2*d^2*e^4*x - 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2)
 + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*d^5*e*x*(b^2 - 4*a*c)^
(1/2) + 3*a*b^2*c^2*d*e^5*x - 3*a*b^3*c*d^2*e^4*x + 4*a^3*b*c*d^4*e^2*x + 3
*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^4*x*(b^2 - 4*a*c)^(1
/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^5*d + b^4*d*(b^2 - 4*a*c)
^(1/2) - 4*a^2*c^3*e - b^4*c*e - 6*a*b^3*c*d - b^3*c*e*(b^2 - 4*a*c)^(1/2)
+ 8*a^2*b*c^2*d + 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2
*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^
2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3*b^3*d*e - 4*a^4*b*c*d
*e)) + x^2/(2*a*e) - (x*(a*d + b*e))/(a^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.63 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=176

$$\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}$$

```
[Out] x/a/e-d^3*ln(e*x+d)/e^2/(a*d^2-e*(b*d-c*e))+1/2*(-a*c*d+b^2*d-b*c*e)*ln(a*x
^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))+(-3*a*b*c*d+2*a*c^2*e+b^3*d-b^2*c*e)*arct
anh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] time = 0.29, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-3abcd+2ac^2e-b^2ce+b^3d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/Sq
rt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*Lo
g[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log[c
+ b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
```

```
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^3}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{ae}+\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{a\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{c+bx+ax^2}\,dx}{a\left(ad^2-bde+ce^2\right)}$$

$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-3abcd-b^2ce\right.}{2a^2\left(ad^2\right.}$$

$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^3d-3abcd\right.}{}$$

$$= \frac{x}{ae}+\frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2\right.}{}$$

Mathematica [A] time = 0.19, size = 178, normalized size = 1.01

$$\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-bde+ce^2\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^2\sqrt{4ac-b^2}\left(-ad^2+bde-ce^2\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-bde+ce^2\right)}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b + 2*a*x)/Sqr
t[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^
3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2)) + ((b^2*d - a*c*d - b*c*e)*Lo
g[c + b*x + a*x^2])/(2*a^2*(a*d^2 - b*d*e + c*e^2))
```

fricas [A] time = 10.87, size = 596, normalized size = 3.39

$$\left[-\frac{2\left(a^2b^2-4a^3c\right)d^3\log(ex+d)-\left(\left(b^3-3abc\right)de^2-\left(b^2c-2ac^2\right)e^3\right)\sqrt{b^2-4ac}\,\log\left(\frac{2a^2x^2+2abx+b^2-2ac+\sqrt{b^2-4ac}\,(2\right.}{ax^2+bx+c}\right.}{2\left(\left(a^3b^2-4a^4c\right)d^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - ((b^3 - 3*a*b*c)*d*e^2 - (b
^2*c - 2*a*c^2)*e^3)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^2*b^2 - 4*a^
3*c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b
^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*
x + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b
^2*c - 4*a^3*c^2)*e^4), -1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - 2*((
b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*((a^2*b^2 - 4*a^3*c)*d^2*e
- (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b^4 - 5*a*b^
2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*x + c))/((a
^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^
3*c^2)*e^4)]
```

giac [A] time = 0.40, size = 185, normalized size = 1.05

$$-\frac{d^3\log\left(|xe+d|\right)}{ad^2e^2-bde^3+ce^4}+\frac{xe^{(-1)}}{a}+\frac{\left(b^2d-acd-bce\right)\log\left(ax^2+bx+c\right)}{2\left(a^3d^2-a^2bde+a^2ce^2\right)}-\frac{\left(b^3d-3\,abcd-b^2ce+2\,ac^2e\right)\arctan\left(\frac{2\,a\ldots}{\sqrt{-b\ldots}}\right.}{\left(a^3d^2-a^2bde+a^2ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^3*log(abs(x*e + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + x*e^(-1)/a + 1/2*(b^
2*d - a*c*d - b*c*e)*log(a*x^2 + b*x + c)/(a^3*d^2 - a^2*b*d*e + a^2*c*e^2)
 - (b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a^3*d^2 - a^2*b*d*e + a^2*c*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [B] time = 0.01, size = 388, normalized size = 2.20

$$\frac{3bcd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a}-\frac{2c^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a}-\frac{b^3d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{b^2ce\,a\ldots}{\left(a\,d^2-deb\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] x/a/e-1/2/(a*d^2-b*d*e+c*e^2)/a*ln(a*x^2+b*x+c)*c*d+1/2/(a*d^2-b*d*e+c*e^2)
/a^2*ln(a*x^2+b*x+c)*b^2*d-1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b*c*
e+3/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1
/2))*b*c*d-2/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*e*c^2-1/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2))*b^3*d+1/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*e-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*l
n(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.34, size = 1367, normalized size = 7.77

$$\frac{x}{a\,e}-\frac{\ln\left(c^3\,e^5\,\sqrt{b^2-4\,a\,c}\;-b\,c^3\,e^5-4\,a^3\,c\,d^5+a^2\,b^2\,d^5+b^4\,d^3\,e^2+3\,b^2\,c^2\,d\,e^4-3\,b^3\,c\,d^2\,e^3-b^3\,d^3\,e^2\,\sqrt{b^2-4\,a\,c}\;\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e) - (log(c^3*e^5*(b^2 - 4*a*c)^(1/2) - b*c^3*e^5 - 4*a^3*c*d^5 + a^2*
b^2*d^5 + b^4*d^3*e^2 + 3*b^2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 - b^3*d^3*e^2*(b^
2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x - a^2*
b*d^5*(b^2 - 4*a*c)^(1/2) - 2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e
*x + 4*a^2*c*d^4*e*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2)
+ 9*a*b*c^2*d^2*e^3 - 5*a*b^2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x - 3*a*c^2*d^2*e
^3*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^
2*e^3*x - 2*a*b^2*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c*d^3*e^2*x*(b^2 -
4*a*c)^(1/2) + 3*a*b*c^2*d*e^4*x + a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 2*a^
2*b*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a
*b^2*c*d^2*e^3*x + a^2*b*c*d^3*e^2*x + 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2
))*(b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d
 + 4*a*b*c^2*e - 2*a*c^2*e*(b^2 - 4*a*c)^(1/2) + b^2*c*e*(b^2 - 4*a*c)^(1/2
) + 3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c
^2*e^2 - a^2*b^2*c*e^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) - (log(a^2*b^2*d^5 -
 b*c^3*e^5 - c^3*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5 + b^4*d^3*e^2 + 3*b^
2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 + b^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2
*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x + a^2*b*d^5*(b^2 - 4*a*c)^(1/2) +
2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e*x - 4*a^2*c*d^4*e*(b^2 - 4*
a*c)^(1/2) + 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^2*d^2*e^3 - 5*a*b^
2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x + 3*a*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 3*b
^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^2*e^3*x + 2*a*b^2*d^3*e^2*x*
(b^2 - 4*a*c)^(1/2) - 3*a^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*e
^4*x - a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d^4*e*x*(b^2 - 4*a*c)^(1
/2) + 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^3*x + a^2*b*c*d
^3*e^2*x - 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d + b^3*d*(b^2 - 4*a
*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d + 4*a*b*c^2*e + 2*a*c^2*e*(
b^2 - 4*a*c)^(1/2) - b^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c^2*e^2 - a^2*b^2*c*e^2 + a^2
*b^3*d*e - 4*a^3*b*c*d*e)) - (d^3*log(d + e*x))/(c*e^4 + a*d^2*e^2 - b*d*e^
3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=149

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

```
[Out] d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)-1/2*(b*d-c*e)*ln(a*x^2+b*x+c)/a/(a*d^2-
e*(b*d-c*e))-(-2*a*c*d+b^2*d-b*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a
/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1445, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d + e*x])/(e*(a*d^2 - b
*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d -
c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1445

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
```

```
, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^2}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{-cd-(bd-ce)x}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} + \frac{\int \frac{-cd-(bd-ce)x}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} - \frac{(bd-ce)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^2d-2acd-bce\right)\int \frac{1}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} - \frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)} - \frac{\left(b^2d-2acd-bce\right)\text{Subst}\left(\int \frac{}{b^2}\right.}{a\left(ad^2-e(bd\right.}$$

$$= -\frac{\left(b^2d-2acd-bce\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} + \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} - \frac{(bd-ce)\log\left(c+bx\right.}{2a\left(ad^2-e(bd-\right.}$$

Mathematica [A] time = 0.12, size = 132, normalized size = 0.89

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ce)\log(x(ax+b)+c)-2ad^2\log(d+ex)\right)+2e\left(2acd+b^2(-d)+bce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2ae\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -1/2*(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c
]] + Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c*e)*Log[c + x*(b
 + a*x)]))/(a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(-(b*d) + c*e)))
```

fricas [A] time = 3.57, size = 405, normalized size = 2.72

$$\left[\frac{2\left(ab^2-4a^2c\right)d^2\log(ex+d)+\left(bce^2-\left(b^2-2ac\right)de\right)\sqrt{b^2-4ac}\log\left(\frac{2a^2x^2+2abx+b^2-2ac+\sqrt{b^2-4ac}(2ax+b)}{ax^2+bx+c}\right)-\left(\left(b^3-\right.\right.}{2\left(\left(a^2b^2-4a^3c\right)d^2e-\left(ab^3-4a^2bc\right)de^2+\left(ab^2c-4a^2c^2\right)e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x + d) + (b*c*e^2 - (b^2 - 2*a*c)*d*e)*
sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c
```

```
)*(2*a*x + b))/(a*x^2 + b*x + c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a*c^2
)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*
c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x
 + d) + 2*(b*c*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^
2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a
*c^2)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^
2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

giac [A] time = 0.37, size = 149, normalized size = 1.00

$$\frac{d^2\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2\left(a^2d^2-abde+ace^2\right)}+\frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2d^2-abde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^2*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*(b*d - c*e)*log(a*x
^2 + b*x + c)/(a^2*d^2 - a*b*d*e + a*c*e^2) + (b^2*d - 2*a*c*d - b*c*e)*arc
tan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*d^2 - a*b*d*e + a*c*e^2)*sqrt(-b^
2 + 4*a*c))
```

maple [A] time = 0.01, size = 275, normalized size = 1.85

$$\frac{b^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a}-\frac{bce\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a}-\frac{2cd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}-\frac{bd\ln\left(a\,x^2+\right.}{2\left(a\,d^2-deb\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)/a*ln(a*x^2+b*x+c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/a*ln
(a*x^2+b*x+c)*c*e-2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/
(4*a*c-b^2)^(1/2))*c*d+1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))/a*b^2*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arc
tan((2*a*x+b)/(4*a*c-b^2)^(1/2))/a*b*c*e+d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.67, size = 966, normalized size = 6.48

$$\frac{d^2\,\ln\left(d+e\,x\right)}{a\,d^2\,e-b\,d\,e^2+c\,e^3}-\frac{\ln\left(a\,b^2\,d^4-2\,c^3\,e^4-4\,a^2\,c\,d^4+b^3\,d^3\,e+c^2\,e^4\,x\,\sqrt{b^2-4\,a\,c}+10\,a\,c^2\,d^2\,e^2-4\,b^2\,c\,d^2\,e^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^2*log(d + e*x))/(c*e^3 + a*d^2*e - b*d*e^2) - (log(a*b^2*d^4 - 2*c^3*e^4
 - 4*a^2*c*d^4 + b^3*d^3*e + c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*c^2*d^2*e
^2 - 4*b^2*c*d^2*e^2 - b^3*d^2*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^
2*d*e^3 - b*c^2*e^4*x + b^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 -
4*a*c)^(1/2) + 2*a^2*d^4*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*d^3*e*x + 6*a*c^2*
d*e^3*x - 10*a^2*c*d^3*e*x - 2*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d^
3*e + b^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) -
 a*b*d^3*e*x*(b^2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 -
 4*a*c)^(1/2))*(e*((b^2*c)/2 - 2*a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^
3*d)/2 - (b^2*d*(b^2 - 4*a*c)^(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) + 2*a*b*
c*d))/(4*a^3*c*d^2 - a^2*b^2*d^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2
- 4*a^2*b*c*d*e) + (log(2*c^3*e^4 - a*b^2*d^4 + 4*a^2*c*d^4 - b^3*d^3*e + c
^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2 + 4*b^2*c*d^2*e^2 + b^3*d^2
*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^3 + b*c^2*e^4*x + b^2*d^
3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^2*d^4*x*(b^
2 - 4*a*c)^(1/2) - 3*a*b^2*d^3*e*x - 6*a*c^2*d*e^3*x + 10*a^2*c*d^3*e*x - 2
*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^3*e + b^2*d^2*e^2*x*(b^2 - 4*a
*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) - a*b*d^3*e*x*(b^2 - 4*a*c)^(1/
2) - a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*d)/2 + e*
(2*a*c^2 - (b^2*c)/2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^2*d*(b^2 - 4*a*c)^
(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d))/(4*a^3*c*d^2 - a^2*b^2*d
^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2 - 4*a^2*b*c*d*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.65 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$

Optimal. Leaf size=124

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

```
[Out] -d*ln(e*x+d)/(a*d^2-e*(b*d-c*e))+1/2*d*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))+
(b*d-2*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4*a
*c+b^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(b*d - c*e)) + (d*Log
[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \int \frac{x}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$
$$= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)} + \frac{ce+adx}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$
$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{\int \frac{ce+adx}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$
$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)} + \frac{(-bd+2ce)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}$$
$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)} + \frac{(bd-2ce)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\right.}{ad^2-e(bd-ce)}$$
$$= \frac{(bd-2ce)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} - \frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.86

$$\frac{d\sqrt{4ac-b^2}\,(2\log(d+ex)-\log(x(ax+b)+c))+2(bd-2ce)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] (2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c
]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2
) + e*(b*d - c*e)))
```

fricas [A] time = 1.49, size = 305, normalized size = 2.46

$$\left[\frac{\left(b^2-4\,ac\right)d\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)d\log\left(ex+d\right)-\sqrt{b^2-4\,ac}\,(bd-2\,ce)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4}}{ax^2+bx+c}\right.}{2\left(\left(ab^2-4\,a^2c\right)d^2-\left(b^3-4\,abc\right)de+\left(b^2c-4\,ac^2\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d)
 - sqrt(b^2 - 4*a*c)*(b*d - 2*c*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c -
```

```
sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
- (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), 1/2*((b^2 - 4*a*c)*d*log(a*
x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(b*d
 - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2 -
4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

giac [A] time = 0.39, size = 127, normalized size = 1.02

$$-\frac{de\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(bd-2\,ce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -d*e*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*log(a*x^2 + b*x
+ c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 169, normalized size = 1.36

$$-\frac{bd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}+\frac{2ce\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}-\frac{d\ln\left(ex+d\right)}{a\,d^2-deb+c\,e^2}+\frac{d\ln\left(a\,x^2+bx+c\right)}{2a\,d^2-2deb+2c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d),x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)*d*ln(a*x^2+b*x+c)-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*d+2/(a*d^2-b*d*e+c*e^2)/(4*a*c
-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c*e-d/(a*d^2-b*d*e+c*e^2)*l
n(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.41, size = 801, normalized size = 6.46

$$\frac{\ln\left(a\,e\,x-\frac{\left(d\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\left(x\left(d\,a^2\,e+b\,a\,e^2\right)+\frac{\left(d\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\left(x\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2-6\,c\,a^2\,e^3+2\,a\,b^2\,e^3\cdots\right.\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\cdots}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(a*e*x - ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - c*e*(b^2 - 4
*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) + ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*
c + b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^2*e^3 - 6*a^2*c*e^3 + 2*a^3
*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^2 + a^2*b*d^2*e - 8*a^2*c*d
*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b
*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*
c*e^2 - b^3*d*e + 4*a*b*c*d*e))*(d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2
/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^
2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(((d*(2*a*c + (b*(b^2 - 4*a*c)^(1/2)
)/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) - ((d*(2*a*c
 + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^
2*e^3 - 6*a^2*c*e^3 + 2*a^3*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^
2 + a^2*b*d^2*e - 8*a^2*c*d*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 +
b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2
*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*e*x)*(d*(2*a*
c + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d
^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (d*lo
g(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

```
[Out] Timed out
```

3.66 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$

Optimal. Leaf size=123

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

```
[Out] e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)-1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)-(
2*a*d-b*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4*a*
c+b^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1569, 705, 31, 634, 618, 206, 628}

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) - (e*
Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx=\int \frac{1}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\frac{e^2\int \frac{1}{d+ex}\,dx}{ad^2-bde+ce^2}+\frac{\int \frac{ad-be-aex}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$

$$=\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)}+\frac{(2ad-be)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}$$

$$=\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(2ad-be)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{ad^2-e(bd-ce)}$$

$$=-\frac{(2ad-be)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.85

$$\frac{e\sqrt{4ac-b^2}\,(\log(x(ax+b)+c)-2\log(d+ex))+(2be-4ad)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] ((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*
c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d
^2) + e*(b*d - c*e)))
```

fricas [A] time = 1.45, size = 305, normalized size = 2.48

$$\left[-\frac{\left(b^2-4\,ac\right)e\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)e\log\left(ex+d\right)+\sqrt{b^2-4\,ac}\,(2\,ad-be)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-}}{ax^2+bx+c}\right.}{2\left(\left(ab^2-4\,a^2c\right)d^2-\left(b^3-4\,abc\right)de+\left(b^2c-4\,ac^2\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d
) + sqrt(b^2 - 4*a*c)*(2*a*d - b*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c
```

```
+ sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), -1/2*((b^2 - 4*a*c)*e*log(
a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(2
*a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2
- 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

giac [A] time = 0.34, size = 126, normalized size = 1.02

$$-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e^2\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{(2\,ad-be)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*e*log(a*x^2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) + e^2*log(abs(x*e + d))
/(a*d^2*e - b*d*e^2 + c*e^3) + (2*a*d - b*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 168, normalized size = 1.37

$$\frac{2ad\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}-\frac{be\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}+\frac{e\ln\left(ex+d\right)}{a\,d^2-deb+c\,e^2}-\frac{e\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x)
```

```
[Out] -1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)+2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*
c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*e+e*ln(e*x+d)/(a*d^2-b*d
*e+c*e^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.82, size = 521, normalized size = 4.24

$$\frac{\ln\left(3\,a^2\,e^2\,x+a\,b\,e^2+a^2\,d\,e-\frac{a\,e\left(\frac{b^2\,e}{2}-2\,a\,c\,e+a\,d\,\sqrt{b^2-4\,a\,c}-\frac{b\,e\,\sqrt{b^2-4\,a\,c}}{2}\right)\left(2\,x\,a^2\,d^2+a\,b\,d^2-2\,x\,a\,b\,d\,e-8\,c\,a\,d\,e-6\,c\,x\,a\,e^2+b^2\,d\,e+2\,x\,b^2\,e^2\right.}{\left(4\,a\,c-b^2\right)\left(a\,d^2-b\,d\,e+c\,e^2\right)}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*e - (a*e*((b^2*e)/2 - 2*a*c*e + a*d*(b^2
 - 4*a*c)^(1/2) - (b*e*(b^2 - 4*a*c)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x +
 a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*
```

```
a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(e*(2*a*c + (b*(b^2 - 4*a*c)^(1/2))/2
- b^2/2) - a*d*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2
 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*
e - (a*e*((b^2*e)/2 - 2*a*c*e - a*d*(b^2 - 4*a*c)^(1/2) + (b*e*(b^2 - 4*a*c
)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x + a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*
c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
)*(e*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - a*d*(b^2 - 4*a*c)^(1/2))
)/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*
e) + (e*log(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

3.67 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$

Optimal. Leaf size=158

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)}{d\left(ad^2-bde+ce^2\right)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)}{cd}$$

```
[Out] ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)-1/2*(a*d-b*e)*ln(a*x^2+b*x+c)
/c/(a*d^2-e*(b*d-c*e))+(a*b*d+2*a*c*e-b^2*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)
^(1/2))/c/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])/(
d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*d^2
 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \int \frac{1}{x(d+ex)\left(c+bx+ax^2\right)}\,dx$$
$$= \int \left(\frac{1}{cdx}+\frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{b^2e-a(bd+ce)-a(ad-be)x}{c\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right) d$$
$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{b^2e-a(bd+ce)-a(ad-be)x}{c+bx+ax^2}\,dx}{c\left(ad^2-bde+ce^2\right)}$$
$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\left(-abd+b^2e-2ace\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c\left(ad^2-bde+ce^2\right)}-\frac{(ad-be)\int}{2c\left(ad^2-e\right.}$$
$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)}-\frac{\left(-abd+b^2e-2ac\right.}{}$$
$$= \frac{\left(abd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-bde+ce^2\right)}+\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)}{2c\left(a\right.}$$

Mathematica [A] time = 0.19, size = 152, normalized size = 0.96

$$-\frac{\sqrt{4ac-b^2}\left(-2\log(x)\left(ad^2+e(ce-bd)\right)+d(ad-be)\log(x(ax+b)+c)+2ce^2\log(d+ex)\right)+2d\left(abd+2ace+\right.}{2cd\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] -1/2*(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]
+ Sqrt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x] + 2*c*e^2*Log[d
+ e*x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(c*Sqrt[-b^2 + 4*a*c]*d*(a*d^
2 + e*(-(b*d) + c*e)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 164, normalized size = 1.04

$$-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2\left(acd^2-bcde+c^2e^2\right)}-\frac{e^3\log\left(|xe+d|\right)}{ad^3e-bd^2e^2+cde^3}-\frac{\left(abd-b^2e+2\,ace\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(acd^2-bcde+c^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(|x|\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*d - b*e)*log(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2) - e^3*l
og(abs(x*e + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - (a*b*d - b^2*e + 2*a*c*e
)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - b*c*d*e + c^2*e^2)*sqr
t(-b^2 + 4*a*c)) + log(abs(x))/(c*d)
```

maple [A] time = 0.01, size = 285, normalized size = 1.80

$$-\frac{abd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c}-\frac{2ae\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}+\frac{b^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c}-\frac{ad\ln\left(a\,x^2\right.}{2\left(a\,d^2-del\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)/c*a*ln(a*x^2+b*x+c)*d+1/2/(a*d^2-b*d*e+c*e^2)/c*ln
(a*x^2+b*x+c)*b*e-1/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*a*b*d-2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((
2*a*x+b)/(4*a*c-b^2)^(1/2))*a*e+1/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*e+ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b
*d*e+c*e^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.40, size = 2399, normalized size = 15.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(b^3*c^3*e^5 - 6*a^4*c^2*d^5 + 2*a^3*b^2*c*d^5 + 8*a^2*c^4*d*e^4 - b^4*
c^2*d*e^4 - 2*b^5*c*d^2*e^3 + 2*a^3*b^3*d^5*x + 8*a^2*c^4*e^5*x + b^4*c^2*e
^5*x - 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*d^3*e
^2 - 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*e^3*(b
^2 - 4*a*c)^(1/2) - 7*a^4*b*c*d^5*x - b^5*c*d*e^4*x - 27*a^2*b^2*c^2*d^3*e^
2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(1/2) +
 2*a*b^2*c^3*d*e^4 + 6*a*b^4*c*d^3*e^2 - 6*a^2*b^3*c*d^4*e + 21*a^3*b*c^2*d
^4*e - 6*a*b^2*c^3*e^5*x + 6*a*b^5*d^3*e^2*x - 6*a^2*b^4*d^4*e*x - 14*a^4*c
^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2 - 4*a
```

```
*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^2 - 4*
a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2 - 4*a
*c)^(1/2) + 13*a*b^3*c^2*d^2*e^3 - 21*a^2*b*c^3*d^2*e^3 + 10*a^3*c^3*d^2*e^
3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*(b^2 -
 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*b^3*c*d^3*e^
2*x + 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 13
*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4*a*c)^
(1/2) - 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) - 4*
a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 5*a
*b^3*c^2*d*e^4*x + 14*a*b^4*c*d^2*e^3*x - 4*a^2*b*c^3*d*e^4*x + 26*a^3*b^2*
c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^4*x*(b
^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2
*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2)
)*(d*((a*b^2)/2 - 2*a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/2) - (b^3*e)/2 - (b^2
*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*e))/(4*a*c^
3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b*c^2*d
*e) - (log(6*a^4*c^2*d^5 - b^3*c^3*e^5 - 2*a^3*b^2*c*d^5 - 8*a^2*c^4*d*e^4
+ b^4*c^2*d*e^4 + 2*b^5*c*d^2*e^3 - 2*a^3*b^3*d^5*x - 8*a^2*c^4*e^5*x - b^4
*c^2*e^5*x + 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3
*d^3*e^2 + 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*
e^3*(b^2 - 4*a*c)^(1/2) + 7*a^4*b*c*d^5*x + b^5*c*d*e^4*x + 27*a^2*b^2*c^2*
d^3*e^2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(
1/2) - 2*a*b^2*c^3*d*e^4 - 6*a*b^4*c*d^3*e^2 + 6*a^2*b^3*c*d^4*e - 21*a^3*b
*c^2*d^4*e + 6*a*b^2*c^3*e^5*x - 6*a*b^5*d^3*e^2*x + 6*a^2*b^4*d^4*e*x + 14
*a^4*c^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2
 - 4*a*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^
2 - 4*a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2
 - 4*a*c)^(1/2) - 13*a*b^3*c^2*d^2*e^3 + 21*a^2*b*c^3*d^2*e^3 - 10*a^3*c^3*
d^2*e^3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2
- 4*a*c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*
(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*b^3*c*
d^3*e^2*x - 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2
) - 13*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4
*a*c)^(1/2) + 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2
) - 4*a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2)
 - 5*a*b^3*c^2*d*e^4*x - 14*a*b^4*c*d^2*e^3*x + 4*a^2*b*c^3*d*e^4*x - 26*a^
3*b^2*c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^
4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2
*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)
^(1/2))*((b^3*e)/2 + d*(2*a^2*c - (a*b^2)/2 + (a*b*(b^2 - 4*a*c)^(1/2))/2)
- (b^2*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*e))/(
4*a*c^3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b
*c^2*d*e) - (e^2*log(d + e*x))/(a*d^3 - b*d^2*e + c*d*e^2) + log(x)/(c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

3.68 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

Optimal. Leaf size=193

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}$$

```
[Out] -1/c/d/x-(b*d+c*e)*ln(x)/c^2/d^2+e^3*ln(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))+1/2*
(a*b*d+a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))+(2*a^2*c*d+b^3*
e-a*b*(b*d+3*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b*d-
c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(b + 2*a*x)
/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((b*
d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e)
)) + ((a*b*d - b^2*e + a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d
- c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx=\int \frac{1}{x^2(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\int \left(\frac{1}{cdx^2}+\frac{-bd-ce}{c^2d^2x}+\frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-a^2cd-b^3e+ab(bd+2c}{c^2\left(ad^2-e(bd-c\right.}\right.$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-a^2cd-b^3e+ab(bd+2ce)+a\left(abd-b^2e+\right.}{c+bx+ax^2}}{c^2\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd-b^2e+ace\right)\int \frac{b+2ax}{c+bx+ax^2}\,d}{2c^2\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd-b^2e+ace\right)\log\left(c+bx\right.}{2c^2\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{1}{cdx}+\frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{}{d^2\left(\right.}$$

Mathematica [A] time = 0.17, size = 194, normalized size = 1.01

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log(x(ax+b)+c)}{2c^2\left(ad^2+e(ce-bd)\right)}+\frac{e^3\log(d+ex)}{ad^4+d^2e(ce-bd)}-\frac{\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) -
((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(a*d^4 + d^2*e*(-(b*d)
+ c*e)) + ((a*b*d - b^2*e + a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*
(-(b*d) + c*e)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.34, size = 210, normalized size = 1.09

$$\frac{\left(abd - b^2e + ace\right)\log\left(ax^2 + bx + c\right)}{2\left(ac^2d^2 - bc^2de + c^3e^2\right)} + \frac{e^4\log\left(|xe + d|\right)}{ad^4e - bd^3e^2 + cd^2e^3} + \frac{\left(ab^2d - 2a^2cd - b^3e + 3abce\right)\arctan\left(\frac{2ax+b}{\sqrt{-b^2+4ac}}\right)}{\left(ac^2d^2 - bc^2de + c^3e^2\right)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(a*b*d - b^2*e + a*c*e)*log(a*x^2 + b*x + c)/(a*c^2*d^2 - b*c^2*d*e + c
^3*e^2) + e^4*log(abs(x*e + d))/(a*d^4*e - b*d^3*e^2 + c*d^2*e^3) + (a*b^2*
d - 2*a^2*c*d - b^3*e + 3*a*b*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/(
(a*c^2*d^2 - b*c^2*d*e + c^3*e^2)*sqrt(-b^2 + 4*a*c)) - (b*d + c*e)*log(abs
(x))/(c^2*d^2) - 1/(c*d*x)
```

maple [B] time = 0.01, size = 412, normalized size = 2.13

$$-\frac{2a^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2 - deb + ce^2\right)\sqrt{4ac - b^2}\,c} + \frac{ab^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2 - deb + ce^2\right)\sqrt{4ac - b^2}\,c^2} + \frac{3abe\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2 - deb + ce^2\right)\sqrt{4ac - b^2}\,c} - \frac{b^3e}{\left(ad^2 - de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/c
*a*ln(a*x^2+b*x+c)*e-1/2/(a*d^2-b*d*e+c*e^2)/c^2*ln(a*x^2+b*x+c)*b^2*e-2/(a
*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a
^2*d+1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^
2)^(1/2))*a*b^2*d+3/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*a*b*e-1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arct
an((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e-1/c/d/x-1/c^2/d*ln(x)*b-1/c/d^2*ln(x)
*e+e^3/(a*d^2-b*d*e+c*e^2)/d^2*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 20.39, size = 2388, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (e^3*log(d + e*x))/(a*d^4 + c*d^2*e^2 - b*d^3*e) + (log((a^4*e^4*x)/(c^2*d^
2) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^
2*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 -
```

```
a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e
+ b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*
d) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*
a^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c
*d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2
) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*
c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(
4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c
^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x -
8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b
*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x
 - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^
4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a
*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*
a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
 + (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e +
b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c
*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*
e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4
*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*
b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a
*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2
- a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) + (log((a^4*e^4*x)/(c^2*d^2
) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^2
*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 -
a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e +
 b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*d
) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*a
^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c*
d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2)
 + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e + a*b^2*d*(b^2 - 4*a*c
)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(4
*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^
2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8
*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*
c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x
- 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4
*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*
b^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a
*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
+ (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e - b
^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*
e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e
*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*
e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b
^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*
b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2 -
 a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - 1/(c*d*x) - (log(x)*(b*d +
 c*e))/(c^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

3.69 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx$

Optimal. Leaf size=252

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\log$$

```
[Out] -1/2/c/d/x^2+(b*d+c*e)/c^2/d^2/x+(b^2*d^2+b*c*d*e-c*(a*d^2-c*e^2))*ln(x)/c^
3/d^3-e^4*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c*d+b^3*e-a*b*(b*d+2*c
*e))*ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b*d-c*e))-(b^4*e+a^2*c*(3*b*d+2*c*e)-a*b
^2*(b*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*d^2-e*(b*d-c*e
))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\log$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e)
- a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^
2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 - c*e^
2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))) +
((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2
- e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx &= \int \frac{1}{x^3(d+ex)\left(c+bx+ax^2\right)}\,dx\\
&= \int \left(\frac{1}{cdx^3}+\frac{-bd-ce}{c^2d^2x^2}+\frac{b^2d^2+bcde-c\left(ad^2-ce^2\right)}{c^3d^3x}+\frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right)(d+\right.\\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}-\frac{\left(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+
\end{aligned}$$

Mathematica [A] time = 0.22, size = 252, normalized size = 1.00

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}+\frac{\log(x)\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e
) - a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[
-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e + c*(-(a*d^
2) + c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(a*d^5 + d^3*e*(-(b*d)
+ c*e)) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])/(2*c
^3*(a*d^2 + e*(-(b*d) + c*e)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 279, normalized size = 1.11

$$-\frac{\left(ab^2d-a^2cd-b^3e+2\,abce\right)\log\left(ax^2+bx+c\right)}{2\left(ac^3d^2-bc^3de+c^4e^2\right)}-\frac{e^5\log\left(|xe+d|\right)}{ad^5e-bd^4e^2+cd^3e^3}-\frac{\left(ab^3d-3\,a^2bcd-b^4e+4\,ab^2ce-2\,a\right.}{\left(ac^3d^2-bc^3de+c^4e^2\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^2*d - a^2*c*d - b^3*e + 2*a*b*c*e)*log(a*x^2 + b*x + c)/(a*c^3*d^
2 - b*c^3*d*e + c^4*e^2) - e^5*log(abs(x*e + d))/(a*d^5*e - b*d^4*e^2 + c*d
^3*e^3) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*arcta
n((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c^3*d^2 - b*c^3*d*e + c^4*e^2)*sqrt(-
b^2 + 4*a*c)) + (b^2*d^2 - a*c*d^2 + b*c*d*e + c^2*e^2)*log(abs(x))/(c^3*d^
3) - 1/2*(c^2*d^2 - 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)
```

maple [B] time = 0.01, size = 562, normalized size = 2.23

$$\frac{3a^2bd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c^2}+\frac{2a^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c}-\frac{a\,b^3d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c^3}-\frac{4a\,b^2e}{\left(a\,d^2-de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)/c^2*a^2*ln(a*x^2+b*x+c)*d-1/2/(a*d^2-b*d*e+c*e^2)/c
^3*a*ln(a*x^2+b*x+c)*b^2*d-1/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*e+
1/2/(a*d^2-b*d*e+c*e^2)/c^3*ln(a*x^2+b*x+c)*b^3*e+3/(a*d^2-b*d*e+c*e^2)/c^2
/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*d+2/(a*d^2-b*d
*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*e-1/(
a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)
)*a*b^3*d-4/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a
*c-b^2)^(1/2))*a*b^2*e+1/(a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e-1/2/c/d/x^2+1/c^2/d/x*b+1/c/d^2/x*e-1/c^2
/d*ln(x)*a+1/c^3/d*ln(x)*b^2+1/c^2/d^2*ln(x)*b*e+1/c/d^3*ln(x)*e^2-e^4/(a*d
^2-b*d*e+c*e^2)/d^3*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 26.16, size = 3530, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x)*(a + b/x + c/x^2)),x)
```

[Out] (log((a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b*c*d*e))/(c^4*d^4) - (((((a*e
*(a^2*b^3*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e +
 b^3*c^2*d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^
4*d^4*e - 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2
*c^2*d^2*e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*
a^4*c*d^5 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4
*e + 8*a^2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*
a^3*b*c*d^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e
^3 + 10*a^2*b^2*c*d^3*e^2))/(c^2*d^2) - (a*e*(b^4*e*(b^2 - 4*a*c)^(1/2) - b
^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) -
5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c
d(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e
+ b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4
*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x -
2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6
*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2
*e^2*x))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c
)^(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*
c)^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
+ 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^
3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(a^3*b^3*d^6 + b^3*c^3*e^6
+ b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*e + 2*b^4*c^2*d*e^5 + 2*b^5*c
*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3*a*b*c^4*e^6 + 9*a^2*b^2*c^2*
d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^3 - 9*a*b^3*c^2*d^2*e^4 + 7*a
^2*b*c^3*d^2*e^4))/(c^4*d^4) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4*c^
2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^4*e
^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b^4*
c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^
5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) - 5
*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b
^2)*(a*d^2 + c*e^2 - b*d*e)) - (a^5*e^5*x)/(c^3*d^3))*(b^4*e*(b^2 - 4*a*c)^
(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)
^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) +
3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a
*c^5*e^2 + 4*a^2*c^4*d^2 - b^2*c^4*e^2 - a*b^2*c^3*d^2 + b^3*c^3*d*e - 4*a*
b*c^4*d*e)) - (e^4*log(d + e*x))/(a*d^5 + c*d^3*e^2 - b*d^4*e) - (log((((a*
e*(a^3*b^3*d^6 + b^3*c^3*e^6 + b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*
e + 2*b^4*c^2*d*e^5 + 2*b^5*c*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3
*a*b*c^4*e^6 + 9*a^2*b^2*c^2*d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^
3 - 9*a*b^3*c^2*d^2*e^4 + 7*a^2*b*c^3*d^2*e^4))/(c^4*d^4) - (((a*e*(a^2*b^3
*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e + b^3*c^2*
d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^4*d^4*e -
 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2*c^2*d^2*
e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*a^4*c*d^5
 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4*e + 8*a^
2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*a^3*b*c*d
^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e^3 + 10*a
^2*b^2*c*d^3*e^2))/(c^2*d^2) + (a*e*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*
a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^2
*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*d*(b^2 -
 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2
*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*
x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c
*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*
d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/
(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b

```
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4
*c^2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^
4*e^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b
^4*c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b
*c*d*e))/(c^4*d^4) - (a^5*e^5*x)/(c^3*d^3))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1
/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) + 5
*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^2 +
 4*a^2*c^4*d^2 - b^2*c^4*e^2 - a*b^2*c^3*d^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)
) - (1/(2*c*d) - (x*(b*d + c*e))/(c^2*d^2))/x^2 + (log(x)*(c^2*e^2 - d^2*(a
*c - b^2) + b*c*d*e))/(c^3*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

3.70 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=343

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}-\frac{x(2ad+be)}{a^2e^3}+\frac{\left(-4a^2c^3de-b\right.}{}$$

```
[Out] -(2*a*d+b*e)*x/a^2/e^3+1/2*x^2/a/e^2+d^5/e^4/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^
4*(3*a*d^2-e*(4*b*d-5*c*e))*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))^2+1/2*(b^4*d^
2-2*b^3*c*d*e+4*a*b*c^2*d*e+a*c^2*(a*d^2-c*e^2)-b^2*c*(3*a*d^2-c*e^2))*ln(a
*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))^2+(b^5*d^2-2*b^4*c*d*e+8*a*b^2*c^2*d*e-
4*a^2*c^3*d*e+a*b*c^2*(5*a*d^2-3*c*e^2)-b^3*c*(5*a*d^2-c*e^2))*arctanh((2*a
*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-4a^2c^3de+8ab^2c^2de-b^3c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3*d
*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a*d^2 - c*e^2))*ArcTanh[(b + 2*
a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2)
+ (d^4*(3*a*d^2 - e*(4*b*d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c
*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) -
 b^2*c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*
e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^5}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{-2ad-be}{a^2e^3}+\frac{x}{ae^2}+\frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)(d+ex)^2}+\frac{d^4\left(3ad^2-e(4bd-\right.}{e^3\left(ad^2-e(bd-ce)\right)^{\cdots}}\right.$$

$$= -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{d^4\left(3ad^2-e(4bd-5ce\right.}{e^4\left(ad^2-e(bd-\right.}$$

$$= -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{d^4\left(3ad^2-e(4bd-5ce\right.}{e^4\left(ad^2-e(bd-\right.}$$

$$= -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{d^4\left(3ad^2-e(4bd-5ce\right.}{e^4\left(ad^2-e(bd-\right.}$$

$$= -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\left(b^5d^2-2b^4cde+8ab^2c\right.}{}$$

Mathematica [A] time = 0.36, size = 338, normalized size = 0.99

$$\frac{\left(b^2c\left(ce^2-3ad^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)^2}-\frac{x(2ad+be)}{a^2e^3}-\frac{\left(-4a^2c^3de+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^
3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(-5*a*d^2 + c*e^2))*ArcTan[(b +
 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c
*e))^2) + ((3*a*d^6 + d^4*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 + e
*(-(b*d) + c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^
2 - c*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2
+ e*(-(b*d) + c*e))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.42, size = 565, normalized size = 1.65

$$\frac{d^5e^4}{\left(ad^2e^8 - bde^9 + ce^{10}\right)(xe+d)} + \frac{\left(b^5d^2e^2 - 5\,ab^3cd^2e^2 + 5\,a^2bc^2d^2e^2 - 2\,b^4cde^3 + 8\,ab^2c^2de^3 - 4\,a^2c^3de^3 + b^3c^2e^4 - \right.}{\left(a^5d^4 - 2\,a^4bd^3e + a^3b^2d^2e^2 + 2\,a^4cd^2e^2 - 2\,a^3bcde^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] d^5*e^4/((a*d^2*e^8 - b*d*e^9 + c*e^10)*(x*e + d)) + (b^5*d^2*e^2 - 5*a*b^3
*c*d^2*e^2 + 5*a^2*b*c^2*d^2*e^2 - 2*b^4*c*d*e^3 + 8*a*b^2*c^2*d*e^3 - 4*a^
2*c^3*d*e^3 + b^3*c^2*e^4 - 3*a*b*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
)*e^(-2)/((a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*
a^3*b*c*d*e^3 + a^3*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2 - 2*(3*a^2*d*e
+ a*b*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-4)/a^3 + 1/2*(b^4*d^2 - 3*a*b^
2*c*d^2 + a^2*c^2*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + b^2*c^2*e^2 - a*c^3*e
^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x
*e + d)^2 - c*e^2/(x*e + d)^2)/(a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 +
 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^3 + a^3*c^2*e^4) - (3*a^2*d^2 + 2*a*b*d*e
+ b^2*e^2 - a*c*e^2)*e^(-4)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^3
```

maple [B] time = 0.01, size = 943, normalized size = 2.75

$$-\frac{5b\,c^2d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}+\frac{4c^3de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}+\frac{5b^3c\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a^2}-\frac{8b^2c^2}{\left(a\,d^2-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] 1/2*x^2/a/e^2-2/a/e^3*x*d-1/a^2/e^2*b*x+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^
2+b*x+c)*c^2*d^2-3/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b^2*c*d^2+2/
(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*d*e-1/2/(a*d^2-b*d*e+c*e^2)
^2/a^2*ln(a*x^2+b*x+c)*c^3*e^2+1/2/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c
)*b^4*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c)*b^3*c*d*e+1/2/(a*d^2-
b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c)*b^2*c^2*e^2-5/(a*d^2-b*d*e+c*e^2)^2/a/(4
*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^2+4/(a*d^2-b*d*
e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3*d*e+
5/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^
(1/2))*b^3*c*d^2-8/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*d*e+3/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*e^2-1/(a*d^2-b*d*e+c*e^2)^
2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*d^2+2/(a*d^
2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*
b^4*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4
*a*c-b^2)^(1/2))*b^3*c^2*e^2+3/e^4*d^6/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a-4/
```

```
e^3*d^5/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b+5/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*l
n(e*x+d)*c+1/e^4*d^5/(a*d^2-b*d*e+c*e^2)/(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.04, size = 3503, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(3*a*d^6 + 5*c*d^4*e^2 - 4*b*d^5*e))/(c^2*e^8 + a^2*d^4*e^4 +
 b^2*d^2*e^6 - 2*b*c*d*e^7 - 2*a*b*d^3*e^5 + 2*a*c*d^2*e^6) - (log(12*a^5*c
*d^8 - 2*a*c^5*e^8 - 3*a^4*b^2*d^8 + b^2*c^4*e^8 + b^6*d^4*e^4 + 4*a^3*b^3*
d^7*e - 4*b^3*c^3*d*e^7 - 4*b^5*c*d^3*e^5 + b^5*d^4*e^4*(b^2 - 4*a*c)^(1/2)
 + 12*a^2*c^4*d^2*e^6 - 22*a^3*c^3*d^4*e^4 + 8*a^4*c^2*d^6*e^2 + 6*b^4*c^2*
d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*e^8*(b^2 - 4*a*c)^(1/2) -
 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) + 12*a*b*c^4*d*e^7 - 16*a^4*b*c*d^7*e - 4*
a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) + a*b*c^4*e^8*x + 24*a^5*c*d^7*e*
x + 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 - 4*a*c)^(1/2) + 12*a^4*c*d
^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^4*c*d^4*
e^4 + a*b^5*d^4*e^4*x - 6*a^4*b^2*d^7*e*x + 8*a^2*c^4*d*e^7*x + 4*a^3*b^2*d
^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 - 4*a*c)^(1/2) - 4*b^4*c*d^
3*e^5*(b^2 - 4*a*c)^(1/2) - 24*a*b^2*c^3*d^2*e^6 + 20*a*b^3*c^2*d^3*e^5 - 2
0*a^2*b*c^3*d^3*e^5 - 4*a^2*b^3*c*d^5*e^3 + 16*a^3*b*c^2*d^5*e^3 - 2*a^3*b^
2*c*d^6*e^2 - 4*a^2*b^4*d^5*e^3*x + 11*a^3*b^3*d^6*e^2*x - 8*a^3*c^3*d^3*e^
5*x + 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b^2 - 4*a*c)^(1/2) - 4*a*b
^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*e^2*(b^2 - 4*a*c)^(1/2) +
 a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6*e^2*x*(b^2 - 4*a*c)^(1/2
) + 6*a*b^3*c^2*d^2*e^6*x - 18*a^2*b*c^3*d^2*e^6*x - 15*a^3*b*c^2*d^4*e^4*x
 + 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b^2 - 4*a*c)^(1/2) - 2*a^2
*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a^3*b^2*d^6*e^2*x*(b^2 - 4
*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 11*a^3*c^2*d^4*e^4*
x*(b^2 - 4*a*c)^(1/2) + 16*a^2*b^2*c^2*d^3*e^5*x + 14*a^4*b*d^7*e*x*(b^2 -
4*a*c)^(1/2) - 4*a*b^2*c^3*d*e^7*x - 4*a*b^4*c*d^3*e^5*x - 44*a^4*b*c*d^6*e
^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^3*e^5*x*(b^2 - 4
*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^6
*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*
b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 + b^5*d^2*(b^2 - 4*a*c)^(1/2)
 - 4*a^3*c^3*d^2 + 4*a^2*c^4*e^2 + b^4*c^2*e^2 - 5*a*b^2*c^3*e^2 + b^3*c^2*
e^2*(b^2 - 4*a*c)^(1/2) - 2*b^5*c*d*e + 13*a^2*b^2*c^2*d^2 - 7*a*b^4*c*d^2
+ 12*a*b^3*c^2*d*e - 16*a^2*b*c^3*d*e - 5*a*b^3*c*d^2*(b^2 - 4*a*c)^(1/2) -
 3*a*b*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d*e*(b^2 - 4*a*c)^(1/2) + 5*
a^2*b*c^2*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^4*c*d*e*(b^2 - 4*a*c)^(1/2) + 8*a*b
^2*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^4 - a^5*b^2*d^4 + 4*a^4*c^3*
e^4 + 2*a^4*b^3*d^3*e - a^3*b^2*c^2*e^4 - a^3*b^4*d^2*e^2 + 8*a^5*c^2*d^2*e
^2 - 8*a^5*b*c*d^3*e + 2*a^3*b^3*c*d*e^3 - 8*a^4*b*c^2*d*e^3 + 2*a^4*b^2*c*
```

```
d^2*e^2)) - (log(2*a*c^5*e^8 - 12*a^5*c*d^8 + 3*a^4*b^2*d^8 - b^2*c^4*e^8 -
 b^6*d^4*e^4 - 4*a^3*b^3*d^7*e + 4*b^3*c^3*d*e^7 + 4*b^5*c*d^3*e^5 + b^5*d^
4*e^4*(b^2 - 4*a*c)^(1/2) - 12*a^2*c^4*d^2*e^6 + 22*a^3*c^3*d^4*e^4 - 8*a^4
*c^2*d^6*e^2 - 6*b^4*c^2*d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*
e^8*(b^2 - 4*a*c)^(1/2) - 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^4*d*e^
7 + 16*a^4*b*c*d^7*e - 4*a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^
4*e^8*x - 24*a^5*c*d^7*e*x - 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 -
4*a*c)^(1/2) + 12*a^4*c*d^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*c*d^4*e^4 - a*b^5*d^4*e^4*x + 6*a^4*b^2*d^7*e*x - 8*a^2*
c^4*d*e^7*x + 4*a^3*b^2*d^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 -
4*a*c)^(1/2) - 4*b^4*c*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 24*a*b^2*c^3*d^2*e^6 -
 20*a*b^3*c^2*d^3*e^5 + 20*a^2*b*c^3*d^3*e^5 + 4*a^2*b^3*c*d^5*e^3 - 16*a^3
*b*c^2*d^5*e^3 + 2*a^3*b^2*c*d^6*e^2 + 4*a^2*b^4*d^5*e^3*x - 11*a^3*b^3*d^6
*e^2*x + 8*a^3*c^3*d^3*e^5*x - 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b
^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*
e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6
*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*d^2*e^6*x + 18*a^2*b*c^3*d^2*e^6*x
 + 15*a^3*b*c^2*d^4*e^4*x - 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b
^2 - 4*a*c)^(1/2) - 2*a^2*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a
^3*b^2*d^6*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1
/2) + 11*a^3*c^2*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 16*a^2*b^2*c^2*d^3*e^5*x +
 14*a^4*b*d^7*e*x*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^3*d*e^7*x + 4*a*b^4*c*d^3
*e^5*x + 44*a^4*b*c*d^6*e^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a
*b^3*c*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1
/2) + 6*a*b^2*c^2*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^
2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 - b^
5*d^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^3*d^2 + 4*a^2*c^4*e^2 + b^4*c^2*e^2 - 5
*a*b^2*c^3*e^2 - b^3*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*b^5*c*d*e + 13*a^2*b^2
*c^2*d^2 - 7*a*b^4*c*d^2 + 12*a*b^3*c^2*d*e - 16*a^2*b*c^3*d*e + 5*a*b^3*c*
d^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^3*e^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e
*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*b^4*c*d*e*(b
^2 - 4*a*c)^(1/2) - 8*a*b^2*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^4 -
 a^5*b^2*d^4 + 4*a^4*c^3*e^4 + 2*a^4*b^3*d^3*e - a^3*b^2*c^2*e^4 - a^3*b^4*
d^2*e^2 + 8*a^5*c^2*d^2*e^2 - 8*a^5*b*c*d^3*e + 2*a^3*b^3*c*d*e^3 - 8*a^4*b
*c^2*d*e^3 + 2*a^4*b^2*c*d^2*e^2)) + x^2/(2*a*e^2) - (x*(b*e^2 + 2*a*d*e))/
(a^2*e^4) + (a^2*d^5)/(e*(a^2*d*e^3 + a^2*e^4*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.71 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=274

$$-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-2\right.}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

```
[Out] x/a/e^2-d^4/e^3/(a*d^2-e*(b*d-c*e))/(e*x+d)-d^3*(2*a*d^2-e*(3*b*d-4*c*e))*l
n(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))^2-1/2*(b*d-c*e)*(-2*a*c*d+b^2*d-b*c*e)*ln(
a*x^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))^2-(b^4*d^2-2*b^3*c*d*e+6*a*b*c^2*d*e+2
*a*c^2*(a*d^2-c*e^2)-b^2*c*(4*a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^
(1/2))/a^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi **[A]** time = 0.56, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 1628, 634, 618, 206, 628}

$$-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{(bd-ce)\left(-2acd+b^2d\right.}{2a^2\left(ad^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^3
*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2))
*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*
d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*
x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^4}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{ae^2} + \frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)^2} + \frac{d^3\left(-2ad^2+e(3bd-4ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{-c\left(b^2\right.}{}\right.$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2} + \int \frac{-c}{}$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2} - \frac{((bd}{}$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2} - \frac{(bd}{}$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{\left(b^4d^2-2b^3cde+6abc^2de+2ac^2\left(ad^2-ce^2\right)\right.}{a^2\sqrt{b^2-4ac}\left(ad^2\right.}$$

Mathematica [A] time = 0.29, size = 269, normalized size = 0.98

$$\frac{(bd-ce)\left(2acd+b^2(-d)+bce\right)\log(x(ax+b)+c)}{2a^2\left(ad^2+e(ce-bd)\right)^2} + \frac{\left(b^2c\left(ce^2-4ad^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3c\right.}{a^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^4*d^2 - 2*
b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b^2*c*(-4*a*d^2 + c*e
^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(a*d^2
 + e*(-(b*d) + c*e))^2) - ((2*a*d^5 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e*x])
/(e^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d + b*
c*e)*Log[c + x*(b + a*x)])/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

fricas [B] time = 103.93, size = 2139, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b
^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4
*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c
- 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*e^6)*x^2 + ((b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)
*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e
^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^
3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a^2*b
^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*c^2
- 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b^4*c
 - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b^5 -
 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e
^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2 - 4
*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)*d^4
*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + 4*(
a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d^5*e
^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*
d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*d
*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 + (a^
2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^
7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x), -1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*
(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b
^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^
2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4
*a^2*c^3)*e^6)*x^2 + 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c -
3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*
c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*sq
rt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*
((a^3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a
^2*b^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*
c^2 - 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b
^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)
*d*e^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2
 - 4*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)
*d^4*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 +
 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d
^5*e^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c
^2)*d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^
3)*d*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 +
 (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
d*e^7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x)]
```

giac [A] time = 0.40, size = 476, normalized size = 1.74

$$-\frac{d^4e^3}{\left(ad^2e^6 - bde^7 + ce^8\right)(xe+d)} - \frac{\left(b^4d^2e^2 - 4\,ab^2cd^2e^2 + 2\,a^2c^2d^2e^2 - 2\,b^3cde^3 + 6\,abc^2de^3 + b^2c^2e^4 - 2\,ac^3e^4\right)\mathrm{ar}}{\left(a^4d^4 - 2\,a^3bd^3e + a^2b^2d^2e^2 + 2\,a^3cd^2e^2 - 2\,a^2bcde^3 + a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -d^4*e^3/((a*d^2*e^6 - b*d*e^7 + c*e^8)*(x*e + d)) - (b^4*d^2*e^2 - 4*a*b^2
*c*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3 + b^2*c^2*
e^4 - 2*a*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e
```

```
+ d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^4*d^4 - 2*a
^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*
e^4)*sqrt(-b^2 + 4*a*c)) + (x*e + d)*e^(-3)/a - 1/2*(b^3*d^2 - 2*a*b*c*d^2
- 2*b^2*c*d*e + 2*a*c^2*d*e + b*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(
x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^4*d^
4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a
^2*c^2*e^4) + (2*a*d + b*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^2
```

maple [B] time = 0.01, size = 765, normalized size = 2.79

$$-\frac{4b^2c\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}+\frac{6b\,c^2de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}-\frac{2c^3e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}+\frac{b^4d^2}{\left(a\,d^2-de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] x/a/e^2+1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b*c*d^2-1/(a*d^2-b*d*e+c*
e^2)^2/a*ln(a*x^2+b*x+c)*c^2*d*e-1/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x
+c)*b^3*d^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b^2*c*d*e-1/2/(a*d^
2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*e^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^2-4/(a*d^2-b*d*e+c*
e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d^2+6/
(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))*b*c^2*d*e-2/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(
4*a*c-b^2)^(1/2))*c^3*e^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d^2-2/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c
-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*e+1/(a*d^2-b*d*e+c*
e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*e^
2-1/e^3*d^4/(a*d^2-b*d*e+c*e^2)/(e*x+d)-2/e^3*d^5/(a*d^2-b*d*e+c*e^2)^2*ln(
e*x+d)*a+3/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-4/e*d^3/(a*d^2-b*d*e+c
*e^2)^2*ln(e*x+d)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.00, size = 2495, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e^2) - (log(d + e*x)*(2*a*d^5 + 4*c*d^3*e^2 - 3*b*d^4*e))/(c^2*e^7 + a
^2*d^4*e^3 + b^2*d^2*e^5 - 2*b*c*d*e^6 - 2*a*b*d^3*e^4 + 2*a*c*d^2*e^5) + (
log(8*a^4*c*d^7 + b*c^4*e^7 + c^4*e^7*(b^2 - 4*a*c)^(1/2) - 2*a^3*b^2*d^7 +
 b^5*d^4*e^3 + 3*a^2*b^3*d^6*e - 4*b^2*c^3*d*e^6 - 4*b^4*c*d^3*e^4 + b^4*d^
4*e^3*(b^2 - 4*a*c)^(1/2) - 24*a^2*c^3*d^3*e^4 + 8*a^3*c^2*d^5*e^2 + 6*b^3*
c^2*d^2*e^5 + 8*a*c^4*d*e^6 + 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/
2) - 4*a^4*d^7*x*(b^2 - 4*a*c)^(1/2) - 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^
```

```
3*(b^2 - 4*a*c)^(1/2) + 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 16*a^4*c*d^
6*e*x + 8*a^3*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/
2) - 18*a*b*c^3*d^2*e^5 - 8*a*b^3*c*d^4*e^3 - 2*a*b^4*d^4*e^3*x - 4*a^3*b^2
*d^6*e*x + 3*a^2*b^2*d^6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a
*c)^(1/2) - 4*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*d^3*e^4 + 17
*a^2*b*c^2*d^4*e^3 - 2*a^2*b^2*c*d^5*e^2 + 8*a^2*b^3*d^5*e^2*x - 12*a^2*c^3
*d^2*e^5*x + 34*a^3*c^2*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) -
 18*a^2*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(
1/2) - 4*a^3*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^5*x - 4*a^
2*b*c^2*d^3*e^4*x - 8*a^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^3*d*e
^6*x + 12*a^2*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4
*a*c)^(1/2) - 4*a*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 32*a^3*b*c*d^5*e^2*x +
6*a*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)
^(1/2))*(b^5*d^2 + b^4*d^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*e^2 + 8*a^2*b*c^2*
d^2 + 2*a^2*c^2*d^2*(b^2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) -
 2*b^4*c*d*e - 6*a*b^3*c*d^2 - 4*a*b*c^3*e^2 - 8*a^2*c^3*d*e - 2*a*c^3*e^2*
(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2)
- 2*b^3*c*d*e*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*
(4*a^5*c*d^4 - a^4*b^2*d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*
e^4 - a^2*b^4*d^2*e^2 + 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d
*e^3 - 8*a^3*b*c^2*d*e^3 + 2*a^3*b^2*c*d^2*e^2)) - (log(c^4*e^7*(b^2 - 4*a*
c)^(1/2) - b*c^4*e^7 - 8*a^4*c*d^7 + 2*a^3*b^2*d^7 - b^5*d^4*e^3 - 3*a^2*b^
3*d^6*e + 4*b^2*c^3*d*e^6 + 4*b^4*c*d^3*e^4 + b^4*d^4*e^3*(b^2 - 4*a*c)^(1/
2) + 24*a^2*c^3*d^3*e^4 - 8*a^3*c^2*d^5*e^2 - 6*b^3*c^2*d^2*e^5 - 8*a*c^4*d
*e^6 - 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/2) - 4*a^4*d^7*x*(b^2 -
 4*a*c)^(1/2) + 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^3*(b^2 - 4*a*c)^(1/2) +
 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 16*a^4*c*d^6*e*x + 8*a^3*c*d^6*e*(
b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) + 18*a*b*c^3*d^2*e^5
 + 8*a*b^3*c*d^4*e^3 + 2*a*b^4*d^4*e^3*x + 4*a^3*b^2*d^6*e*x + 3*a^2*b^2*d^
6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 4*b^3*c*d^3
*e^4*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*d^3*e^4 - 17*a^2*b*c^2*d^4*e^3 + 2*
a^2*b^2*c*d^5*e^2 - 8*a^2*b^3*d^5*e^2*x + 12*a^2*c^3*d^2*e^5*x - 34*a^3*c^2
*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) - 18*a^2*b*c*d^5*e^2*(b^
2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*e^2*
x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d^2*e^5*x + 4*a^2*b*c^2*d^3*e^4*x - 8*a
^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 4*a*b*c^3*d*e^6*x + 12*a^2*c^2*d^3*e
^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) - 4*a*c^3*d
*e^6*x*(b^2 - 4*a*c)^(1/2) + 32*a^3*b*c*d^5*e^2*x + 6*a*b*c^2*d^2*e^5*x*(b^
2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2*(b^2 -
 4*a*c)^(1/2) - b^5*d^2 - b^3*c^2*e^2 - 8*a^2*b*c^2*d^2 + 2*a^2*c^2*d^2*(b^
2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*b^4*c*d*e + 6*a*b^3*
c*d^2 + 4*a*b*c^3*e^2 + 8*a^2*c^3*d*e - 2*a*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 1
0*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^3*c*d*e*(b^2 - 4*
a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^4 - a^4*b^2*
d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*e^4 - a^2*b^4*d^2*e^2 +
 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d*e^3 - 8*a^3*b*c^2*d*e^
3 + 2*a^3*b^2*c*d^2*e^2)) - (a*d^4)/(e*(a*d*e^2 + a*e^3*x)*(a*d^2 + c*e^2 -
 b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.72 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=246

$$\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de+b^3d^2-2b^2cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

```
[Out] d^3/e^2/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^2*(a*d^2-e*(2*b*d-3*c*e))*ln(e*x+d)/e
^2/(a*d^2-e*(b*d-c*e))^2+1/2*(b^2*d^2-2*b*c*d*e-c*(a*d^2-c*e^2))*ln(a*x^2+b
*x+c)/a/(a*d^2-e*(b*d-c*e))^2+(b^3*d^2-2*b^2*c*d*e+4*a*c^2*d*e-b*c*(3*a*d^2
-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a/(a*d^2-e*(b*d-c*e))^2/(-4*
a*c+b^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de-2b^2cde+b^3d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*a
*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d - 3
*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*
e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^3}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)}{e\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{cd(bd-2\cdots}{(ac\cdots}\right.$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \int \frac{cd(bd-2\cdots}{(\cdots}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(b^2d^2-2\cdots\right.}{\cdots}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(b^2d^2-2\cdots\right.}{\cdots}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^3d^2-2b^2cde+4ac^2de-bc\left(3ad^2-ce^2\right)\right)\tan\cdots}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}\cdots$$

Mathematica [A] time = 0.23, size = 207, normalized size = 0.84

$$\frac{\frac{\left(c\left(ce^2-ad^2\right)+b^2d^2-2bcde\right)\log(x(ax+b)+c)}{a} - \frac{2\left(bc\left(ce^2-3ad^2\right)+4ac^2de+b^3d^2-2b^2cde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}} + \frac{2\log(d+ex)\left(ad^4+d^2e(3ce-2bd)\right)}{e^2} + \frac{2d^3\cdots}{\cdots}}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2 - 2*b^2*c
*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 +
 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4 + d^2*e*(-2*b*d + 3*c*e))*Log[
d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[c + x*(b
+ a*x)])/a)/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

fricas [B] time = 36.70, size = 1465, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c
- 4*a^2*c^2)*d^3*e^2 + (b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c -
2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a*c^
2)*d*e^4)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqr
t(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((b^4 - 5*a*b^2*c + 4*a^2*
c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 +
((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2
*c^2 - 4*a*c^3)*e^5)*x)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 -
 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2
- 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*
d^2*e^3)*x)*log(e*x + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3
*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d
^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2
)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*
x), 1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2
*c - 4*a^2*c^2)*d^3*e^2 + 2*(b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2
*c - 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2
*a*c^2)*d*e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)
/(b^2 - 4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b
*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*
d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)*log(a*x
^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e +
3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4
*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*d^2*e^3)*x)*log(e*x + d))/((a^3
*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*
b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2
 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c
)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*
b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*x)]
```

giac [A] time = 0.42, size = 412, normalized size = 1.67

$$\frac{1}{2}\left(\frac{2\,d^3e^2}{\left(ad^2e^3 - bde^4 + ce^5\right)(xe + d)} + \frac{2\left(b^3d^2e^3 - 3\,abcd^2e^3 - 2\,b^2cde^4 + 4\,ac^2de^4 + bc^2e^5\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d}\right.}{\sqrt{-b^2+4\,a}}\right.}{\left(a^3d^4 - 2\,a^2bd^3e + ab^2d^2e^2 + 2\,a^2cd^2e^2 - 2\,abcde^3 + ac^2e^4\right)\sqrt{-b^2 + }}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(x*e + d)) + 2*(b^3*d^2*e^3 -
 3*a*b*c*d^2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^2*e^5)*arctan(-(2*a*
d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)
/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^3*d^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a
^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + (b^2*d^2*e
- a*c*d^2*e - 2*b*c*d*e^2 + c^2*e^3)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e
+ d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^3*d^4 -
2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4
) - 2*e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a)*e^(-1)
```

maple [B] time = 0.01, size = 580, normalized size = 2.36

$$-\frac{b^3d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}+\frac{2b^2cde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}-\frac{b\,c^2e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;a}+\frac{3bc\,d^2}{\left(a\,d^2-de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*c*d^2+1/2/(a*d^2-b*d*e+c*e^2)^2/
a*ln(a*x^2+b*x+c)*b^2*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b*c*d*e
+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*e^2+3/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d^2-4/(a*d^2-b*
d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d*e-
1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))/a*b^3*d^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*
a*c-b^2)^(1/2))/a*b^2*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arcta
n((2*a*x+b)/(4*a*c-b^2)^(1/2))/a*b*c^2*e^2+d^4/(a*d^2-b*d*e+c*e^2)^2/e^2*ln
(e*x+d)*a-2*d^3/(a*d^2-b*d*e+c*e^2)^2/e*ln(e*x+d)*b+3*d^2/(a*d^2-b*d*e+c*e^
2)^2*ln(e*x+d)*c+1/e^2*d^3/(a*d^2-b*d*e+c*e^2)/(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.11, size = 2037, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(a*d^4 + 3*c*d^2*e^2 - 2*b*d^3*e))/(c^2*e^6 + a^2*d^4*e^2 + b
^2*d^2*e^4 - 2*b*c*d*e^5 - 2*a*b*d^3*e^3 + 2*a*c*d^2*e^4) - (log(a^2*b^2*d^
6 - 4*a^3*c*d^6 - 2*c^4*e^6 - b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2) +
 24*a*c^3*d^2*e^4 + 6*b^3*c*d^3*e^3 + 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2 -
4*a*c)^(1/2) - 10*a^2*c^2*d^4*e^2 - 9*b^2*c^2*d^2*e^4 - 2*a*b^3*d^5*e + 4*b
*c^3*d*e^5 - b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b^2
 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d^5*e + 8*a*c
^3*d*e^5*x - 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*
d^5*e*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^2*d^3*e^3 + 6*a*b^2*c*d^4*e^2 - 6*a*b^
3*d^4*e^2*x + 2*a^2*b^2*d^5*e*x - 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b^2
 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(b^
2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*c^2*d^3*e^3
*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*a*
c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2 -
 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b^2
 - 4*a*c)^(1/2) + 6*a*b*c^2*d^2*e^4*x + 2*a*b^2*c*d^3*e^3*x + 23*a^2*b*c*d^
4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 + b
^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 5*
a*b^2*c*d^2 + b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e - 3*a*b*c*d^2*(
b^2 - 4*a*c)^(1/2) + 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e*(b^2 - 4
*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e^4
 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3 -
8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) - (log(2*c^4*e^
6 + 4*a^3*c*d^6 - a^2*b^2*d^6 + b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2)
 - 24*a*c^3*d^2*e^4 - 6*b^3*c*d^3*e^3 - 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2
```

```
- 4*a*c)^(1/2) + 10*a^2*c^2*d^4*e^2 + 9*b^2*c^2*d^2*e^4 + 2*a*b^3*d^5*e - 4
*b*c^3*d*e^5 + b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b
^2 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c*d^5*e - 8*a
*c^3*d*e^5*x + 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d^5*e*(b^2 - 4*a*c)^(1/2) + 20*a*b*c^2*d^3*e^3 - 6*a*b^2*c*d^4*e^2 + 6*a*
b^3*d^4*e^2*x - 2*a^2*b^2*d^5*e*x + 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(
b^2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*c^2*d^3*e
^3*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*
a*c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2
 - 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b
^2 - 4*a*c)^(1/2) - 6*a*b*c^2*d^2*e^4*x - 2*a*b^2*c*d^3*e^3*x - 23*a^2*b*c*
d^4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 -
 b^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e -
5*a*b^2*c*d^2 - b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e + 3*a*b*c*d^2
*(b^2 - 4*a*c)^(1/2) - 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d*e*(b^2 -
 4*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e
^4 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3
- 8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) + d^3/(e^2*(d
 + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.73 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2} dx$

Optimal. Leaf size=194

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+\right.}{2\left(ad^2-e(bd-ce)\right)^2}$$

```
[Out] -d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)*ln(e*x+d)/(a*d^2-e*(b*d-c*
e))^2-1/2*d*(b*d-2*c*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(b^2*d^2-2*b*
c*d*e-2*c*(a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*
d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1445, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+\right.}{2\left(ad^2-e(bd-ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c*
(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*Log[d + e*x])/(a*d^2 - e*(b*
d - c*e))^2 - (d*(b*d - 2*c*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c
*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1445

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^2}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)^2}+\frac{de(bd-2ce)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)}+\frac{-c\left(ad^2-ce^2\right)\cdots}{\left(ad^2-e(bd-ce)\cdots\right.}\right.$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}+\frac{\int \frac{-c\left(ad^2-ce^2\right)-ad(bd-2ce)x}{c+bx+ax^2}\,d\cdots}{\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}-\frac{(d(bd-2ce))\int \frac{b+2ax}{c+bx+ax^2}\cdots}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}-\frac{d(bd-2ce)\log\left(c+bx\cdots\right.}{2\left(ad^2-e(bd-ce)\cdots\right.}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}-\frac{\left(b^2d^2-2bcde-2c\left(ad^2-ce^2\right)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}\cdots$$

Mathematica [A] time = 0.23, size = 159, normalized size = 0.82

$$\frac{\frac{2\left(2c\left(ce^2-ad^2\right)+b^2d^2-2bcde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{2d^2\left(ad^2+e(ce-bd)\right)}{e(d+ex)}-d(bd-2ce)\log(x(ax+b)+c)+2d(bd-2ce)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2 - 2*b*c*d*
e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b
^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x] - d*(b*d - 2*c*e)*Log[c + x*(b
 + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

fricas [B] time = 13.25, size = 1120, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c
^2)*d^2*e^2 + (2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d
*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x
^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x +
c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*
c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(a*x^2 + b*x + c) - 2*((b^3 -
 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 -
2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*
(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^
3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)
*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^
2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*c^2 - 4*a*c^3)*e^6)*x), -1/2*(2*
(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c^2)*d^2*e
^2 - 2*(2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d*e^3 -
2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4
*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*
log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e
^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))
/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^
2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c
^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*
c^2 - 4*a*c^3)*e^6)*x)]
```

giac [A] time = 0.35, size = 331, normalized size = 1.71

$$\frac{\left(b^2d^2e^2 - 2\,acd^2e^2 - 2\,bcde^3 + 2\,c^2e^4\right)\arctan\left(\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4\right)\sqrt{-b^2+4\,ac}} - \frac{d^2e}{\left(ad^2e^2 - bde^3 + ce^4\right)(xe+d)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + 2*c^2*e^4)*arctan((2*a*d - 2*a
*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-
b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
- 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - d^2*e/((a*d^2*e^2 - b*d*e^3
+ c*e^4)*(x*e + d)) - 1/2*(b*d^2 - 2*c*d*e)*log(a - 2*a*d/(x*e + d) + a*d^2
/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2*
d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)
```

maple [B] time = 0.01, size = 389, normalized size = 2.01

$$-\frac{2ac\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}} + \frac{b^2d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}} - \frac{2bcde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}} + \frac{2c^2e^2\,\mathrm{ar}}{\left(a\,d^2-deb\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*b*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln
(a*x^2+b*x+c)*c*d*e-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x
+b)/(4*a*c-b^2)^(1/2))*a*c*d^2+1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^
2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*e+2/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*e^2-d^2/e/(a*d^
```

```
2-b*d*e+c*e^2)/(e*x+d)+d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-2*d/(a*d^2-b*d
*e+c*e^2)^2*ln(e*x+d)*c*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.09, size = 1585, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(2*a*b^3*d^4 + b*c^3*e^4 - c^3*e^4*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d^3
*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4*x + b^2*c^2*e^4*x - b^4*
d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3*c*d^4*x - 2*a*c^3*e^4*x
+ 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) - a^2*c*d^4*(b^2 - 4*a*c)^(1/2) - 6*a*b^2
*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x - 2*b*c^2*d*e^3*(b^2 - 4*a*c)^
(1/2) + 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) - b*c^2*e^4*x*(b^2 - 4*a*c)^(1/2)
 + 10*a*b*c^2*d^2*e^2 + 14*a*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^
2*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 28*a^2*c^2*d^2*
e^2*x - 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^2*d*e^3*x - 12*a^2*b*
c*d^3*e*x - 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) + 8*a*c^2*d*e^3*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e^3*x*(b^2 - 4
*a*c)^(1/2) + 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*(d^2*(b^3/2 + (b^2*(b^
2 - 4*a*c)^(1/2))/2) - c*(d^2*(2*a*b + a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e +
b*e*(b^2 - 4*a*c)^(1/2))) + c^2*(e^2*(b^2 - 4*a*c)^(1/2) + 4*a*d*e)))/(4*a^
3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2
*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*
e + 2*a*b^2*c*d^2*e^2) - (log(2*a*b^3*d^4 + b*c^3*e^4 + c^3*e^4*(b^2 - 4*a*
c)^(1/2) + 16*a^2*c^2*d^3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4
*x + b^2*c^2*e^4*x - b^4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3
*c*d^4*x - 2*a*c^3*e^4*x - 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) + a^2*c*d^4*(b^2
 - 4*a*c)^(1/2) - 6*a*b^2*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x + 2*b
*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) + b*c^2*
e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2 - 14*a*c^2*d^2*e^2*(b^2 - 4*
a*c)^(1/2) - b^2*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*x*(b^2 - 4*a*c
)^(1/2) + 28*a^2*c^2*d^2*e^2*x + 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*
b*c^2*d*e^3*x - 12*a^2*b*c*d^3*e*x + 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) -
8*a*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) +
 2*b^2*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2
))*(c*(d^2*(2*a*b - a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e - b*e*(b^2 - 4*a*c)^(
1/2))) - d^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + c^2*(e^2*(b^2 - 4*a*c)
^(1/2) - 4*a*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4
- b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c
^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) + (log(d + e*x)*(b*d^2 - 2*
c*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*
c*d^2*e^2) - d^2/(e*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.74 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$

Optimal. Leaf size=183

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{\left(ad^2-ce^2\right)l}{\left(ad^2-e(b\right.}$$

```
[Out] d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^2
+1/2*(a*d^2-c*e^2)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2+(b*c*e^2+a*d*(b*d-
4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))^2/(-4*a*c
+b^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{\left(ad^2-ce^2\right)l}{\left(ad^2-e(b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*ArcT
anh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*
e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*d^2
 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \int \frac{x}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e\left(-ad^2+ce^2\right)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{ce(2ad-\ldots}{\left(ad^2-e(b\ldots\right.}\right.$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{ce(2ad-be)+a\left(ad^2-ce^2\right)x}{c+bx+ax^2}\,d\ldots}{\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(ad^2-ce^2\right)\int \frac{b+2ax}{c+bx+ax^2}\ldots}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(ad^2-ce^2\right)\log\left(c+bx\ldots\right.}{2\left(ad^2-e(bd-ce\ldots\right.}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\left(bce^2+ad(bd-4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(ad^2-\ldots\right.}{\left(ad^2\ldots\right.}$$

Mathematica [A] time = 0.24, size = 148, normalized size = 0.81

$$\frac{-\frac{2\left(ad(bd-4ce)+bce^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(ad^2-ce^2\right)\log(x(ax+b)+c)+\frac{2d\left(ad^2+e(ce-bd)\right)}{d+ex}+\left(2ce^2-2ad^2\right)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] ((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(b*d - 4*c*
e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-2*a*d^2
+ 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log[c + x*(b + a*x)])/(2*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

fricas [B] time = 14.16, size = 1059, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^
2)*d*e^2 + (a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 +
b*c*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt
(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b
^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x
)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2
 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2
*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^
2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4
+ ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^
3)*e^5)*x), 1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2
*c - 4*a*c^2)*d*e^2 + 2*(a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4
*a*c*d*e^2 + b*c*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a
*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 +
 ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a*x^2 + b*x + c)
- 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d
^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2
*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3
*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*
d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

giac [A] time = 0.37, size = 323, normalized size = 1.77

$$-\frac{1}{2}\left(\frac{2\left(abd^2e-4\,acde^2+bce^3\right)\arctan\left(\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4\right)\sqrt{-b^2+4\,ac}}-\frac{\left(ad^2-ce^2\right)\log\left(a-\frac{2\,ad}{xe+d}+\frac{ad^2}{(xe+d)^2}+\frac{b\cdots}{xe+\cdots}\right.}{a^2d^4e-2\,abd^3e^2+b^2d^2e^3+2\,acd^2e\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*arctan((2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
)*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^
3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - (a*d^2 - c*e^2)*log(a - 2*a*d/(x*e + d)
+ a*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2
)/(a^2*d^4*e - 2*a*b*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 +
c^2*e^5) - 2*d*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)))*e
```

maple [A] time = 0.01, size = 328, normalized size = 1.79

$$-\frac{ab\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}+\frac{4acde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}-\frac{bc\,e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}-\frac{a\,d^2\ln\left(ex+\cdots\right.}{\left(a\,d^2-deb+c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*d^2-1/2/(a*d^2-b*d*e+c*e^2)^2*l
n(a*x^2+b*x+c)*c*e^2-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2+4/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*e^2+d/(a*d^2-b*d*e+c*e^2)
/(e*x+d)-1/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln
(e*x+d)*c*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.07, size = 1768, normalized size = 9.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] d/((d + e*x)*(a*d^2 + c*e^2 - b*d*e)) - (log(56*a^3*b^2*c*d^4 - 96*a^4*c^2*
d^4 - 96*a^2*c^4*e^4 - 8*b^4*c^2*e^4 - 8*a^2*b^4*d^4 + 56*a*b^2*c^3*e^4 - 4
*a^3*b^3*d^4*x + 320*a^3*c^3*d^2*e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*
d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e^4*x*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4*x +
 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2) - 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3
*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*d*e^3*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d
^4*x - 112*a^2*b^2*c^2*d^2*e^2 - 8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*
c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*
e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 16*a*b^3*c^
2*d*e^3 + 8*a*b^4*c*d^2*e^2 - 64*a^2*b*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e - 64*
a^3*b*c^2*d^3*e + 60*a*b^3*c^2*e^4*x - 112*a^2*b*c^3*e^4*x + 4*a*b^5*d^2*e^
2*x - 8*a^2*b^4*d^3*e*x + 256*a^3*c^3*d*e^3*x - 256*a^4*c^2*d^3*e*x - 6*a*b
^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 160*a^2*b^2*c^2*d*e^3*x - 56*a^2*b^3*c*d
^2*e^2*x + 160*a^3*b*c^2*d^2*e^2*x + 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(
b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*d^3*e*x)*(b^2*((a*d^2)/2 - (c*e^2)/2) - b
*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2) - 2*a^2*c*
d^2 + 2*a*c^2*e^2 + 2*a*c*d*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*
e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3
*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*
e^2) - (log(8*a^2*b^4*d^4 + 96*a^4*c^2*d^4 + 96*a^2*c^4*e^4 + 8*b^4*c^2*e^4
 - 56*a^3*b^2*c*d^4 - 56*a*b^2*c^3*e^4 + 4*a^3*b^3*d^4*x - 320*a^3*c^3*d^2*
e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e
^4*x*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4*x + 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2)
- 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*
d*e^3*x*(b^2 - 4*a*c)^(5/2) - 16*a^4*b*c*d^4*x + 112*a^2*b^2*c^2*d^2*e^2 -
8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*
a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3
*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 16*a*b^3*c^2*d*e^3 - 8*a*b^4*c*d^2*e^2 + 64*
a^2*b*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e + 64*a^3*b*c^2*d^3*e - 60*a*b^3*c^2*e^
4*x + 112*a^2*b*c^3*e^4*x - 4*a*b^5*d^2*e^2*x + 8*a^2*b^4*d^3*e*x - 256*a^3
*c^3*d*e^3*x + 256*a^4*c^2*d^3*e*x - 6*a*b^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2)
+ 160*a^2*b^2*c^2*d*e^3*x + 56*a^2*b^3*c*d^2*e^2*x - 160*a^3*b*c^2*d^2*e^2*
x - 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(b^2 - 4*a*c)^(3/2) - 96*a^3*b^2*c
*d^3*e*x)*(b*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2
) + b^2*((a*d^2)/2 - (c*e^2)/2) - 2*a^2*c*d^2 + 2*a*c^2*e^2 - 2*a*c*d*e*(b^
2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 -
 b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^
2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(a*d^2 - c*e
^2))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*c*d
^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.75 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{e}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+$$

```
[Out] -e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*(2*a*d-b*e)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^
2-1/2*e*(2*a*d-b*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(2*a^2*d^2+b^2*e^
2-2*a*e*(b*d+c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e)
)^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1569, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{e}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2 - 2*a*e*(b
*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^
2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[d + e*x])/(a*d^2 - e*(b*d - c*
e))^2 - (e*(2*a*d - b*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx=\int \frac{1}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$=-\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{\int \frac{ad-be-aex}{(d+ex)\left(c+bx+ax^2\right)}\,dx}{ad^2-bde+ce^2}$$

$$=-\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{\int \left(\frac{e^2(2ad-be)}{\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right.}{ad^2-bde+ce^2}$$

$$=-\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}+\frac{\int \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(}{c+bx+ax^2}}{\left(ad^2-e(bd-ce\right.}$$

$$=-\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}-\frac{(e(2ad-be))\int \frac{b+2ax}{c+bx+ax^2}}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$=-\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}-\frac{e(2ad-be)\log\left(c+bx\right.}{2\left(ad^2-e(bd-ce\right.}$$

$$=-\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)}-\frac{\left(2a^2d^2+b^2e^2-2ae(bd+ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+$$

Mathematica [A] time = 0.21, size = 151, normalized size = 0.80

$$\frac{\frac{2\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{2e\left(ad^2+e(ce-bd)\right)}{d+ex}+e(be-2ad)\log(x(ax+b)+c)-2e(be-2ad)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] ((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^2*e^2 - 2*
a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*d + b*e)*Log[c + x*(b + a*x)])
/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

fricas [B] time = 7.01, size = 1079, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a
*c^2)*e^3 + (2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e -
 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a
*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + (2
*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a^2*c)*
d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^
4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3
 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^
2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c
^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), -1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e -
2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a*c^2)*e^3 + 2*(2*a^2*d^3 - 2*a*b*d^
2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)
) + (2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*
c)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a
^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4
*a*b*c)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b
*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d
^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3
- 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4
*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

giac [A] time = 0.35, size = 331, normalized size = 1.75

$$-\frac{\left(2\,a^2d^2e^2 - 2\,abde^3 + b^2e^4 - 2\,ace^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4\right)\sqrt{-b^2+4\,ac}} - \frac{\left(2\,ade - be^2\right)\log\left(-a + \frac{2\,ad}{xe+d}\right.}{2\left(a^2d^4 - 2\,abd^3e + b^2d^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*a*c*e^4)*arctan(-(2*a*d - 2*a*d
^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^
2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 -
2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*a*d*e - b*e^2)*log(-a +
 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 -
c*e^2/(x*e + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2
*b*c*d*e^3 + c^2*e^4) - e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d))
```

maple [B] time = 0.01, size = 386, normalized size = 2.04

$$\frac{2a^2d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}} - \frac{2abde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}} - \frac{2ac\,e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}} + \frac{b^2e^2\,\mathrm{arcta}}{\left(a\,d^2-deb+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x)
```

```
[Out] -1/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*ln
(a*x^2+b*x+c)*b*e^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x
+b)/(4*a*c-b^2)^(1/2))*a^2*d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^
2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*e^2+1/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2-e/(a*d^2-b*
d*e+c*e^2)/(e*x+d)+2*e/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a*d-e^2/(a*d^2-b*d*e
+c*e^2)^2*ln(e*x+d)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.11, size = 1782, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(c*e^4*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4 - 8*b^6*e^4*x - 4*a^3*d^4*(b^2
 - 4*a*c)^(3/2) - 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a*c)^(3/2) + 60*a*b^
3*c^2*e^4 - 112*a^2*b*c^3*e^4 + 4*a*b^5*d^2*e^2 - 8*a^2*b^4*d^3*e + 256*a^3
*c^3*d*e^3 - 256*a^4*c^2*d^3*e - 8*a^4*b^2*d^4*x + 32*a^3*c^3*e^4*x + 10*b*
d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d^4
+ 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*b^2*c*e^4*(b^2 - 4*
a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^3*x*(b^2 - 4*a*c)^(
5/2) + 24*a*b^4*c*d*e^3 + 64*a*b^4*c*e^4*x + 32*a*b^5*d*e^3*x - 8*a^2*b*d^3
*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*
d^3*e + 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 160*a^2
*b^2*c^2*d*e^3 - 56*a^2*b^3*c*d^2*e^2 + 160*a^3*b*c^2*d^2*e^2 - 136*a^2*b^2
*c^2*e^4*x - 40*a^2*b^4*d^2*e^2*x - 448*a^4*c^2*d^2*e^2*x + 48*a^2*b*d^2*e^
2*x*(b^2 - 4*a*c)^(3/2) + 272*a^3*b^2*c*d^2*e^2*x - 64*a^4*b*c*d^3*e*x - 24
*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 240*a^2*b^3*c*d*e^3*x + 448*a^3*b*c^2*
d*e^3*x)*(a*(e^2*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d - b*d*(b^2 - 4*
a*c)^(1/2))) - e^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + a^2*(d^2*(b^2 -
4*a*c)^(1/2) - 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^
2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8
*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(b*e^
2 - 2*a*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3
+ 2*a*c*d^2*e^2) - (log(c*e^4*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4 + 8*b^6*e^4
*x - 4*a^3*d^4*(b^2 - 4*a*c)^(3/2) + 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a
*c)^(3/2) - 60*a*b^3*c^2*e^4 + 112*a^2*b*c^3*e^4 - 4*a*b^5*d^2*e^2 + 8*a^2*
b^4*d^3*e - 256*a^3*c^3*d*e^3 + 256*a^4*c^2*d^3*e + 8*a^4*b^2*d^4*x - 32*a^
3*c^3*e^4*x + 10*b*d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2
) - 16*a^4*b*c*d^4 - 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*
b^2*c*e^4*(b^2 - 4*a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^
3*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^4*c*d*e^3 - 64*a*b^4*c*e^4*x - 32*a*b^5*d*
```

```
e^3*x - 8*a^2*b*d^3*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3
/2) - 96*a^3*b^2*c*d^3*e - 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a
*c)^(3/2) + 160*a^2*b^2*c^2*d*e^3 + 56*a^2*b^3*c*d^2*e^2 - 160*a^3*b*c^2*d^
2*e^2 + 136*a^2*b^2*c^2*e^4*x + 40*a^2*b^4*d^2*e^2*x + 448*a^4*c^2*d^2*e^2*
x + 48*a^2*b*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 272*a^3*b^2*c*d^2*e^2*x + 64*a
^4*b*c*d^3*e*x - 24*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 240*a^2*b^3*c*d*e^3
*x - 448*a^3*b*c^2*d*e^3*x)*(e^2*(b^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2) - a*
(e^2*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d + b*d*(b^2 - 4*a*c)^(1/2)))
 + a^2*(d^2*(b^2 - 4*a*c)^(1/2) + 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a
^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e
+ 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) -
e/((d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.76 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$

Optimal. Leaf size=248

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{}{d(}$$

```
[Out] e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)+ln(x)/c/d^2-e^2*(3*a*d^2-e*(2*b*d-c*e))*l
n(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))^2-1/2*(a^2*d^2+b^2*e^2-a*e*(2*b*d+c*e))*ln
(a*x^2+b*x+c)/c/(a*d^2-e*(b*d-c*e))^2+(b^3*e^2-a*b*e*(2*b*d+3*c*e)+a^2*d*(b
*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*d^2-e*(b*d-c*e))^2/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 893, 634, 618, 206, 628}

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{}{d(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c*
e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b
^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2 - e
*(2*b*d - c*e))*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 +
 b^2*e^2 - a*e*(2*b*d + c*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \int \frac{1}{x(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int\left(\frac{1}{cd^2x} + \frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e^3\left(-3ad^2+e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2(d+ex)}\right.$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\tanh}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Mathematica [A] time = 0.25, size = 246, normalized size = 0.99

$$\frac{\left(-a^2d^2+ae(2bd+ce)-b^2e^2\right)\log(x(ax+b)+c)}{2c\left(ad^2+e(ce-bd)\right)^2} - \frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2} + \frac{}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*(2*b*d + 3
*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c*Sqr
t[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*
d^2 + e*(-2*b*d + c*e))*Log[d + e*x])/(a*d^3 + d*e*(-(b*d) + c*e))^2 + ((-(
a^2*d^2) - b^2*e^2 + a*e*(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 391, normalized size = 1.58

$$-\frac{\left(a^2bd^2e^2-2\,ab^2de^3+4\,a^2cde^3+b^3e^4-3\,abce^4\right)\arctan\left(\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2cd^4-2\,abcd^3e+b^2cd^2e^2+2\,ac^2d^2e^2-2\,bc^2de^3+c^3e^4\right)\sqrt{-b^2+4\,ac}}-\frac{\left(a^2d^2-2\,abde+b^2e^2\right.}{2\left(a^2cd^4-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b*d^2*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b*c*e^4)*ar
ctan((2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e +
d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d
^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4)*sqrt(-b^2 + 4*a*c)) - 1
/2*(a^2*d^2 - 2*a*b*d*e + b^2*e^2 - a*c*e^2)*log(a - 2*a*d/(x*e + d) + a*d^
2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2
*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 +
c^3*e^4) + e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(x*e + d)) + log(abs(-d/(
x*e + d) + 1))/(c*d^2)
```

maple [B] time = 0.01, size = 589, normalized size = 2.38

$$-\frac{a^2b\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;c}-\frac{4a^2de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}+\frac{2a\,b^2de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;c}+\frac{3ab\,e^2\,\text{ar}}{\left(a\,d^2-deb\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)^2/c*a^2*ln(a*x^2+b*x+c)*d^2+1/(a*d^2-b*d*e+c*e^2)^
2/c*a*ln(a*x^2+b*x+c)*b*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*e^2
-1/2/(a*d^2-b*d*e+c*e^2)^2/c*ln(a*x^2+b*x+c)*b^2*e^2-1/(a*d^2-b*d*e+c*e^2)^
2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*d^2-4/(a*d^
2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*
d*e+2/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2
)^(1/2))*a*b^2*d*e+3/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2))*a*b*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2+ln(x)/c/d^2+e^2/d/(a*d^2-b*d*e+c
*e^2)/(e*x+d)-3*e^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a+2*e^3/(a*d^2-b*d*e+c*
e^2)^2/d*ln(e*x+d)*b-e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*ln(e*x+d)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 25.28, size = 3510, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log((a^4*e^4)/(d*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*
e^2 - b*d*e)^2) - (((a*e^3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d
^2*e^2 - 7*a^2*b^2*d^3*e + 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e
- a*b^2*c*d*e^3 - 8*a^2*b*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((
a*e*(a^3*b*d^5 - 4*a*c^3*e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2
- 3*a^2*b^2*d^4*e - 8*a^2*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b
*c^2*d*e^4 + 6*a*b^2*c*d^2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d
*e^2) + (a*e*x*(3*a^4*d^5 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4
*a^2*c^2*d*e^4 + 19*a^3*c*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a
^3*b*d^4*e + 15*a*b^2*c*d*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d
*e^2) - (a*e*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2
*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c
)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^
2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*
c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e
^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^
2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c
^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x
))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 +
b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5
*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*
(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 -
4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*
a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4
*a*b^2*c*e^4 - 4*a*b^3*d*e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2
- 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2
+ a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)
^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))
)/(2*(4*a*c^4*e^4 + 4*a^3*c^2*d^4 - b^2*c^3*e^4 - a^2*b^2*c*d^4 + 2*b^3*c^2
*d*e^3 - b^4*c*d^2*e^2 + 8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^
3*e - 8*a^2*b*c^2*d^3*e + 2*a*b^2*c^2*d^2*e^2)) + (log((a^4*e^4)/(d*(a*d^2
+ c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e^
3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d^2*e^2 - 7*a^2*b^2*d^3*e
+ 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e - a*b^2*c*d*e^3 - 8*a^2*b
*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(a^3*b*d^5 - 4*a*c^3*
e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2 - 3*a^2*b^2*d^4*e - 8*a^2
*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b*c^2*d*e^4 + 6*a*b^2*c*d^
2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d*e^2) + (a*e*x*(3*a^4*d^5
 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4*a^2*c^2*d*e^4 + 19*a^3*c
*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a^3*b*d^4*e + 15*a*b^2*c*d
*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d*e^2) - (a*e*(b^4*e^2 - 4
*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*
a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e +
 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2
 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*
c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e
```

```
*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*
d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c*(4*a*c - b^2)*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*a^4*d^4 + b^4*e^4 + 2*a^2
*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*a*b^3*d*
e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2))*
(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*
c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a
^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1
/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 -
 b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*
d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)
^(1/2) + 8*a^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2
 - 4*a*c)^(1/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^4 + 4*a^3
*c^2*d^4 - b^2*c^3*e^4 - a^2*b^2*c*d^4 + 2*b^3*c^2*d*e^3 - b^4*c*d^2*e^2 +
8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e - 8*a^2*b*c^2*d^3*e +
 2*a*b^2*c^2*d^2*e^2)) - (log(d + e*x)*(c*e^4 + 3*a*d^2*e^2 - 2*b*d*e^3))/(
a^2*d^6 + b^2*d^4*e^2 + c^2*d^2*e^4 - 2*a*b*d^5*e + 2*a*c*d^4*e^2 - 2*b*c*d
^3*e^3) + log(x)/(c*d^2) + e^2/(d*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.77 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

Optimal. Leaf size=291

$$\frac{\left(2a^3cd^2-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{(ad-be)\left(abd+2ace+\right.}{2c^2\left(ad^2-\right.}$$

```
[Out] -1/c/d^2/x-e^3/d^2/(a*d^2-e*(b*d-c*e))/(e*x+d)-(b*d+2*c*e)*ln(x)/c^2/d^3+e^
3*(4*a*d^2-e*(3*b*d-2*c*e))*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))^2+1/2*(a*d-b*
e)*(a*b*d+2*a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))^2+(2*a^3*c
*d^2-b^4*e^2+2*a*b^2*e*(b*d+2*c*e)-a^2*(b^2*d^2+6*b*c*d*e+2*c^2*e^2))*arcta
nh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/
2)
```

Rubi [A] time = 0.56, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2a^3cd^2+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{(ad-be)\left(abd+2ace\right.}{2c^2\left(ad^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d^
2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^
2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 -
 e*(b*d - c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(
3*b*d - 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e
)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \int \frac{1}{x^2(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^2}+\frac{-bd-2ce}{c^2d^3x}+\frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)^2}+\frac{e^4\left(4ad^2-e(3bd-\right.}{d^3\left(ad^2-e(bd-ce)\right)}\right.$$

$$= -\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{(bd+2ce)\log(x)}{c^2d^3}+\frac{e^3\left(4ad^2-e(3bd-\right.}{d^3\left(ad^2-e\right.}$$

$$= -\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{(bd+2ce)\log(x)}{c^2d^3}+\frac{e^3\left(4ad^2-e(3bd-\right.}{d^3\left(ad^2-e\right.}$$

$$= -\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{(bd+2ce)\log(x)}{c^2d^3}+\frac{e^3\left(4ad^2-e(3bd-\right.}{d^3\left(ad^2-e\right.}$$

$$= -\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\left(2a^3cd^2-b^4e^2+2ab^2e(bd+2ce)-a^2\right.}{c^2\sqrt{b^2-4ac}\left(\right.}$$

Mathematica [A] time = 0.34, size = 287, normalized size = 0.99

$$\frac{\left(-2a^3cd^2+a^2\left(b^2d^2+6bcde+2c^2e^2\right)-2ab^2e(bd+2ce)+b^4e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right.}{2c^2\left(ad^2+e(ce-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((-2*a^3*
c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^
2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(a*
d^2 + e*(-(b*d) + c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d
^2 + e*(-3*b*d + 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))^2) +
```

```
((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2
+ e*(-(b*d) + c*e))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 487, normalized size = 1.67

$$-\frac{\left(a^2b^2d^2e^2 - 2\,a^3cd^2e^2 - 2\,ab^3de^3 + 6\,a^2bcde^3 + b^4e^4 - 4\,ab^2ce^4 + 2\,a^2c^2e^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1}}{\sqrt{-b^2+4\,ac}}\right.}{\left(a^2c^2d^4 - 2\,abc^2d^3e + b^2c^2d^2e^2 + 2\,ac^3d^2e^2 - 2\,bc^3de^3 + c^4e^4\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b^2*d^2*e^2 - 2*a^3*c*d^2*e^2 - 2*a*b^3*d*e^3 + 6*a^2*b*c*d*e^3 + b^4
*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) -
b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(
-2)/((a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2
*b*c^3*d*e^3 + c^4*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2*b*d^2 - 2*a*b^2*d*e
+ 2*a^2*c*d*e + b^3*e^2 - 2*a*b*c*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*
e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^2*
d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*b*c^3*d*e^3 +
 c^4*e^4) - e^7/((a*d^4*e^4 - b*d^3*e^5 + c*d^2*e^6)*(x*e + d)) - (b*d*e +
2*c*e^2)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^2*d^3) + e/(c*d^3*(d/(x*e + d
) - 1))
```

maple [B] time = 0.01, size = 791, normalized size = 2.72

$$-\frac{2a^3d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;c}+\frac{a^2b^2d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;c^2}+\frac{6a^2bde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}\;c}+\frac{2a}{\left(a\,d^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x)
```

```
[Out] 1/2/c^2/(a*d^2-b*d*e+c*e^2)^2*a^2*ln(a*x^2+b*x+c)*b*d^2+1/c/(a*d^2-b*d*e+c*
e^2)^2*a^2*ln(a*x^2+b*x+c)*d*e-1/c^2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c
)*b^2*d*e-1/c/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*b*e^2+1/2/c^2/(a*d^2-
b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*b^3*e^2-2/c/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d^2+1/c^2/(a*d^2-b*d*e+c*e^
2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2*d^2+6/c/
(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*a^2*b*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*a^2*e^2-2/c^2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan(
(2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*d*e-4/c/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^2+1/c^2/(a*d^2-b*d*e+c*
e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^2-1/c/d^
2/x-1/c^2/d^2*ln(x)*b-2/c/d^3*ln(x)*e-e^3/(a*d^2-b*d*e+c*e^2)/d^2/(e*x+d)+4
*e^3/(a*d^2-b*d*e+c*e^2)^2/d*ln(e*x+d)*a-3*e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*ln
(e*x+d)*b+2*e^5/(a*d^2-b*d*e+c*e^2)^2/d^3*ln(e*x+d)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 31.16, size = 4948, normalized size = 17.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(2*c*e^5 + 4*a*d^2*e^3 - 3*b*d*e^4))/(a^2*d^7 + b^2*d^5*e^2 +
 c^2*d^3*e^4 - 2*a*b*d^6*e + 2*a*c*d^5*e^2 - 2*b*c*d^4*e^3) - (1/(c*d) + (x
*(2*c*e^3 + a*d^2*e - b*d*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))/(d*x + e*x
^2) - (log((((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*e^5 - 2*a*b^5*d^4*e^
4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^7 - b^5*c*d^2*e^6 +
a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5 + a^4*c^2*d^5*e^3 -
12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5 + 4*a*b^2*c^3*d*e^7
 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*c^3*d^2*e^6 + a^2*b^
3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) - (
((a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6 + b^5*d^3*e^3
- 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d^2*e^4 + 8*a^2
*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*c*d^5*e - 7*a*
b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 + 12*a^2*b^2*c*d
^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b^5*e^2 + b^4*e^2*(b^2 - 4
*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/
2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*
b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*
d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) +
 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*
c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d
^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*
b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8
*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c
 - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (2*a*e*x*(a*d - b*e)*(a^3*b*d^5 + 8*a*
c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 - a^2*b^2*d^4*e + 16*
a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b*c^2*d*e^4 - 8*a*b^2
*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e^2
+ b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2
*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*
a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/
2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^
2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)
*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c^4*e^8 + 4*b^4*c^2*e
^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 + 2*a^5*c*d^6*e^2 + a
^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 + 18*a^4*c^2*d^4*e^4 -
 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6 + 8*a*b^3*c^2*d*e^7
 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d^5*e^3 + 10*a^2*b^3*
c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2))*(b^
5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b
^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*
```

```
e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*
c)^(1/2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d
*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e
^2 - 3*b*d*e))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d - b*
e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2))*(b^5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(
1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*
a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e
^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*d*e - 4
*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*
b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*e^4
 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e^2
- 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*d^2
*e^2)) + (log((a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e^2 - 3*b*d*e))/(c^2*d^
4*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*
e^5 - 2*a*b^5*d^4*e^4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^
7 - b^5*c*d^2*e^6 + a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5
+ a^4*c^2*d^5*e^3 - 12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5
 + 4*a*b^2*c^3*d*e^7 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*
c^3*d^2*e^6 + a^2*b^3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c
*e^2 - b*d*e)^2) - (((a*e*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*
d^2 - 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^
2 - 4*a*c)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*
d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b
^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 -
4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*
d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6
*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*
d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*
a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2
 - b*d*e)^2) - (a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6
+ b^5*d^3*e^3 - 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d
^2*e^4 + 8*a^2*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*
c*d^5*e - 7*a*b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 +
12*a^2*b^2*c*d^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (2*a*e*x*(a*d - b*
e)*(a^3*b*d^5 + 8*a*c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 -
 a^2*b^2*d^4*e + 16*a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b
*c^2*d*e^4 - 8*a*b^2*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2
- b*d*e)))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*d^2 - 8*a^2*b*c
^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2
) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*d*e - 2*a^3*c*d
^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/
2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/
(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c
^4*e^8 + 4*b^4*c^2*e^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 +
 2*a^5*c*d^6*e^2 + a^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 +
18*a^4*c^2*d^4*e^4 - 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6
 + 8*a*b^3*c^2*d*e^7 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d
^5*e^3 + 10*a^2*b^3*c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*
e^2 - b*d*e)^2))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*d^2 - 8*a
^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c
)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*d*e - 2*a
^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*
c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1
/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d -
 b*e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) -
b^5*e^2 - a^2*b^3*d^2 - 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) +
 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*
c*e^2 + 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e
```

```
- 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a
^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*
e^4 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e
^2 - 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*
d^2*e^2)) - (log(x)*(b*d + 2*c*e))/(c^2*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.78 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

Optimal. Leaf size=372

$$\frac{\left(a^3cd^2-a^2\left(b^2d^2+4bcde+c^2e^2\right)+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-a^3cd(3bd+4ce)+a^2b\right.}{}$$

```
[Out] -1/2/c/d^2/x^2+(b*d+2*c*e)/c^2/d^3/x+e^4/d^3/(a*d^2-e*(b*d-c*e))/(e*x+d)+(b
^2*d^2+2*b*c*d*e-c*(a*d^2-3*c*e^2))*ln(x)/c^3/d^4-e^4*(5*a*d^2-e*(4*b*d-3*c
*e))*ln(e*x+d)/d^4/(a*d^2-e*(b*d-c*e))^2+1/2*(a^3*c*d^2-b^4*e^2+a*b^2*e*(2*
b*d+3*c*e)-a^2*(b^2*d^2+4*b*c*d*e+c^2*e^2))*ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b
*d-c*e))^2+(b^5*e^2-a^3*c*d*(3*b*d+4*c*e)-a*b^3*e*(2*b*d+5*c*e)+a^2*b*(b^2*
d^2+8*b*c*d*e+5*c^2*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*d^2-
e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(-a^2\left(b^2d^2+4bcde+c^2e^2\right)+a^3cd^2+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2b\left(b^2d^2+8bcde+5c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d - c
*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d + 5*
c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^
2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2
+ 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*(4
*b*d - 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^2
- b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e^2))*
Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \int \frac{1}{x^3(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^3} + \frac{-bd-2ce}{c^2d^3x^2} + \frac{b^2d^2+2bcde-c\left(ad^2-3ce^2\right)}{c^3d^4x} + \frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right.}\right.$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-\right.\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-\right.\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-\right.\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^5e^2-a^3cd(3bd+4ce)\right.}{}$$

Mathematica [A] time = 0.43, size = 370, normalized size = 0.99

$$-\frac{\left(-a^3cd^2+a^2\left(b^2d^2+4bcde+c^2e^2\right)-ab^2e(2bd+3ce)+b^4e^2\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)^2}+\frac{\left(a^3cd(3bd+4ce)-a^2b\left(b^2d^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) + ((-(b^5*e^2) + a^3*c*d*(3*b*d + 4*c*e) + a*b^3*e*(2*
b*d + 5*c*e) - a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2)
```

```
+ ((b^2*d^2 + 2*b*c*d*e + c*(-(a*d^2) + 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*
(5*a*d^2 + e*(-4*b*d + 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 + e*(-(b*d) + c*e)
)^2) - ((-(a^3*c*d^2) + b^4*e^2 - a*b^2*e*(2*b*d + 3*c*e) + a^2*(b^2*d^2 +
4*b*c*d*e + c^2*e^2))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(b*d) + c*e
))^2)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.49, size = 587, normalized size = 1.58

$$\frac{\left(a^2b^3d^2e^2 - 3\,a^3bcd^2e^2 - 2\,ab^4de^3 + 8\,a^2b^2cde^3 - 4\,a^3c^2de^3 + b^5e^4 - 5\,ab^3ce^4 + 5\,a^2bc^2e^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - b\cdots\right.}{\sqrt{-\cdots}}\right.}{\left(a^2c^3d^4 - 2\,abc^3d^3e + b^2c^3d^2e^2 + 2\,ac^4d^2e^2 - 2\,bc^4de^3 + c^5e^4\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (a^2*b^3*d^2*e^2 - 3*a^3*b*c*d^2*e^2 - 2*a*b^4*d*e^3 + 8*a^2*b^2*c*d*e^3 -
4*a^3*c^2*d*e^3 + b^5*e^4 - 5*a*b^3*c*e^4 + 5*a^2*b*c^2*e^4)*arctan(-(2*a*d
 - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/
sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^
2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(a
^2*b^2*d^2 - a^3*c*d^2 - 2*a*b^3*d*e + 4*a^2*b*c*d*e + b^4*e^2 - 3*a*b^2*c*
e^2 + a^2*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e
+ d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^3*d^4 - 2*a*b*c^3*d^3*
e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4) + e^9/((a*
d^5*e^5 - b*d^4*e^6 + c*d^3*e^7)*(x*e + d)) + (b^2*d^2*e - a*c*d^2*e + 2*b*
c*d*e^2 + 3*c^2*e^3)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^3*d^4) + 1/2*(2*b
*c*d*e + 5*c^2*e^2 - 2*(b*c*d^2*e^2 + 3*c^2*d*e^3)*e^(-1)/(x*e + d))/(c^3*d
^4*(d/(x*e + d) - 1)^2)
```

maple [**B**] time = 0.02, size = 993, normalized size = 2.67

$$\frac{3a^3b\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}\;c^2} + \frac{4a^3de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}\;c} - \frac{a^2b^3d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}\;c^3} - \frac{8a^2\cdots}{\left(a\,d^2 - \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x)
```

```
[Out] -8/(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)
^(1/2))*a^2*b^2*d*e-1/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(1/2)*arctan((2
*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3*d^2-5/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^2+5/(a*d^2-b*d*e+c*e^2)
^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^2-2/(a
*d^2-b*d*e+c*e^2)^2/c^2*a^2*ln(a*x^2+b*x+c)*b*d*e+1/(a*d^2-b*d*e+c*e^2)^2/c
^3*a*ln(a*x^2+b*x+c)*b^3*d*e+3/(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*b*d^2+4/(a*d^2-b*d*e+c*e^2)^2/c/(4*
a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d*e-1/c^2/d^2*ln(x)*
```

```
a+1/c^3/d^2*ln(x)*b^2+3/c/d^4*ln(x)*e^2+1/c^2/d^2/x*b+2/c/d^3/x*e+e^4/(a*d^
2-b*d*e+c*e^2)/d^3/(e*x+d)+2/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(1/2)*ar
ctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2/c^2*a
^3*ln(a*x^2+b*x+c)*d^2-1/2/c/d^2/x^2-1/2/(a*d^2-b*d*e+c*e^2)^2/c*a^2*ln(a*x
^2+b*x+c)*e^2-1/2/(a*d^2-b*d*e+c*e^2)^2/c^3*ln(a*x^2+b*x+c)*b^4*e^2+2/c^2/d
^3*ln(x)*b*e-5*e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*ln(e*x+d)*a+4*e^5/(a*d^2-b*d*e
+c*e^2)^2/d^3*ln(e*x+d)*b-3*e^6/(a*d^2-b*d*e+c*e^2)^2/d^4*ln(e*x+d)*c-1/2/(
a*d^2-b*d*e+c*e^2)^2/c^3*a^2*ln(a*x^2+b*x+c)*b^2*d^2+3/2/(a*d^2-b*d*e+c*e^2
)^2/c^2*a*ln(a*x^2+b*x+c)*b^2*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 45.61, size = 7144, normalized size = 19.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] ((x*(2*b*d + 3*c*e))/(2*c^2*d^2) - 1/(2*c*d) + (x^2*(3*c^2*e^4 - b^2*d^2*e^
2 + a*b*d^3*e - b*c*d*e^3 + 2*a*c*d^2*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e
)))/(d*x^2 + e*x^3) - (log(d + e*x)*(3*c*e^6 + 5*a*d^2*e^4 - 4*b*d*e^5))/(a
^2*d^8 + b^2*d^6*e^2 + c^2*d^4*e^4 - 2*a*b*d^7*e + 2*a*c*d^6*e^2 - 2*b*c*d^
5*e^3) + (log((((27*a^2*b*c^6*e^11 - 9*a*b^3*c^5*e^11 - a*b^8*d^5*e^6 - a^6
*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^7*d^6*e^5 - a^3*b^6*d^7*e^4 - a^4
*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*c^5*d^3*e^8 + 4*a^5*c^4*d^5*e^6 +
 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a^2*b^3*c^4*d^2*e^9 - 15*a^2*b^4*c
^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^3*b^2*c^4*d^3*e^8 + 7*a^3*b^3*c^3
*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4*b^2*c^3*d^5*e^6 + 33*a^4*b^3*c^2
*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4*c^4*d*e^10 - 2*a*b^7*c*d^4*e^7 +
 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 + 12*a^2*b^6*c*d^5*e^6 + 51*a^3*b*
c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^4*b*c^4*d^4*e^7 + 6*a^4*b^4*c*d^7
*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*d^8*e^3 - a^6*b*c^2*d^8*e^3 - 4*a
^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(12*a*c^5*e^
7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*e^3 - 3*a*b^5*d^5*e^2 + 3*a^2*b^4
*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 + b^5*c*d^3*e^4 + 8*a^2*c^4*d^2*e^
5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 + 2*a^4*b*c*d^7 - 4*a*b*c^4*d*e^6 +
 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^3 - 10*a^3*b^2*c*d^6*e - 6*a*b^2*
c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2*b*c^3*d^3*e^4 + 15*a^2*b^3*c*d^5
*e^2 - 16*a^3*b*c^2*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(4*a
^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*
e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a
*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^
2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x -
6*a*b^2*c*d^2*e^2*x)*(b^6*e^2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 +
 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c
)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d
*e - 16*a^3*b*c^2*d*e - 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(
b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b
```

^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2
- 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (a*e*x*(
2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e
^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 - 5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 +
32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 - 24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e
- 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d*e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b
^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24*a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2
*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)))*(b^6*e^
2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e
^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a
^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^
3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c
^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*
d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a
c - b^2)(a*d^2 + c*e^2 - b*d*e)^2) - (x*(18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^
11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e
^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2*a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*
e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6*e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*
b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 - 85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b
^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 - 42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b
^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 - 6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^
3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a
^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 + 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*
c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5*b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*
e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 +
b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2
- 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*
b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3*b
*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*
d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e
*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a
^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4 + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^
3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e - 12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d
^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*
b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*
b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2
+ b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^
2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^
2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3
*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^
2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d
e(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^6*
e^4 + 4*a^3*c^4*d^4 - b^2*c^5*e^4 + 2*b^3*c^4*d*e^3 - a^2*b^2*c^3*d^4 + 8*a
^2*c^5*d^2*e^2 - b^4*c^3*d^2*e^2 - 8*a*b*c^5*d*e^3 + 2*a*b^3*c^3*d^3*e - 8*
a^2*b*c^4*d^3*e + 2*a*b^2*c^4*d^2*e^2)) + (log((((27*a^2*b*c^6*e^11 - 9*a*b
^3*c^5*e^11 - a*b^8*d^5*e^6 - a^6*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^
7*d^6*e^5 - a^3*b^6*d^7*e^4 - a^4*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*
c^5*d^3*e^8 + 4*a^5*c^4*d^5*e^6 + 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a
^2*b^3*c^4*d^2*e^9 - 15*a^2*b^4*c^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^
3*b^2*c^4*d^3*e^8 + 7*a^3*b^3*c^3*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4
*b^2*c^3*d^5*e^6 + 33*a^4*b^3*c^2*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4
*c^4*d*e^10 - 2*a*b^7*c*d^4*e^7 + 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 +
12*a^2*b^6*c*d^5*e^6 + 51*a^3*b*c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^
4*b*c^4*d^4*e^7 + 6*a^4*b^4*c*d^7*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*
d^8*e^3 - a^6*b*c^2*d^8*e^3 - 4*a^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2
- b*d*e)^2) + (((a*e*(12*a*c^5*e^7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*
e^3 - 3*a*b^5*d^5*e^2 + 3*a^2*b^4*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 +
b^5*c*d^3*e^4 + 8*a^2*c^4*d^2*e^5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 +

2*a^4*b*c*d^7 - 4*a*b*c^4*d*e^6 + 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^
3 - 10*a^3*b^2*c*d^6*e - 6*a*b^2*c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2
*b*c^3*d^3*e^4 + 15*a^2*b^3*c*d^5*e^2 - 16*a^3*b*c^2*d^5*e^2))/(c^2*d^3*(a*
d^2 + c*e^2 - b*d*e)) + (a*e*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e
^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*
a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3
*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^
2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x)*(b^6*e^2 - b^5*e^2*(b
^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2
*c*d^2 - a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2
 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2
 - 4*a*c)^(1/2) + 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 -
4*a*c)^(1/2) - 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a
*c)^(1/2) - 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d
^2 + c*e^2 - b*d*e)^2) - (a*e*x*(2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^
7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 -
5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 + 32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 -
24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e - 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d*
e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24
*a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2))/(c^2*d^3*
(a*d^2 + c*e^2 - b*d*e)))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*
d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 -
4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^
3*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*
e^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e
^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*
(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (x*
(18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a
^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2
*a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6*
e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 -
85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 -
42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 -
6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^
9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 +
 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5
*b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^
2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2
 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4*a
*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c
*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*e^2
*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e^2*
(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*(b^
2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e
^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4
 + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e -
12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^
2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c
^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d
^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4
*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3
*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*e
^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e^
2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*(
b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^6*e^4 + 4*a^3*c^4*d^4 - b^2*c^5*e^4 + 2*b^3*
c^4*d*e^3 - a^2*b^2*c^3*d^4 + 8*a^2*c^5*d^2*e^2 - b^4*c^3*d^2*e^2 - 8*a*b*c
^5*d*e^3 + 2*a*b^3*c^3*d^3*e - 8*a^2*b*c^4*d^3*e + 2*a*b^2*c^4*d^2*e^2)) +
(log(x)*(3*c^2*e^2 - d^2*(a*c - b^2) + 2*b*c*d*e))/(c^3*d^4)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.79 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+ex}\,dx$

Optimal. Leaf size=981

$$\frac{2}{11}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,\sqrt{d+ex}\,x^5+\frac{2(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,(d+ex)^{7/2}x}{99ae^4}-\frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{b}{x}+}}{693a^2e^4}$$

```
[Out] 2/3465*(233*a^3*d^3+48*b^3*e^3+a*b*e^2*(67*b*d-157*c*e)+4*a^2*d*e*(18*b*d-3
7*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/a^3/e^4-2/693*(29*a^2*d^2+8*b^2
*e^2+a*e*(19*b*d-18*c*e))*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a^2/e^4+2/99*
(a*d+b*e)*x*(e*x+d)^(7/2)*(a+c/x^2+b/x)^(1/2)/a/e^4-2/3465*(187*a^4*d^4+64*
b^4*e^4+4*a*b^2*e^3*(7*b*d-69*c*e)-4*a^3*d^2*e*(2*b*d+3*c*e)+3*a^2*e^2*(3*b
^2*d^2-29*b*c*d*e+50*c^2*e^2))*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^4/e^4+
2/11*x^5*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3465*(128*a^5*d^5+128*b^5*e^5-
4*a^4*d^3*e*(14*b*d-27*c*e)-8*a*b^3*e^4*(7*b*d+87*c*e)-a^2*b*e^3*(37*b^2*d^
2-258*b*c*d*e-771*c^2*e^2)-a^3*d*e^2*(37*b^2*d^2-135*b*c*d*e+156*c^2*e^2))*
x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/
(-4*a*c+b^2))^(1/2)/a^5/e^5/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)-2/3465*(a*d^2-e*(b*d-c*e))*(128*a^4*d^4-64*b^4*e^4-4*a*b^
2*e^3*(7*b*d-69*c*e)+4*a^3*d^2*e*(2*b*d+3*c*e)-3*a^2*e^2*(3*b^2*d^2-29*b*c*
d*e+50*c^2*e^2))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+
c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a
^5/e^5/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A] time = 6.17, antiderivative size = 981, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1573, 918, 1653, 843, 718, 424, 419}

$$\frac{2}{11}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,\sqrt{d+ex}\,x^5+\frac{2(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,(d+ex)^{7/2}x}{99ae^4}-\frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{b}{x}+}}{693a^2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

```
[Out] (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*a^3*d^2*e*
(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a +
 c/x^2 + b/x]*x*Sqrt[d + e*x])/(3465*a^4*e^4) + (2*Sqrt[a + c/x^2 + b/x]*x^
5*Sqrt[d + e*x])/11 + (2*(233*a^3*d^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*
c*e) + 4*a^2*d*e*(18*b*d - 37*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2)
)/(3465*a^3*e^4) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(693*a^2*e^4) + (2*(a*d + b*e)*Sqrt[a
+ c/x^2 + b/x]*x*(d + e*x)^(7/2))/(99*a*e^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d - 27*c*e) - 8*a*b^3*e^4*(7*
b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 - 258*b*c*d*e - 771*c^2*e^2) - a^3*d*
```

```
e^2*(37*b^2*d^2 - 135*b*c*d*e + 156*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[
d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3465*a^5*e^5*Sqrt[(a*(d
+ e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (2*Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(128*a^4*d^4 - 64*b^4*e^4 - 4*a
*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*
d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(3465*a^5*e^5*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
```

```
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int x^3\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{x^3\left(-3cd-2(bd+ce)x-(ad+be)x^2\right)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,d}{11\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} + \frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4} - \frac{\left(2\sqrt{a+\right.}{}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} - \frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{c}{x^2}}}{693a^2e^4}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} + \frac{2\left(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+\right.}{3465}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b\right.\right.}{3465a^4e^4}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b\right.\right.}{3465a^4e^4}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b\right.\right.}{3465a^4e^4}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b\right.\right.}{3465a^4e^4}$$

Mathematica [C] time = 14.20, size = 10904, normalized size = 11.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
[Out] Result too large to show
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex+d}\,x^4\sqrt{\frac{ax^2+bx+c}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x^4*sqrt((a*x^2 + b*x + c)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

maple [B] time = 0.16, size = 11938, normalized size = 12.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Timed out
```

3.80 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^3 \sqrt{d+ex}\, dx$

Optimal. Leaf size=778

$$-\frac{4x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,\left(8a^2d^2+ae(4bd-7ce)+3b^2e^2\right)}{315a^2e^3}+\frac{2x\sqrt{d+ex}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,\left(19a^3d^3-6a^2cde^2+3a\cdots\right.}{315a^3e^3}$$

```
[Out] -4/315*(8*a^2*d^2+3*b^2*e^2+a*e*(4*b*d-7*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/x
)^(1/2)/a^2/e^3+2/63*(a*d+b*e)*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a/e^3+2/
315*(19*a^3*d^3-6*a^2*c*d*e^2+8*b^3*e^3+3*a*b*e^2*(b*d-9*c*e))*x*(a+c/x^2+b
/x)^(1/2)*(e*x+d)^(1/2)/a^3/e^3+2/9*x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)-2
/315*(8*a^4*d^4+8*b^4*e^4-a^3*d^2*e*(4*b*d-9*c*e)-4*a*b^2*e^3*(b*d+9*c*e)-3
*a^2*e^2*(b^2*d^2-5*b*c*d*e-7*c^2*e^2))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x
)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^4/e^4/(a*x^2+
b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(16*a^3*d^3
+6*a^2*c*d*e^2-8*b^3*e^3-3*a*b*e^2*(b*d-9*c*e))*(a*d^2-e*(b*d-c*e))*x*Ellip
ticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-
2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(
a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a^4/e^4/(a*x^2+b*x+c)/(e*
x+d)^(1/2)
```

Rubi [A] time = 2.35, antiderivative size = 778, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1573, 918, 1653, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\,\sqrt{d+ex}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,\left(-3a^2e^2\left(b^2d^2-5bcde-7c^2e^2\right)-a^3d^2e(4bd-9ce)+8a\cdots\right.}{315a^4e^4\left(ax^2+bx+c\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c*e))*Sqrt[
a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sqrt[a + c/x^2 + b/x]*
x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*e^2 + a*e*(4*b*d - 7*c*e))*Sqr
t[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a
 + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(8*a^4*d^4 + 8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9
*c*e) - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*
x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[
(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*
Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e
^2*(b*d - 9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))
```

```
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt
[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
4*a*c])*e)])/(315*a^4*e^4*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^3\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{x^2\left(-3cd-2(bd+ce)x-(ad+be)x^2\right)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{9\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+ex} + \frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{5/2}}{63ae^3} - \frac{\left(2\sqrt{a+\right.}{}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+ex} - \frac{4\left(8a^2d^2+3b^2e^2+ae(4bd-7ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{315a^2e^3}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}$$

Mathematica [C] time = 13.74, size = 7531, normalized size = 9.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
[Out] Result too large to show
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex+d}\,x^3\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*x^3*sqrt((a*x^2 + b*x + c)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

maple [B] time = 0.07, size = 9182, normalized size = 11.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Timed out
```

3.81 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^2\sqrt{d+ex}\, dx$

Optimal. Leaf size=636

$$-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{105a^2e^2}-\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\cdot}}{}$$

```
[Out] -2/105*x*(4*a^2*d^2+4*b^2*e^2-a*e*(2*b*d-5*c*e)-3*a*e*(a*d-4*b*e)*x)*(a+c/x
^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^2/e^2+2/7*x*(a*x^2+b*x+c)*(a+c/x^2+b/x)^(1/2)
*(e*x+d)^(1/2)/a+1/105*(8*a^3*d^3+8*b^3*e^3-a^2*d*e*(5*b*d-16*c*e)-a*b*e^2*
(5*b*d+29*c*e))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2
))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-
a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^3/e^3/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/105*(8*a^2*d^2-4*b^2*e^2-a*e*(b*d-10*c*
e))*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x
^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)/a^3/e^3/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1573, 832, 814, 843, 718, 424, 419}

$$-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{105a^2e^2}-\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\cdot}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2 - a*e*(2*b
*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (2*Sqrt[a + c/x^2 + b
/x]*x*Sqrt[d + e*x]*(c + b*x + a*x^2))/(7*a) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*b*d - 16*c*e) - a*b*e^2*(5*b*d + 29*c*e)
)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(105*a^3*e^3*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^2*d^2 - 4*b^2*e^2 -
a*e*(b*d - 10*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a
*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(105*a^3*e^3*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
```

```
2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && !IntegerQ[p] &&
!IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^2\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int x\sqrt{d+ex}\,\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\left(c+bx+ax^2\right)}{7a} + \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\left(\frac{1}{2}(-3bd-ce)+\frac{1}{2}(a}{\sqrt{}}}{7a\sqrt{c+bx+ax}}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2}$$

Mathematica [C] time = 13.05, size = 5350, normalized size = 8.41

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
[Out] Result too large to show
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex+d}\,x^2\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*x^2*sqrt((a*x^2 + b*x + c)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

maple [B] time = 0.05, size = 6302, normalized size = 9.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

```
[Out] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

3.82 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\, dx$

Optimal. Leaf size=550

$$\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{\ldots}}{\sqrt{b^2-\ldots}}}}{\sqrt{2}}\ldots\right.\right.}{15a^2e^2\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

```
[Out] 2/5*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/e-2/15*(2*a*d-b*e)*x*(a+c/x^2+b/x)^
(1/2)*(e*x+d)^(1/2)/a/e-2/15*(a^2*d^2+b^2*e^2-a*e*(b*d+3*c*e))*x*EllipticE(
1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(
-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)
)^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)+2/15*(2*a*d-b*e)*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2
*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+
b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4
*a*c+b^2)^(1/2))))^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1573, 734, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{\ldots}}{\sqrt{b^2-\ldots}}}}{\sqrt{2}}\ldots\right.\right.}{15a^2e^2\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (-2*(2*a*d - b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(15*a*e) + (2*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(5*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x
]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sq
rt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x +
a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \sqrt{d+ex}\,\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\,(bd-2ce+(2ad-be)x)}{\sqrt{c+bx+ax^2}}\,dx}{5e\sqrt{c+bx+ax^2}}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} - \frac{\left(2\sqrt{a+\right.}{}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} + \frac{\left((2ad-\right.}{}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} - \frac{\left(2\sqrt{2}\,\sqrt{b\right.}{}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} - \frac{2\sqrt{2}\,\sqrt{b^2}}{}$$

Mathematica [C] time = 11.77, size = 693, normalized size = 1.26

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(\frac{4e^2\left(-a^2d^2+ae(bd+3ce)-b^2e^2\right)}{\sqrt{d+ex}} + \frac{i(d+ex)\sqrt{1-\frac{2\left(ad^2+e(ce-bd)\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}+2ad-be\right)}}\sqrt{\frac{4\left(ad^2+e(ce-bd)\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}-2ad+be\right)}+2}\left(\left(a^2d\left(8ce^2-d\sqrt{e^2\left(b^2-\right.}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*((4*e^2*(-(a^2*d^2) - b^2*e^2 + a*e*(b*d + 3*c*e
)))/Sqrt[d + e*x] + 2*a*e^2*Sqrt[d + e*x]*(b*e + a*(d + 3*e*x)) + (I*(d + e
*x)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*e)))/((2*a*d - b*e + Sqrt[(b^2 - 4*a
*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(a*d^2 + e*(-(b*d) + c*e)))/((-2*a*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)
*e^2])*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*EllipticE[I*ArcSinh[(Sqrt[2]
*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sq
rt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2]))] + (b^2*e^2*(b*e - Sqrt[(b^2 - 4*a*c)*e^2]) + a^2*d*
(8*c*e^2 - d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e*(-2*b^2*d*e - 4*b*c*e^2 + b*d*S
qrt[(b^2 - 4*a*c)*e^2] + 3*c*e*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d -
```

```
 b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*
d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))))/(15*a^2*e^3)
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex+d}\,x\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x*sqrt((a*x^2 + b*x + c)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

maple [B] time = 0.05, size = 4361, normalized size = 7.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] -1/15*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(2^(1/2)*(-a*(e*x+d)/((-4*a
*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c
+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1
/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1
/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3+2*2^(1/2)*(-a*(e*x+d)
/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(
(-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4
*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+
b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+
b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*d^3*e+8*2^(1/2)*(-a*
(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/
2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2
))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((
-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((
-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a^2*b*d^3*e+8*2^(1/2)*(-a*(e*x+d)/((
-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4
*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*
c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2
)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2
)^(1/2)*e+2*a*d-b*e))^(1/2))*a^2*c*d^2*e^2-8*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b
^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2
)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(
1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*
e+2*a*d-b*e))^(1/2))*a*b^2*d^2*e^2-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a
```

```
*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*
e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*
e))^(1/2))*(-4*a*c+b^2)^(1/2)*b*c*e^4-12*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^
(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1
/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)
*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*
a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2))*a^2*c*d^2*e^2+3*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d
+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)
)^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e)
)^(1/2))*a*b^2*d^2*e^2-6*x^4*a^3*e^4-8*x^3*a^3*d*e^3-8*x^3*a^2*b*e^4-2*x^2*
a^3*d^2*e^2-6*x^2*a^2*c*e^4-2*x^2*a*b^2*e^4-2*a^2*c*d^2*e^2-8*x*a^2*c*d*e^3
-2*x*a*b^2*d*e^3-2*x*a*b*c*e^4-12*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e
-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2
*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*
d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e
))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e
))^(1/2))*a*c^2*e^4-3*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2
)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-(
(-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*b^
3*d*e^3+3*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-
2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2
*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(
2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)
^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*b^2*c*e^4+12*2
^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a
*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*
c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a
*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*
a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a*c^2*e^4+4*2^(1/2)*(-a*(
e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2
)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)
)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-
4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*b^3*d*e^3-4*2^(1/2)*(-a*(e*x+d)/((-4*
a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*
c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b
^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(
1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(
1/2)*e+2*a*d-b*e))^(1/2))*b^2*c*e^4-2*a*b*c*d*e^3-4*2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*
a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)
^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)
^(1/2)*e+2*a*d-b*e))^(1/2))*a^3*d^4-10*x^2*a^2*b*d*e^3-2*x*a^2*b*d^2*e^2+2*
2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*
a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a
*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-
a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2
*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*a*c*d
*e^3-8*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a
*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*
x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(
1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1
/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a*b*c*d*e^3+12*2^
```

```
(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*
c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c
+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*
(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a
*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a*b*c*d*e^3-3*2^(1/2)*(-a*
(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/
2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2
))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((
-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((
-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*d^2*e^2)/a^2/
(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/e^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Integral(x*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

3.83 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

Optimal. Leaf size=955

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x\ \ 2}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}$$

```
[Out] 2/3*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3*(a*d+b*e)*x*EllipticE(1/2*((b+2
*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^
2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2
)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a
/e/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*d*(
a*d+b*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1
/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a
*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/e/(a*x^
2+b*x+c)/(e*x+d)^(1/2)+4/3*(b*d+c*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e
*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(
1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2)/a/(a*x^2+b*x+c)/(e*x+d)^(1/2)-c*x*EllipticPi(2^(1/2)*a^(
1/2)*(e*x+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(e*(-4*a*c+b^
2)^(1/2)+2*a*d-b*e)/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+
b^2)^(1/2)))^(1/2))*2^(1/2)*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-
(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(
e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)/a^(1/2)
```

Rubi [A] time = 3.31, antiderivative size = 955, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1449, 918, 6742, 718, 419, 934, 169, 538, 537, 843, 424}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x\ \ 2}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a
*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*a*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a +
c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)
) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(
```

```
a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[2]*c*Sqrt[2*a
*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d +
e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e
)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^
2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)])/(Sqrt[a]*(c + b*x + a*x^2))
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1449

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_
)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^n + c/x^(2*
n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[((d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p)/x^(2*n*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q
}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] &&
 PosQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}{x}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{-3cd-2(bd+ce)x-(ad+be)x^2}{x\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}\,dx}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \left(-\frac{2(bd+ce)}{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}} - \frac{3cd}{x\sqrt{d+ex}\,\sqrt{c+bx+\cdots}}\cdots\right.}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{1}{x\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}\,dx}{\sqrt{c+bx+ax^2}} - \frac{\left((-ad-b\cdots\right.}{\cdots}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{b-\sqrt{b^2-4ac}+2ax}\,\sqrt{b+\sqrt{b^2-\cdots}}\right.}{c+b\cdots}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{4\sqrt{2}\,\sqrt{b^2-4ac}\,(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4\cdots}\right.}}}{3a\sqrt{d+\cdots}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a(c\cdots}{\cdots}}}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4\cdots}\right.}}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a(c\cdots}{\cdots}}}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4\cdots}\right.}}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a(c\cdots}{\cdots}}}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4\cdots}\right.}}}$$

Mathematica [C] time = 10.53, size = 1258, normalized size = 1.32

$$\frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{4(ad+be)\sqrt{\frac{ad^2+e(ce-bd)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}\,(c+x(b+ax))e^2}{(d+ex)^2}+\frac{6i\sqrt{2}\,ac\sqrt{\frac{-2ce^2+2adxe+b(d-ex)e+\sqrt{(b^2-4ac)e^2}\,(d+ex)}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{2ce^2-2adxe+b(ex-d)e+\sqrt{(b^2\cdots}}{\left(-2ad+be+\sqrt{(b^2-4ac)e^2}\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2])/3 + (x*(d + e*x)^(3/2)*Sqrt[a +
 (c + b*x)/x^2]*((4*e^2*(a*d + b*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 - (I*Sqrt
[2]*(a*d + b*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e^2 + 2*
a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d
+ e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c
*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
/Sqrt[d + e*x] + (I*Sqrt[2]*(b*e*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2]) + a*(3*
b*d*e - 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + 2*a*d*e*x +
b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + S
qrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*
a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d +
e*x] + ((6*I)*Sqrt[2]*a*c*e^2*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) +
Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c
)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ell
ipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) +
 c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*a*e^
2*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
*(c + x*(b + a*x)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

maple [B] time = 0.05, size = 3023, normalized size = 3.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] 1/3*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)/a*(2^(1/2)*(-a*(e*x+d)/((-4*a
*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c
+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1
/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1
/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*a*d^2*e-2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*
a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)
^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)
^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2-2*2^(1/2)*(-a*(e*x+d
)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/
((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c
+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3+3*2^(1/2)*(-a*(e*
x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-
b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/
((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*
a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*
a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a*b*d^2*e+6*2^(1/2)*(-a*(e*x+d)/((-4*a*
c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+
b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2
)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/
2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/
2)*e+2*a*d-b*e))^(1/2))*a*c*d*e^2-3*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)
*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e
+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*
a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b
*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b
*e))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e
))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))
^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1
/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(
-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*
a^2*d^3-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-
2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2
*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(
2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)
^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a*c*d*e^2+2*2^
(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*
c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c
+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*
(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a
*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-a*(e
*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)
-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))
/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4
*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4
```

```
*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*b*c*e^3+3*2^(1/2)*(-a*(e*x+d)/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b
^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)
^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/
2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b
^2)^(1/2)*c*e^3-6*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/
2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)
*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*El
lipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*(
(-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4
*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a*c*d*e^2+3*2^(1/2)*(-a*(e*x+d)/((-4*a
*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c
+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(
1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a
*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*b*c*e^3
+2*x^3*a^2*e^3+2*x^2*a^2*d*e^2+2*x^2*a*b*e^3+2*x*a*b*d*e^2+2*x*a*c*e^3+2*a*
c*d*e^2)/e^2/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Timed out
```

3.84 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{x}\,dx$

Optimal. Leaf size=929

$$\frac{3\sqrt{b^2-4ac}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\left(ax^2+bx+c\right)}-\frac{3\sqrt{2}\sqrt{b^2-4ac}\,c}{}$$

```
[Out] -(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+3/2*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(
e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/(a*x^2+b*x+c)*2^(1/2)/(a
*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-3*d*x*EllipticF(1/2*((b+2*
a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2
)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)/(e*x+d)^(1/2)+2*(a*d+b*e)*
x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^
(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/(a*x^2+b*x+c)/(e
*x+d)^(1/2)-1/2*(b*d+c*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d
-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e)/a/d,(
(b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c/
x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)
```

Rubi [A] time = 2.72, antiderivative size = 929, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1573, 916, 6742, 718, 419, 934, 169, 538, 537, 843, 424}

$$\frac{3\sqrt{b^2-4ac}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\left(ax^2+bx+c\right)}-\frac{3\sqrt{2}\sqrt{b^2-4ac}\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqrt[a + c/x^
2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*
Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2))
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))
/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4
*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(Sqrt[d + e*x]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d +
```

```
 b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*Sqrt[d + e*x]*(c + b
*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a
+ c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticP
i[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt
[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]*d*(c +
 b*x + a*x^2))
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\sqrt{c+bx+ax^2}}{x^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{bd+ce+2(ad+be)x+3aex^2}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \left(\frac{2(ad+be)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}} + \frac{bd+ce}{x\sqrt{d+ex}\sqrt{c+bx+}\cdots}\right.}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{x}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}} + \frac{\left((ad+\cdots\right.}{\cdots}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b\cdots}\right.}{2\left(\cdots\right.}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{2\sqrt{2}\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-\cdots}\right.}}}{a\sqrt{d\cdots}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}\,E\cdots}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\cdots}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}\,E\cdots}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\cdots}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}\,E\cdots}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\cdots}$$

Mathematica [C] time = 11.41, size = 1372, normalized size = 1.48

$$\frac{x(d+ex)^{3/2}\sqrt{a+\frac{c+bx}{x^2}}\left(12d\sqrt{\frac{ad^2+e(ce-bd)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}\left(a\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ce}{d+ex}\right)}{d+ex}\right)-\frac{3i\sqrt{2}d\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{-\frac{2c}{d+}\cdots}}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]) + (x*(d + e*x)^(3/2)*Sqrt[a + (c +
 b*x)/x^2]*(12*d*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])]*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(
d + e*x)))/(d + e*x)) - ((3*I)*Sqrt[2]*d*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(
d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(
d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] +
(I*Sqrt[2]*(4*a*d^2 - b*d*e - 2*c*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(
Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) +
b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(
Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) +
b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Ellipt
icF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + ((2*I)*Sqrt[
2]*e*(b*d + c*e)*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*
d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*
d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])]*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*
(a*d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2
)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqr
t[d + e*x]))/(4*d*e*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])]*Sqrt[c + b*x + a*x^2]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

maple [B] time = 0.05, size = 3553, normalized size = 3.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b
^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2
)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(
1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*
e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x*a*d^2*e-2*2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*
a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)
^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)
^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x*b*d*e^2+4*2^(1/2)*(-a*(e*x
+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b
)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(
(-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a
*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a
*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x*a^2*d^3-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b
^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2
)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(
1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*
e+2*a*d-b*e))^(1/2))*x*a*b*d^2*e+6*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a
*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*
e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*
e))^(1/2))*x*a*c*d*e^2-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*
e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e)
)^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(
1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),
(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))
*x*b^2*d*e^2-6*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*
(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e
*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*Ellip
ticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x*a^2*d^3
+6*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(
-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-
4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)
*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x*a*b*d^2*e-6*2^(1/2)
*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2
```

```
)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)
^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+
d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*
e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x*a*c*d*e^2+2^(1/2)*(-a*(e*x+d)
/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(
(-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4
*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(
-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*
(-4*a*c+b^2)^(1/2)*x*b*d*e^2+2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*
d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-
b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e
))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(
1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*
d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x*c*e^3-
2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-
4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)
*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/
2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*
e+2*a*d-b*e))^(1/2))*x*a*b*d^2*e-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a
*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b
*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x*a*c*d*e^2+2^(1/2)*(
-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^
(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(
1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d
)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+
b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e
))^(1/2))*x*b^2*d*e^2+2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2
)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1
/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/
((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x*b*c*e^3-2*x^3*a^2*d*e^2-2*x^2*a^
2*d^2*e-2*x^2*a*b*d*e^2-2*x*a*b*d^2*e-2*x*a*c*d*e^2-2*a*c*d^2*e)/(a*e*x^3+a
*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/a/e/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x,x)
```

```
[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)
[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x, x)
```

3.85 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

Optimal. Leaf size=1287

$$\frac{\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\Pi\left(\frac{2ad-be+\sqrt{b^2-4ac}\,e}{2ad};\sin^{-1}\left(\cdots\right.\right.}{4\sqrt{2}\sqrt{a}\,cd^2\left(ax^2+bx+c\right)}$$

```
[Out] -1/4*(b*d+c*e)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/c/d-1/2*(a+c/x^2+b/x)^(1/2
)*(e*x+d)^(1/2)/x+1/8*(b*d+c*e)*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(
-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^
(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/c/d/(a*x^2+b*x+c)*2^(1/2)/(a*(e
*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+3/2*e*x*EllipticF(1/2*((b+2*a
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2
+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)*2^(1/2)/(e*x+d)^(1/2)-1/4*(b*d+c*e)
*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4
*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(
a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(a*x^2+b*x+c)*2^(1/2)/(
e*x+d)^(1/2)-1/2*(a*d+b*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e)/a/d,
((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c
/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)+1/8*(b*d+c*e)^2*x*EllipticPi
(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(
e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e)/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*
d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e
*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2
*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/d^2/(a*x^2+b*x+c)*2^(1
/2)/a^(1/2)
```

Rubi [A] time = 5.30, antiderivative size = 1287, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {1573, 916, 6742, 718, 419, 939, 934, 169, 538, 537, 843, 424}

$$\frac{\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\Pi\left(\frac{2ad-be+\sqrt{b^2-4ac}\,e}{2ad};\sin^{-1}\left(\cdots\right.\right.}{4\sqrt{2}\sqrt{a}\,cd^2\left(ax^2+bx+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] -((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(4*c*d) - (Sqrt[a + c/x^
2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x
```

```
^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Elli
pticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(4*Sqrt[
2]*c*d*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a
*x^2)) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(Sqrt[2]*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d + c*
e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[2]*c*Sqrt[d + e*x]
*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*
Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*El
lipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[
a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 -
 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]
*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])
*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*S
qrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b
^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(4*Sqrt[2]*S
qrt[a]*c*d^2*(c + b*x + a*x^2))
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
```

```
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
```

```
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{x^2}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}{x^3}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{bd+ce+2(ad+be)x+3aex^2}{x^2\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}\,dx}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \left(\frac{3ae}{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}} + \frac{bd+ce}{x^2\sqrt{d+ex}\,\sqrt{c+bx+ax^2}} \cdots\right.}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{1}{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}\,dx}{4\sqrt{c+bx+ax^2}} + \frac{\left((ad+be)\cdots\right.}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\cdots}\right.}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}\,e\sqrt{a+\frac{c}{x^2}\cdots}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}\,e\sqrt{a+\frac{c}{x^2}\cdots}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}\,e\sqrt{a+\frac{c}{x^2}\cdots}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}\,e\sqrt{a+\frac{c}{x^2}\cdots}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\sqrt{b^2-4ac}\,(bd+ce)\sqrt{\cdots}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\sqrt{b^2-4ac}\,(bd+ce)\sqrt{\cdots}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\sqrt{b^2-4ac}\,(bd+ce)\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 12.53, size = 811, normalized size = 0.63

$$\frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(-\frac{8cd^3}{x^2}-\frac{8ced^2}{x}-\frac{4(bd+ce)d^2}{x}-\frac{i(d+ex)^{3/2}\sqrt{1-\frac{2(ad^2+e(ce-bd))}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{4(ad^2+e(ce-bd))}{\left(-2ad+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}+2}\left(d(bd+ce)\left(2ad-\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*((-8*c*d^3)/x^2 - (8*c*d^2*e)/x - (4*d^2*(b*d +
c*e))/x - (I*(d + e*x)^(3/2)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*e)))/((2*a*
d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(a*d^2 + e*(-(b*
d) + c*e)))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*(d*(b*d +
 c*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*EllipticE[I*ArcSinh[(Sqrt[2]*
Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqr
t[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2]))] - (b^2*d^2*e + b*d*(-5*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*
e^2]) + c*e*(4*a*d^2 + 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*Ar
cSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a
*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + 2*e*(b^2*d^2 - 2*b*c*d*e + c*(-4*a*
d^2 + c*e^2))*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*
d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(
-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(e*Sq
rt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c
+ x*(b + a*x)))))/(16*c*d^2*Sqrt[d + e*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

maple [B] time = 0.06, size = 4957, normalized size = 3.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x)
```

```
[Out] 1/8*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(-8*x^2*a*b*c*d^2*e^2-4*a*c^2*d
^3*e-6*x*a*b*c*d^3*e+2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)
)^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^
(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/
2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-
((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x
^2*a*c^2*d*e^3-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*
(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e
*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*Ellip
ticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4
*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*
c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*b^2*d^2*e^2-8*2^(1
/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+
b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b
^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(
e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2
*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*
d-b*e))^(1/2))*x^2*a^2*c*d^3*e+2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d
+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e
))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*a*b^2*d^3*e+2*2^(1/
2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b
^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^
2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e
*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*
a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d
-b*e))^(1/2))*x^2*a*c^2*d*e^3+2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a
*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d
-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*
e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(
1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(
1/2))*(-4*a*c+b^2)^(1/2)*x^2*a*b*d^3*e-5*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^
(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1
/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)
*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*
a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*a*c*d^2*e^2-2*x^3*a*b*c*d*e^3+2*2^(
1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c
+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+
b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*
(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a
*d-b*e))^(1/2))*x^2*b^2*c*d*e^3-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e
-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2
*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*
d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e
))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e
))^(1/2))*x^2*a*c^2*d*e^3-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d
+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b
*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)
)^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/
2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/
2))*x^2*a^2*c*d^3*e+2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2
)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-(
```

```
(-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^
2*a*b^2*d^3*e+12*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2
)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*
(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*Ell
ipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a
*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*a^2
*c*d^3*e-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2
*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*
a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2
^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^
(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*a*b^2*d^3*e
-4*x^2*a^2*c*d^3*e-2*x^2*a*b^2*d^3*e-2*x^4*a^2*b*d^2*e^2-2*x^4*a^2*c*d*e^3-
2*x^3*a^2*b*d^3*e-6*x^3*a^2*c*d^2*e^2-2*x^3*a*b^2*d^2*e^2-2*x^2*a*c^2*d*e^3
-6*x*a*c^2*d^2*e^2-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^
(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1
/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)
*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((
-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2
*a^2*b*d^4-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(
-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+
2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticP
i(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^
2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*b*c^2*e^4-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)
^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1
/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)
^(1/2)*x^2*c^2*e^4-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1
/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2
)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*E
llipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*
((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-
4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*b^3*d^2*e^2-5*2^(1/2)*(-a*(e*x+d)
/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(
(-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4
*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+
b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+
b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*a*b*c*d^2*e^2+4*2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*
a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2
)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*
a*c+b^2)^(1/2)*x^2*a*c*d^2*e^2+2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d
+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e
))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*
b*c*d*e^3)/x/a/e/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/c/d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2,x)
[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)
[Out] Timed out
```

3.86 $\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left((fx)^m \left(a+cx^{2n}\right)^p (d+ex^n)^q ,x\right)$$

[Out] Unintegrable((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is Not applicable to the result.

[In] Int[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]

[Out] Defer[Int][(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]

Rubi steps

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is Not applicable to the result.

[In] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]

[Out] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(cx^{2n}+a\right)^p (ex^n+d)^q \left(fx\right)^m ,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n}+a\right)^p (ex^n+d)^q \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)

maple [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \left(c\,x^{2n}+a\right)^p (e\,x^n+d)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)^q*(c*x^(2*n)+a)^p,x)
[Out] int((f*x)^m*(e*x^n+d)^q*(c*x^(2*n)+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n}+a\right)^{p}(ex^{n}+d)^{q}\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(a+c\,x^{2\,n}\right)^p\,\left(f\,x\right)^m\,\left(d+e\,x^n\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q,x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

3.87 $\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=358

$$\frac{d^3(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{3d^2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1}{m+n+1}$$

```
[Out] d^3*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n]
,-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+3*d^2*e*x^(1+n)*(f*x)^m*(a+c*x^(
2*n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+
n)/((1+c*x^(2*n)/a)^p)+3*d*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom(
[-p, 1/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*x^(2*
n)/a)^p)+e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m+3*n)
/n],[1/2*(1+m+5*n)/n],-c*x^(2*n)/a)/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] time = 0.24, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1561, 365, 364, 20}

$$\frac{3d^2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{d^3(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (3
*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2
*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*
n))/a)^p) + (3*d*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F
1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m
+ 2*n)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*
Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n
))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p dx = \int \left(d^3(fx)^m \left(a+cx^{2n}\right)^p + 3d^2ex^n(fx)^m \left(a+cx^{2n}\right)^p + 3de^2x^{2n}(fx)^m \left(a+cx^{2\cdots}\right.\right.$$
$$= d^3 \int (fx)^m \left(a+cx^{2n}\right)^p dx + \left(3d^2e\right) \int x^n(fx)^m \left(a+cx^{2n}\right)^p dx + \left(3de^2\right) \int x^{\cdots}$$
$$= \left(3d^2ex^{-m}(fx)^m\right) \int x^{m+n} \left(a+cx^{2n}\right)^p dx + \left(3de^2x^{-m}(fx)^m\right) \int x^{m+2n} \left(a+cx^{\cdots}\right.$$
$$= \frac{d^3(fx)^{1+m} \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \left(3d^2ex^{-\cdots}\right.$$
$$= \frac{d^3(fx)^{1+m} \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{3d^2ex^{1+\cdots}}{\cdots}$$

Mathematica [A] time = 0.31, size = 249, normalized size = 0.70

$$x(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \left(\frac{d^3\,{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{m+1} + ex^n \left(\frac{3d^2\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1} + e\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^3*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((3*d^2*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n
) + e*x^n*((3*d*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2
*n), -((c*x^(2*n))/a)])/(1 + m + 2*n) + (e*x^n*Hypergeometric2F1[(1 + m + 3
*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 3*n)))))/(1
 + (c*x^(2*n))/a)^p
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^{3\,n} + 3\,de^2x^{2\,n} + 3\,d^2ex^n + d^3\right)\left(cx^{2\,n} + a\right)^p \left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) + a
)^p*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming x near 0Simplificati
on assuming f near 0Simplification assuming x near 0Simplification assuming
 f near 0Unable to divide, perhaps due to rounding error%%%{-16,[1,0,6,3,1,
2,4,4,1]%%%}+%%%{-64,[1,0,6,3,1,2,3,4,1]%%%}+%%%{-96,[1,0,6,3,1,2,2,4,1]%%%
}+%%%{-64,[1,0,6,3,1,2,1,4,1]%%%}+%%%{-16,[1,0,6,3,1,2,0,4,1]%%%}+%%%{-16,[
1,0,6,3,0,2,4,4,1]%%%}+%%%{-64,[1,0,6,3,0,2,3,4,1]%%%}+%%%{-96,[1,0,6,3,0,2
,2,4,1]%%%}+%%%{-64,[1,0,6,3,0,2,1,4,1]%%%}+%%%{-16,[1,0,6,3,0,2,0,4,1]%%%}
 / %%%{16,[0,0,6,4,0,2,4,4,0]%%%}+%%%{64,[0,0,6,4,0,2,3,4,0]%%%}+%%%{96,[0,
0,6,4,0,2,2,4,0]%%%}+%%%{64,[0,0,6,4,0,2,1,4,0]%%%}+%%%{16,[0,0,6,4,0,2,0,4
,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (e\,x^{n}+d)^{3} \left(f x\right)^{m} \left(c\,x^{2n}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)^3*(c*x^(2*n)+a)^p,x)
[Out] int((f*x)^m*(e*x^n+d)^3*(c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^{n}+d)^{3} \left(cx^{2\,n}+a\right)^{p} \left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+c\,x^{2\,n}\right)^{p} \left(f\,x\right)^{m} (d+e\,x^{n})^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3,x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

3.88 $\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=262

$$\frac{d^2(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+}{2}\right.}{m+n+1}$$

```
[Out] d^2*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n]
,-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+2*d*e*x^(1+n)*(f*x)^m*(a+c*x^(2*
n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+n)
/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1
/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*x^(2*n)/a)^
p)
```

Rubi [A] time = 0.16, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1561, 365, 364, 20}

$$\frac{d^2(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+}{2}\right.}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (2
*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n
), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n)
)/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 +
 m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n)
*(1 + (c*x^(2*n))/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

```
&& (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p dx = \int \left(d^2(fx)^m \left(a+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+cx^{2n}\right)^p + e^2x^{2n}(fx)^m \left(a+cx^{2n}\right.\right.$$

$$= d^2 \int (fx)^m \left(a+cx^{2n}\right)^p dx + (2de) \int x^n(fx)^m \left(a+cx^{2n}\right)^p dx + e^2 \int x^{2n}($$

$$= \left(2dex^{-m}(fx)^m\right) \int x^{m+n} \left(a+cx^{2n}\right)^p dx + \left(e^2x^{-m}(fx)^m\right) \int x^{m+2n} \left(a+cx^{2}\right.$$

$$= \frac{d^2(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \left(2dex\right.$$

$$= \frac{d^2(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{2dex^{1}}{}$$

Mathematica [A] time = 0.16, size = 189, normalized size = 0.72

$$x(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \left(\frac{d^2\,{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{m+1} + ex^n \left(\frac{2d\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1} + \right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^2*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((2*d*Hypergeometric2F1[
(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n)
+ (e*x^n*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -(
(c*x^(2*n))/a)])/(1 + m + 2*n))))/(1 + (c*x^(2*n))/a)^p
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^{2n} + 2\,dex^n + d^2\right)\left(cx^{2n} + a\right)^p \left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming x near 0Simplificati
on assuming f near 0Simplification assuming x near 0Simplification assuming
 f near 0Unable to divide, perhaps due to rounding error%%%{4,[0,0,3,2,1,0,
2,3,1]%%%}+%%%{8,[0,0,3,2,1,0,1,3,1]%%%}+%%%{4,[0,0,3,2,1,0,0,3,1]%%%}+%%%{
4,[0,0,3,2,0,0,2,3,1]%%%}+%%%{8,[0,0,3,2,0,0,1,3,1]%%%}+%%%{4,[0,0,3,2,0,0,
```

```
0,3,1]%%%} / %%%{-8,[0,0,4,3,0,1,3,3,0]%%%}+%%%{-24,[0,0,4,3,0,1,2,3,0]%%%}
+%%%{-24,[0,0,4,3,0,1,1,3,0]%%%}+%%%{-8,[0,0,4,3,0,1,0,3,0]%%%} Error: Bad
Argument Value
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(e\,x^{n}+d\right)^{2} \left(f x\right)^{m} \left(c\,x^{2 n}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)^2*(c*x^(2*n)+a)^p,x)
```

```
[Out] int((f*x)^m*(e*x^n+d)^2*(c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(ex^{n} + d\right)}^{2} {\left(cx^{2 \, n} + a\right)}^{p} \left(f x\right)^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+c\,x^{2\,n}\right)}^p\,{\left(f\,x\right)}^m\,{\left(d+e\,x^n\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2,x)
```

```
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.89 $\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=166

$$\frac{d(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+\ldots}{2\ldots}\right.}{m+n+1}$$

```
[Out] d*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n],-
c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^p*
hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+n)/((1+c
*x^(2*n)/a)^p)
```

Rubi [A] time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{1561, 365, 364, 20\}$

$$\frac{d(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+\ldots}{2\ldots}\right.}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (e*x
^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n), -p,
 (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p
)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^m (d+ex^n)\left(a+cx^{2n}\right)^p\,dx = \int \left(d(fx)^m\left(a+cx^{2n}\right)^p + ex^n(fx)^m\left(a+cx^{2n}\right)^p\right)dx$$
$$= d\int (fx)^m\left(a+cx^{2n}\right)^p\,dx + e\int x^n(fx)^m\left(a+cx^{2n}\right)^p\,dx$$
$$= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+cx^{2n}\right)^p\,dx + \left(d\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int (fx)^m \left(\right.$$
$$= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \left(ex^{-m}(fx)^{\cdots}\right.$$
$$= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{ex^{1+n}(fx)^{\cdots}}{\cdots}$$

Mathematica [A] time = 0.08, size = 136, normalized size = 0.82

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(d(m+n+1)\,{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)+e(m+1)x^n\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};\cdots\right.\right.}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*(d*(1 + m + n)*Hypergeometric2F1[(1 + m)/(2*n)
, -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)] + e*(1 + m)*x^n*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)]))/((1 + m)*
(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left((ex^n+d)\left(cx^{2n}+a\right)^p(fx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n+d)\left(cx^{2n}+a\right)^p(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (e\,x^n+d)\,(fx)^m\left(c\,x^{2n}+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)*(c*x^(2*n)+a)^p,x)
```

```
[Out] int((f*x)^m*(e*x^n+d)*(c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)\left(cx^{2n} + a\right)^p \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + c\,x^{2\,n}\right)^p \left(f\,x\right)^m \,(d + e\,x^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n),x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

3.90 $\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx$

Optimal. Leaf size=194

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,1;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)}-\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n}{2n}\cdots\right.}{d^2(m+n+1)}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,1,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d/(1+m)/((1+c*x^(2*n)/a)^p)-e*x^(1+n)*(f*x)^m*(a+c*x^(
2*n))^p*AppellF1(1/2*(1+m+n)/n,1,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^2/(1+m+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,1;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)}-\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n}{2n}\cdots\right.}{d^2(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1 + (c*x^(2*n))/a)^p)
 - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 1
, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m +
n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{d+ex^n}\,dx = \left(x^{-m}(fx)^m\right)\int\left(\frac{dx^m\left(a+cx^{2n}\right)^p}{d^2-e^2x^{2n}}+\frac{ex^{m+n}\left(a+cx^{2n}\right)^p}{-d^2+e^2x^{2n}}\right)dx$$
$$= \left(dx^{-m}(fx)^m\right)\int\frac{x^m\left(a+cx^{2n}\right)^p}{d^2-e^2x^{2n}}\,dx+\left(ex^{-m}(fx)^m\right)\int\frac{x^{m+n}\left(a+cx^{2n}\right)^p}{-d^2+e^2x^{2n}}\,dx$$
$$= \left(dx^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{d^2-e^2x^{2n}}\,dx+\left(ex^{-m}(fx)^m\left(a+cx^{2n}\right.\right.$$
$$= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,1;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(1+m)}-\frac{ex^{1+n}(fx)^m\left(\right.}{}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^{2n}+a\right)^p\left(fx\right)^m}{ex^n+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+a\right)^p\left(fx\right)^m}{ex^n+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m\left(c\,x^{2n}+a\right)^p}{e\,x^n+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d),x)
[Out] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n} + a\right)^p \left(fx\right)^m}{ex^n + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + c\,x^{2n}\right)^p \left(f\,x\right)^m}{d + e\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n),x)
```

```
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] Timed out
```

3.91 $\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=302

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,2;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}+\frac{e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1}{d^4(m+2}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,2,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d^2/(1+m)/((1+c*x^(2*n)/a)^p)-2*e*x^(1+n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,2,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*
x^(2*n)/a)/d^3/(1+m+n)/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*
n))^p*AppellF1(1/2*(1+m+2*n)/n,2,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^4/(1+m+2*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] time = 0.33, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,2;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}-\frac{2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1}{d^3(m+n}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 2, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*
AppellF1[(1 + m + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a),
 (e^2*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx = \left(x^{-m}(fx)^m\right)\int \left(\frac{d^2x^m \left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^2} - \frac{2dex^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^2} + \frac{e^2x^{m+2n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^2}\right) dx$$

$$= \left(d^2x^{-m}(fx)^m\right)\int \frac{x^m\left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^2}\,dx - \left(2dex^{-m}(fx)^m\right)\int \frac{x^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^2}\,dx + \left(e^2x^{-m}\right.$$

$$= \left(d^2x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(d^2-e^2x^{2n}\right)^2}\,dx - \left(2dex^{-m}(fx)^m\left(a+cx^{2n}\right.\right.$$

$$= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,2;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m)} - \frac{2ex^{1+n}(fx)^m\left(a\right.}{}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(cx^{2n}+a\right)^p\left(fx\right)^m}{e^2x^{2n}+2\,dex^n+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+a\right)^p\left(fx\right)^m}{(ex^n+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m\left(c\,x^{2n}+a\right)^p}{\left(e\,x^n+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^2,x)
```

```
[Out] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+a\right)^{p}\left(fx\right)^{m}}{\left(ex^{n}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+c\,x^{2\,n}\right)^{p}\,\left(f\,x\right)^{m}}{\left(d+e\,x^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2,x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Timed out
```

3.92 $\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx$

Optimal. Leaf size=412

$$-\frac{e^3x^{3n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+3n+1}{2n};-p,3;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(m+3n+1)}+\frac{3e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+\right.}{d^5}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,3,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d^3/(1+m)/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,3,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*
x^(2*n)/a)/d^4/(1+m+n)/((1+c*x^(2*n)/a)^p)+3*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(
2*n))^p*AppellF1(1/2*(1+m+2*n)/n,3,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,-c*x^
(2*n)/a)/d^5/(1+m+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*
n))^p*AppellF1(1/2*(1+m+3*n)/n,3,-p,1/2*(1+m+5*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^6/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] time = 0.45, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{1562, 511, 510\}$

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,3;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+1)}-\frac{3ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m\cdots}{}\right.}{d^4(m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 3, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^
p*AppellF1[(1 + m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a
), (e^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^3*x^(
1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2*n), -p, 3, (1
+ m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + m + 3*n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
```

```
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx = \left(x^{-m}(fx)^m\right)\int \left(\frac{d^3x^m \left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^3}+\frac{3d^2ex^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}-\frac{3de^2x^{m+2n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}\right.$$

$$=\left(d^3x^{-m}(fx)^m\right)\int \frac{x^m\left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^3}\,dx+\left(3d^2ex^{-m}(fx)^m\right)\int \frac{x^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}\,dx-\left(3d\right.$$

$$=\left(d^3x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(d^2-e^2x^{2n}\right)^3}\,dx+\left(3d^2ex^{-m}(fx)^m\left(a+\right.\right.$$

$$=\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,3;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m)}-\frac{3ex^{1+n}(fx)^m}{}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^{2n}+a\right)^p\left(fx\right)^m}{e^3x^{3n}+3\,de^2x^{2n}+3\,d^2ex^n+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e
*x^n + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+a\right)^p\left(fx\right)^m}{\left(ex^n+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m\left(c\,x^{2n}+a\right)^p}{\left(e\,x^n+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^3,x)
[Out] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n} + a\right)^p \left(fx\right)^m}{\left(ex^n + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + c\,x^{2\,n}\right)^p \left(f\,x\right)^m}{\left(d + e\,x^n\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3,x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)
[Out] Timed out
```

3.93 $\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x+a)^14
```

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {629}

$$\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (a + b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.18, size = 201, normalized size = 12.56

$$\frac{1}{14}x(b+cx)\left(14a^{13}+91a^{12}x(b+cx)+364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3+2002a^9x^4(b+cx)^4+3003a^8x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c*x)^2 + 10
01*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x)
^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 + 2002*a^5*x^8*(b
+ c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 + 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^
11*(b + c*x)^11 + 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

fricas [B] time = 0.46, size = 1446, normalized size = 90.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + x^26*c^13*a + 26*x^25*c
^11*b^3 + 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 + 78*x^24*c^11*b^2*a + 13/
2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 + 286*x^23*c^10*b^3*a + 78*x^23*c^11*b*a
^2 + 429/2*x^22*c^8*b^6 + 715*x^22*c^9*b^4*a + 429*x^22*c^10*b^2*a^2 + 26*x
^22*c^11*a^3 + 1716/7*x^21*c^7*b^7 + 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^
```

```
3*a^2 + 286*x^21*c^10*b*a^3 + 429/2*x^20*c^6*b^8 + 1716*x^20*c^7*b^6*a + 64
35/2*x^20*c^8*b^4*a^2 + 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 + 143*x
^19*c^5*b^9 + 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 + 4290*x^19*c^8*b
^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10 + 1287*x^18*c^5*b^8*a + 6
006*x^18*c^6*b^6*a^2 + 8580*x^18*c^7*b^4*a^3 + 6435/2*x^18*c^8*b^2*a^4 + 14
3*x^18*c^9*a^5 + 26*x^17*c^3*b^11 + 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*
a^2 + 12012*x^17*c^6*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 + 1287*x^17*c^8*b*a^5
+ 13/2*x^16*c^2*b^12 + 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 + 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 + 5148*x^16*c^7*b^2*a^5 + 429/2
*x^16*c^8*a^6 + x^15*c*b^13 + 78*x^15*c^2*b^11*a + 1430*x^15*c^3*b^9*a^2 +
8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^4 + 12012*x^15*c^6*b^3*a^5 + 1
716*x^15*c^7*b*a^6 + 1/14*x^14*b^14 + 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*
a^2 + 4290*x^14*c^3*b^8*a^3 + 15015*x^14*c^4*b^6*a^4 + 18018*x^14*c^5*b^4*a
^5 + 6006*x^14*c^6*b^2*a^6 + 1716/7*x^14*c^7*a^7 + x^13*b^13*a + 78*x^13*c*
b^11*a^2 + 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 + 18018*x^13*c^4*b
^5*a^5 + 12012*x^13*c^5*b^3*a^6 + 1716*x^13*c^6*b*a^7 + 13/2*x^12*b^12*a^2
+ 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 + 12012*x^12*c^3*b^6*a^5 +
15015*x^12*c^4*b^4*a^6 + 5148*x^12*c^5*b^2*a^7 + 429/2*x^12*c^6*a^8 + 26*x^
11*b^11*a^3 + 715*x^11*c*b^9*a^4 + 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b
^5*a^6 + 8580*x^11*c^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4
+ 1287*x^10*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 + 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 + 143*x^10*c^5*a^9 + 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*
a^6 + 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 + 715*x^9*c^4*b*a^9 + 429
/2*x^8*b^8*a^6 + 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 + 1430*x^8*c^3
*b^2*a^9 + 143/2*x^8*c^4*a^10 + 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 + 1
430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 + 715*x^6*c*b^
4*a^9 + 429*x^6*c^2*b^2*a^10 + 26*x^6*c^3*a^11 + 143*x^5*b^5*a^9 + 286*x^5*
c*b^3*a^10 + 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 + 78*x^4*c*b^2*a^11 + 1
3/2*x^4*c^2*a^12 + 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*x^2*b^2*a^12 +
x^2*c*a^13 + x*b*a^13
```

giac [B] time = 0.43, size = 216, normalized size = 13.50

$$\frac{1}{14}\left(cx^2+bx\right)^{14}+\left(cx^2+bx\right)^{13}a+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2+26\left(cx^2+bx\right)^{11}a^3+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4+143\left(cx^2+bx\right)^9a^5+\frac{4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14 + (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 + 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 + 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 + 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 + 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 + 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 + (c*x^2 + b*x)*a^13
```

maple [B] time = 0.00, size = 46548, normalized size = 2909.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^13,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.43, size = 14, normalized size = 0.88

$$\frac{1}{14}\left(cx^2+bx+a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x + a)^14
```

mupad [B] time = 3.34, size = 1203, normalized size = 75.19

$$x^{12}\left(\frac{429\,a^8\,c^6}{2}+5148\,a^7\,b^2\,c^5+15015\,a^6\,b^4\,c^4+12012\,a^5\,b^6\,c^3+\frac{6435\,a^4\,b^8\,c^2}{2}+286\,a^3\,b^{10}\,c+\frac{13\,a^2\,b^{12}}{2}\right)+x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^13,x)
```

```
[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 + 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148*a^5*b^2*c^7) + x^13*(a*b^13 + 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 + 12012*a^6*b^3*c^5) + x^15*(b^13*c + 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*
b^2*c^2) + x^22*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) + x^18*(143*a^5*c^9 + (1
43*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 + (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 + (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 +
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 + 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 + 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 + x^2*(a^13*c + (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 + 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/2 + b
*c^13*x^27 + (c^12*x^26*(2*a*c + 13*b^2))/2 + a^13*b*x + (143*a^7*b*x^7*(12
*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 13*a^3*b*x
^11*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 + 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 +
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c) + 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 + 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 + 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c + 2*b^2) + 13*b*c^11*x^25*(a*c + 2*b^2)
```

sympy [B] time = 0.35, size = 1326, normalized size = 82.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

```
[Out] a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*b**2*c**12
/2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 + 78*a*
b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 + 286*a*b**3*c**10
+ 143*b**5*c**9) + x**22*(26*a**3*c**11 + 429*a**2*b**2*c**10 + 715*a*b**4*
c**9 + 429*b**6*c**8/2) + x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**9 + 1
287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*a**3*b
**2*c**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/2) + x*
*19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 + 1716*a*b
**7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 + 6435*a**4*b**2*c**8/2 +
8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 + 1287*a*b**8*c**5 + 143*b**10*c*
```

```
*4/2) + x**17*(1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 + 12012*a**3*b**5*c**
6 + 5148*a**2*b**7*c**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a**
6*c**8/2 + 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*a**3*b**6*c**
5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/2) + x**15*(17
16*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 + 8580*a**3*b*
*7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c**2 + b**13*c) + x**14*(1716*a*
*7*c**7/7 + 6006*a**6*b**2*c**6 + 18018*a**5*b**4*c**5 + 15015*a**4*b**6*c*
*4 + 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 + 13*a*b**12*c + b**14/14) +
 x**13*(1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*a**5*b**5*c**4 + 85
80*a**4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b**11*c + a*b**13) + x**1
2*(429*a**8*c**6/2 + 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 + 12012*a**
5*b**6*c**3 + 6435*a**4*b**8*c**2/2 + 286*a**3*b**10*c + 13*a**2*b**12/2) +
 x**11*(1287*a**8*b*c**5 + 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c**3 + 514
8*a**5*b**7*c**2 + 715*a**4*b**9*c + 26*a**3*b**11) + x**10*(143*a**9*c**5
+ 6435*a**8*b**2*c**4/2 + 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 + 1287*
a**5*b**8*c + 143*a**4*b**10/2) + x**9*(715*a**9*b*c**4 + 4290*a**8*b**3*c*
*3 + 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c + 143*a**5*b**9) + x**8*(143*a*
*10*c**4/2 + 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 + 1716*a**7*b**6*c
 + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 + 1430*a**9*b**3*c**2 + 1287*a
**8*b**5*c + 1716*a**7*b**7/7) + x**6*(26*a**11*c**3 + 429*a**10*b**2*c**2
+ 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(78*a**11*b*c**2 + 286*a**10*b*
*3*c + 143*a**9*b**5) + x**4*(13*a**12*c**2/2 + 78*a**11*b**2*c + 143*a**10
*b**4/2) + x**3*(13*a**12*b*c + 26*a**11*b**3) + x**2*(a**13*c + 13*a**12*b
**2/2)
```

3.94 $\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

[Out] 1/28*(c*x^4+b*x^2+a)^14

Rubi [A] time = 0.33, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 629}

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]

[Out] (a + b*x^2 + c*x^4)^14/28

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^2\right)\\&=\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.18, size = 233, normalized size = 12.94

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(14a^{13}+91a^{12}x^2\left(b+cx^2\right)+364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3+2002a^9x^8\left(b+cx^2\right)\right.$$

Antiderivative was successfully verified.

[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]

[Out] (x^2*(b + c*x^2)*(14*a^13 + 91*a^12*x^2*(b + c*x^2) + 364*a^11*x^4*(b + c*x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^8*x^10*(b + c*x^2)^5 + 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x^2)^7 + 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 + 364*a^3*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 + 14*a*x^24*(b + c*x^2)^12 + x^26*(b + c*x^2)^13))/28

fricas [B] time = 0.51, size = 1454, normalized size = 80.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 1/2*x^52*c^13*a + 1
3*x^50*c^11*b^3 + 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*b^4 + 39*x^48*c^11*b
^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^5 + 143*x^46*c^10*b^3*a + 39*x
^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6 + 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^1
0*b^2*a^2 + 13*x^44*c^11*a^3 + 858/7*x^42*c^7*b^7 + 1287/2*x^42*c^8*b^5*a +
 715*x^42*c^9*b^3*a^2 + 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 + 858*x^40
*c^7*b^6*a + 6435/4*x^40*c^8*b^4*a^2 + 715*x^40*c^9*b^2*a^3 + 143/4*x^40*c^
10*a^4 + 143/2*x^38*c^5*b^9 + 858*x^38*c^6*b^7*a + 2574*x^38*c^7*b^5*a^2 +
2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143/4*x^36*c^4*b^10 + 1287/2
*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2 + 4290*x^36*c^7*b^4*a^3 + 6435/4*x^
36*c^8*b^2*a^4 + 143/2*x^36*c^9*a^5 + 13*x^34*c^3*b^11 + 715/2*x^34*c^4*b^9
*a + 2574*x^34*c^5*b^7*a^2 + 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4
+ 1287/2*x^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 + 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 + 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*a^4 + 2574
*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^13 + 39*x^30*c^2*b^11
*a + 715*x^30*c^3*b^9*a^2 + 4290*x^30*c^4*b^7*a^3 + 9009*x^30*c^5*b^5*a^4 +
 6006*x^30*c^6*b^3*a^5 + 858*x^30*c^7*b*a^6 + 1/28*x^28*b^14 + 13/2*x^28*c*
b^12*a + 429/2*x^28*c^2*b^10*a^2 + 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4
*b^6*a^4 + 9009*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 + 858/7*x^28*c^7*a
^7 + 1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 + 715*x^26*c^2*b^9*a^3 + 4290*x^2
6*c^3*b^7*a^4 + 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^6 + 858*x^26*c^
6*b*a^7 + 13/4*x^24*b^12*a^2 + 143*x^24*c*b^10*a^3 + 6435/4*x^24*c^2*b^8*a^
4 + 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24*c^4*b^4*a^6 + 2574*x^24*c^5*b^2*a^
7 + 429/4*x^24*c^6*a^8 + 13*x^22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 + 2574*x^2
2*c^2*b^7*a^5 + 6006*x^22*c^3*b^5*a^6 + 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22
*c^5*b*a^8 + 143/4*x^20*b^10*a^4 + 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 + 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 + 143/2*x^20*c^5*a^
9 + 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 + 2574*x^18*c^2*b^5*a^7 + 2145*
x^18*c^3*b^3*a^8 + 715/2*x^18*c^4*b*a^9 + 429/4*x^16*b^8*a^6 + 858*x^16*c*b
^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 + 715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^
10 + 858/7*x^14*b^7*a^7 + 1287/2*x^14*c*b^5*a^8 + 715*x^14*c^2*b^3*a^9 + 14
3*x^14*c^3*b*a^10 + 429/4*x^12*b^6*a^8 + 715/2*x^12*c*b^4*a^9 + 429/2*x^12*
c^2*b^2*a^10 + 13*x^12*c^3*a^11 + 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*a^10
+ 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 + 39*x^8*c*b^2*a^11 + 13/4*x^8*c^
2*a^12 + 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/4*x^4*b^2*a^12 + 1/2*x^4*
c*a^13 + 1/2*x^2*b*a^13
```

giac [B] time = 0.66, size = 246, normalized size = 13.67

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}+\frac{1}{2}\left(cx^4+bx^2\right)^{13}a+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2+13\left(cx^4+bx^2\right)^{11}a^3+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4+\frac{143}{2}\left(cx^4+bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 + 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 + 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 + 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 + 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 + 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 + 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 + 1/2*(c*x^4 + b*x^2)*a^13
```

maple [B] time = 0.00, size = 46552, normalized size = 2586.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.49, size = 1240, normalized size = 68.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 143/2*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^
13*b*x^2 + 143/14*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b^3 + a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

mupad [B] time = 3.23, size = 1210, normalized size = 67.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 + 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 + 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 + 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 + 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
+ 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 + 2574*a^5*b^2*c^7) + x^26*((a*b
^13)/2 + 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 + 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 + 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 + 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 + 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 + 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 + 13*a^11*c^3 + (715*a^9*b^4*c)/2
+ (429*a^10*b^2*c^2)/2) + x^44*(13*a^3*c^11 + (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 + (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 + (143*a^9*c^5)/2 +
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 + 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
^4)/4) + x^36*((143*a^5*c^9)/2 + (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 + 30
```

```
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 +
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 + 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 + 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 + (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 + 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 + 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 + 858*a*b^6*c^7
+ (6435*a^2*b^4*c^8)/4 + 715*a^3*b^2*c^9) + (c^14*x^56)/28 + x^4*((a^13*c)/
2 + (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 + (c^12*x^52*(2*a*c + 13*b^2))/4 + (143*a^7*b*x^14*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (13*a^3*b*x^2
2*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/2 + (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c + 2*b^2))/2 + (13*b*c^11*x^50*(a*c + 2*b^2
))/2
```

sympy [B] time = 0.34, size = 1384, normalized size = 76.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

```
[Out] a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**13/2 + 13*b
**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c**1
2/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 + 143*a*
b**3*c**10 + 143*b**5*c**9/2) + x**44*(13*a**3*c**11 + 429*a**2*b**2*c**10/
2 + 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 715*a*
*2*b**3*c**9 + 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**1
0/4 + 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 + 429*b*
*8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8 + 2574*a**2*b**
5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(143*a**5*c**9/2 + 6435
*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 + 1287*a*b**8
*c**5/2 + 143*b**10*c**4/4) + x**34*(1287*a**5*b*c**8/2 + 4290*a**4*b**3*c*
*7 + 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**4/2 + 13*b**
11*c**3) + x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 15015*a**4*b**4*c
**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b**10*c**3 + 13
*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**3*c**6 + 9009*a**4*b
**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 + 39*a*b**11*c**2 + b**
13*c/2) + x**28*(858*a**7*c**7/7 + 3003*a**6*b**2*c**6 + 9009*a**5*b**4*c**
5 + 15015*a**4*b**6*c**4/2 + 2145*a**3*b**8*c**3 + 429*a**2*b**10*c**2/2 +
13*a*b**12*c/2 + b**14/28) + x**26*(858*a**7*b*c**6 + 6006*a**6*b**3*c**5 +
 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 + 715*a**3*b**9*c**2 + 39*a**2*b
**11*c + a*b**13/2) + x**24*(429*a**8*c**6/4 + 2574*a**7*b**2*c**5 + 15015*
a**6*b**4*c**4/2 + 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 + 143*a**3*b
**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 + 4290*a**7*b**3*c**4
 + 6006*a**6*b**5*c**3 + 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 + 13*a**3*
b**11) + x**20*(143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 + 4290*a**7*b**4*c*
*3 + 3003*a**6*b**6*c**2 + 1287*a**5*b**8*c/2 + 143*a**4*b**10/4) + x**18*(
715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 + 2574*a**7*b**5*c**2 + 858*a**6*b*
*7*c + 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 + 715*a**9*b**2*c**3 + 64
35*a**8*b**4*c**2/4 + 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14*(143*a**10
*b*c**3 + 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 + 858*a**7*b**7/7) + x**1
2*(13*a**11*c**3 + 429*a**10*b**2*c**2/2 + 715*a**9*b**4*c/2 + 429*a**8*b**
6/4) + x**10*(39*a**11*b*c**2 + 143*a**10*b**3*c + 143*a**9*b**5/2) + x**8*
```

```
(13*a**12*c**2/4 + 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*(13*a**12*b*c
/2 + 13*a**11*b**3) + x**4*(a**13*c/2 + 13*a**12*b**2/4)
```

3.95 $\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^{13} \, dx$

Optimal. Leaf size=18

$$\frac{1}{42} \left(a+b x^3+c x^6\right)^{14}$$

```
[Out] 1/42*(c*x^6+b*x^3+a)^14
```

Rubi [A] time = 0.30, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1468, 629}

$$\frac{1}{42} \left(a+b x^3+c x^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^{13} \, dx &= \frac{1}{3} \text{Subst}\left(\int (b+2 c x) \left(a+b x+c x^2\right)^{13} \, dx,x,x^3\right)\\ &= \frac{1}{42} \left(a+b x^3+c x^6\right)^{14} \end{aligned}$$

Mathematica [B] time = 0.18, size = 233, normalized size = 12.94

$$\frac{1}{42} x^3 \left(b+c x^3\right) \left(14 a^{13}+91 a^{12} x^3 \left(b+c x^3\right)+364 a^{11} x^6 \left(b+c x^3\right)^2+1001 a^{10} x^9 \left(b+c x^3\right)^3+2002 a^9 x^{12} \left(b+c x^3\right)^4 \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^6*(b + c*x
^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a^
8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*x
^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 + 364*a^3
*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x^3)^1
2 + x^39*(b + c*x^3)^13))/42
```

fricas [B] time = 0.64, size = 1454, normalized size = 80.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 1/3*x^78*c^13*a + 2
6/3*x^75*c^11*b^3 + 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 + 26*x^72*c^11
*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 + 286/3*x^69*c^10*b^3*a +
26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 + 715/3*x^66*c^9*b^4*a + 143*x^66*c
^10*b^2*a^2 + 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 + 429*x^63*c^8*b^5*a
+ 1430/3*x^63*c^9*b^3*a^2 + 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 + 57
2*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 + 1430/3*x^60*c^9*b^2*a^3 + 143/
6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 + 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b
^5*a^2 + 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10
 + 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 + 2860*x^54*c^7*b^4*a^3 + 214
5/2*x^54*c^8*b^2*a^4 + 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 + 715/3*x^51
*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 + 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*
b^3*a^4 + 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 + 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 + 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 +
1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 + 26*x^45*c^2*
b^11*a + 1430/3*x^45*c^3*b^9*a^2 + 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^
5*a^4 + 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 + 13/3*
x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 + 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c
^4*b^6*a^4 + 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 + 572/7*x^42*c^7
*a^7 + 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 + 1430/3*x^39*c^2*b^9*a^3 + 286
0*x^39*c^3*b^7*a^4 + 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 + 572*x^
39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 + 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2
*b^8*a^4 + 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 + 1716*x^36*c^5*b^
2*a^7 + 143/2*x^36*c^6*a^8 + 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 + 17
16*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 + 2860*x^33*c^4*b^3*a^7 + 429*x
^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 + 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 + 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 + 143/3*x^30*c^5*a^
9 + 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 + 1716*x^27*c^2*b^5*a^7 + 1430*
x^27*c^3*b^3*a^8 + 715/3*x^27*c^4*b*a^9 + 143/2*x^24*b^8*a^6 + 572*x^24*c*b
^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 + 1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4
*a^10 + 572/7*x^21*b^7*a^7 + 429*x^21*c*b^5*a^8 + 1430/3*x^21*c^2*b^3*a^9 +
 286/3*x^21*c^3*b*a^10 + 143/2*x^18*b^6*a^8 + 715/3*x^18*c*b^4*a^9 + 143*x^
18*c^2*b^2*a^10 + 26/3*x^18*c^3*a^11 + 143/3*x^15*b^5*a^9 + 286/3*x^15*c*b^
3*a^10 + 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 + 26*x^12*c*b^2*a^11 + 13
/6*x^12*c^2*a^12 + 26/3*x^9*b^3*a^11 + 13/3*x^9*c*b*a^12 + 13/6*x^6*b^2*a^1
2 + 1/3*x^6*c*a^13 + 1/3*x^3*b*a^13
```

giac [B] time = 0.61, size = 246, normalized size = 13.67

$$\frac{1}{42}\left(cx^6+bx^3\right)^{14}+\frac{1}{3}\left(cx^6+bx^3\right)^{13}a+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2+\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4+\frac{143}{3}\left(cx^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 + 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 + 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 + 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 + 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 + 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
(c*x^6 + b*x^3)^4*a^10 + 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 + 1/3*(c*x^6 + b*x^3)*a^13
```

maple [B] time = 0.00, size = 46552, normalized size = 2586.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.55, size = 1240, normalized size = 68.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 143/3*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21
*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*b^3 + a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

mupad [B] time = 3.18, size = 1210, normalized size = 67.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 + (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 + 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 + 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 + (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 + 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 + 1716*a^5*b^2*c^7) + x^39*((a*b^1
3)/3 + 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 + 2860*a^4*b^7*
c^3 + 6006*a^5*b^5*c^4 + 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 + 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 + 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 + 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 + (26*a^11*c^3)/3 + (715*a
^9*b^4*c)/3 + 143*a^10*b^2*c^2) + x^66*((26*a^3*c^11)/3 + (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 + 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 + (143*a^9*
c^5)/3 + 429*a^5*b^8*c + 2002*a^6*b^6*c^2 + 2860*a^7*b^4*c^3 + (2145*a^8*b^
2*c^4)/2) + x^54*((143*a^5*c^9)/3 + (143*b^10*c^4)/6 + 429*a*b^8*c^5 + 2002
```

```
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 + (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 + (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 + 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 + 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 + (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 + 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 + (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 + 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 + (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 + x^6*((a^13*c)/3
 + (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 + (c^12*x^78*(2*a*c + 13*b^2))/6 + (143*a^7*b*x^21*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (13*a^3*b*x^3
3*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/3 + (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c + 2*b^2))/3 + (13*b*c^11*x^75*(a*c + 2*b^2
))/3
```

sympy [B] time = 0.34, size = 1394, normalized size = 77.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

```
[Out] a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**13/3 + 13*b
**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2*c*
*12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 + 286*
a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(26*a**3*c**11/3 + 143*a**2*b**2*
c**10 + 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10/3 +
1430*a**2*b**3*c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**
4*c**10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b**6*c**7
 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b**3*c**8 + 1716
*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(143*a**5*c**9
/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**6 + 42
9*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(429*a**5*b*c**8 + 2860*a**4*b**3
*c**7 + 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*c**4/3 + 26*
b**11*c**3/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 + 5005*a**4*b*
*4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a*b**10*c**3/3
+ 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5*b**3*c**6 + 6006*a*
*4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 + 26*a*b**11*c**
2 + b**13*c/3) + x**42*(572*a**7*c**7/7 + 2002*a**6*b**2*c**6 + 6006*a**5*b
**4*c**5 + 5005*a**4*b**6*c**4 + 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2
+ 13*a*b**12*c/3 + b**14/42) + x**39*(572*a**7*b*c**6 + 4004*a**6*b**3*c**5
 + 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*a**3*b**9*c**2/3 + 26*a
**2*b**11*c + a*b**13/3) + x**36*(143*a**8*c**6/2 + 1716*a**7*b**2*c**5 + 5
005*a**6*b**4*c**4 + 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 + 286*a**3
*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 + 2860*a**7*b**3*c**
4 + 4004*a**6*b**5*c**3 + 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3 + 26*a**3
*b**11/3) + x**30*(143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 + 2860*a**7*b**4
*c**3 + 2002*a**6*b**6*c**2 + 429*a**5*b**8*c + 143*a**4*b**10/6) + x**27*(
715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 + 1716*a**7*b**5*c**2 + 572*a**6*b*
*7*c + 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 + 1430*a**9*b**2*c**3/3 +
 2145*a**8*b**4*c**2/2 + 572*a**7*b**6*c + 143*a**6*b**8/2) + x**21*(286*a*
*10*b*c**3/3 + 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c + 572*a**7*b**7/7) +
 x**18*(26*a**11*c**3/3 + 143*a**10*b**2*c**2 + 715*a**9*b**4*c/3 + 143*a**
8*b**6/2) + x**15*(26*a**11*b*c**2 + 286*a**10*b**3*c/3 + 143*a**9*b**5/3)
```

```
+ x**12*(13*a**12*c**2/6 + 26*a**11*b**2*c + 143*a**10*b**4/6) + x**9*(13*a
**12*b*c/3 + 26*a**11*b**3/3) + x**6*(a**13*c/3 + 13*a**12*b**2/6)
```

3.96 $\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

```
[Out] 1/14*(a+b*x^n+c*x^(2*n))^14/n
```

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 629}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^{13}\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx, x, x^n\right)}{n}\\ &= \frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 22, normalized size = 0.96

$$\frac{\left(a+x^n\left(b+cx^n\right)\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
[Out] (a + x^n*(b + c*x^n))^14/(14*n)
```

fricas [B] time = 0.74, size = 1297, normalized size = 56.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 14*a^13*b*x^n + 7*(13*b^2*c^12 +
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 + a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 + 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 + 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 +
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 +
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 + 18*a*b
^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 + 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^(1
4*n) + 14*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) + 182*(2*a^3*b^11 + 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 + 2*a^9*c^5)*x^(10*n) + 2002*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*
b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 + 24*a^7*
b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) + 286*(12*a^7*b
^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^(6*n) + 182*(11*a^9*b^5
 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 + 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) + 182*(2*a^11*b^3 + a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
+ 2*a^13*c)*x^(2*n))/n
```

giac [**B**] time = 1.00, size = 1693, normalized size = 73.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) + 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) + 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) + 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) + 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) + 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) + 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) + 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) + 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) + 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) + 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) + 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) + 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) + 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) + 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) + 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) + 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) + 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) + 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
```

```
) + 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
+ 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) + 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) + 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) + 3432*a^7*c^7*x^(14*n) + 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) + 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) + 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) + 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) + 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) + 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) + 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) + 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) + 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) + 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) + 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) + 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) + 2002*a^9*c
^5*x^(10*n) + 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) + 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) + 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) + 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) + 200
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) + 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) + 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) + 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) + 364*a^11*c^3*x^(6*n) + 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) +
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) + 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) + 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) + 14*a^13*c*x^(2*n) + 14*a^13*b*x^n)/n
```

maple [B] time = 0.06, size = 2042, normalized size = 88.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n)+a)^13,x)
```

```
[Out] 26*b^11*c^3/n*(x^n)^17+1716/7/n*(x^n)^14*a^7*c^7+1716/7*b^7*a^7/n*(x^n)^7+1
43*b^9*c^5/n*(x^n)^19+26*b^3*c^11/n*(x^n)^25+a*b^13/n*(x^n)^13+143*a^5*b^9/
n*(x^n)^9+1716/7*b^7*c^7/n*(x^n)^21+143*b^5*c^9/n*(x^n)^23+143/2*a^10/n*(x^
n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8+143*b^5*a^9/n*(x^n)^5+26*b^11*a^3/n*(x^n)^
11+b^13*c/n*(x^n)^15+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+26*a^
11/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6+143*a^9/n*(x^n)^10*c^5+143/2*a^4/n
*(x^n)^10*b^10+429/2*c^8/n*(x^n)^22*b^6+c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)
^26*b^2+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^12+143*c^9/n*(x^n)^1
8*a^5+143/2*c^4/n*(x^n)^18*b^10+143/2*c^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)
^20*b^8+26*c^11/n*(x^n)^22*a^3+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12
*b^12+26*a^11*b^3/n*(x^n)^3+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*
b^4+a^13/n*(x^n)^2*c+13/2*a^12/n*(x^n)^2*b^2+b*a^13/n*x^n+b*c^13/n*(x^n)^27
+1287*b^5*c^8/n*(x^n)^21*a+78*b*c^11/n*(x^n)^23*a^2+286*b^3*c^10/n*(x^n)^23
*a+286*b*a^10/n*(x^n)^7*c^3+1430*b^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)
^7*c+715*b*c^9/n*(x^n)^19*a^4+4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x
^n)^19*a^2+1716*b^7*c^6/n*(x^n)^19*a+5148*a^7/n*(x^n)^12*b^2*c^5+15015*a^6/
n*(x^n)^12*b^4*c^4+12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n)^12*b^8*c
^2+1/14*c^14/n*(x^n)^28+715*a^9/n*(x^n)^6*b^4*c+1/14/n*(x^n)^14*b^14+6435/2
*a^8/n*(x^n)^10*b^2*c^4+8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6
*c^2+1287*a^5/n*(x^n)^10*b^8*c+1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n
)^8*b^4*c^2+1716*a^7/n*(x^n)^8*b^6*c+78*b*a^11/n*(x^n)^5*c^2+286*b^3*a^10/n
*(x^n)^5*c+1287*b*a^8/n*(x^n)^11*c^5+1716*a^7*b/n*(x^n)^13*c^6+12012*a^6*b^
3/n*(x^n)^13*c^5+18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^13*c^3+1
430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c+715*a^9*b/n*(x^n)^9*c^4
+4290*a^8*b^3/n*(x^n)^9*c^3+5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)
^9*c+286*b*c^10/n*(x^n)^21*a^3+1430*b^3*c^9/n*(x^n)^21*a^2+8580*b^3*a^7/n*(
x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3+5148*b^7*a^5/n*(x^n)^11*c^2+715*b^
9*a^4/n*(x^n)^11*c+1716*b*c^7/n*(x^n)^15*a^6+12012*b^3*c^6/n*(x^n)^15*a^5+1
8018*b^5*c^5/n*(x^n)^15*a^4+8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n
```

```
)^15*a^2+78*b^11*c^2/n*(x^n)^15*a+13*b*c^12/n*(x^n)^25*a+1430*c^9/n*(x^n)^2
0*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4+1716*c^7/n*(x^n)^20*a*b^6+429*c^10/
n*(x^n)^22*a^2*b^2+715*c^9/n*(x^n)^22*a*b^4+13*a^12*b/n*(x^n)^3*c+78*c^11/n
*(x^n)^24*a*b^2+78*a^11/n*(x^n)^4*b^2*c+429*a^10/n*(x^n)^6*b^2*c^2+1287*b*c
^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4+12012*b^5*c^6/n*(x^n)^17*a^3+
5148*b^7*c^5/n*(x^n)^17*a^2+715*b^9*c^4/n*(x^n)^17*a+6006/n*(x^n)^14*a^6*b^
2*c^6+18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4+4290/n*(x^n
)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b^10*c^2+13/n*(x^n)^14*a*b^12*c+286*a^3
/n*(x^n)^12*b^10*c+5148*c^7/n*(x^n)^16*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4
+12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*(x^n)^16*a^2*b^8+286*c^3/n*(x^n)
^16*a*b^10+6435/2*c^8/n*(x^n)^18*a^4*b^2+8580*c^7/n*(x^n)^18*a^3*b^4+6006*c
^6/n*(x^n)^18*a^2*b^6+1287*c^5/n*(x^n)^18*a*b^8
```

<u>**maxima**</u> **[B]** time = 0.86, size = 2041, normalized size = 88.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n + 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n + 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n + 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n + 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n + 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n + 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n + 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n + 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 + 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n + 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n + 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n + 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n + 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n + 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n +
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n + 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n + 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
16*n)/n + b^13*c*x^(15*n)/n + 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n + 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n + 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n + 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n + 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n + 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n + 1716/7*a^7*c^7*x^(14*n)/n + a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n + 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n + 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n + 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n + 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n + 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n + 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n + 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n + 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n + 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n + 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n + 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n + 143*a^9*c^5*x^(10*n)/n + 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n + 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n + 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n + 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n + 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n + 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n + 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
```

```
8*b^6*x^(6*n)/n + 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n + 26
*a^11*c^3*x^(6*n)/n + 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n + 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n + 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n + 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n + a^13*c*x^(2*n)/n + a^13*b*x^n/n
```

mupad [B] time = 5.78, size = 1395, normalized size = 60.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x)
```

```
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 + 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148
*a^5*b^2*c^7))/n + (x^(12*n + 1)*(a*b^13 + 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
5))/n + (x^(14*n + 1)*(b^13*c + 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n + (x^(21*n + 1)*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n + (x^
(17*n + 1)*(143*a^5*c^9 + (143*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
+ (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 + 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 + 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
+ 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
+ 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) + (a^12*x^(n + 1)*(a*c + (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n + (c^12
*x^(25*n + 1)*(a*c + (13*b^2)/2))/n + (a^13*b*x)/n + (143*a^7*b*x^(6*n + 1)
*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 +
30*a^3*b^2*c^3 + 12*a*b^6*c))/n + (13*a^3*b*x^(10*n + 1)*(2*b^10 + 99*a^5*c
^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n + (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 + 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 + 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c + 2*b^2))/n + (13*b*c^11*x^(24*
n + 1)*(a*c + 2*b^2))/n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

3.97 $\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

```
[Out] 1/14*(-c*x^2-b*x+a)^14
```

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (a - b*x - c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Mathematica [B] time = 0.18, size = 201, normalized size = 11.17

$$\frac{1}{14}x(b+cx)\left(-14a^{13}+91a^{12}x(b+cx)-364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3-2002a^9x^4(b+cx)^4+3003a^8x^5(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b + c*x)^2 + 1
001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x
)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 - 2002*a^5*x^8*(b
 + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x
^11*(b + c*x)^11 - 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

fricas [B] time = 0.50, size = 1450, normalized size = 80.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 - x^26*c^13*a + 26*x^25*c
^11*b^3 - 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 - 78*x^24*c^11*b^2*a + 13/
2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 - 286*x^23*c^10*b^3*a + 78*x^23*c^11*b*a
^2 + 429/2*x^22*c^8*b^6 - 715*x^22*c^9*b^4*a + 429*x^22*c^10*b^2*a^2 - 26*x
^22*c^11*a^3 + 1716/7*x^21*c^7*b^7 - 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^
```

```
3*a^2 - 286*x^21*c^10*b*a^3 + 429/2*x^20*c^6*b^8 - 1716*x^20*c^7*b^6*a + 64
35/2*x^20*c^8*b^4*a^2 - 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 + 143*x
^19*c^5*b^9 - 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 - 4290*x^19*c^8*b
^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10 - 1287*x^18*c^5*b^8*a + 6
006*x^18*c^6*b^6*a^2 - 8580*x^18*c^7*b^4*a^3 + 6435/2*x^18*c^8*b^2*a^4 - 14
3*x^18*c^9*a^5 + 26*x^17*c^3*b^11 - 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*
a^2 - 12012*x^17*c^6*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 - 1287*x^17*c^8*b*a^5
+ 13/2*x^16*c^2*b^12 - 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 - 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 - 5148*x^16*c^7*b^2*a^5 + 429/2
*x^16*c^8*a^6 + x^15*c*b^13 - 78*x^15*c^2*b^11*a + 1430*x^15*c^3*b^9*a^2 -
8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^4 - 12012*x^15*c^6*b^3*a^5 + 1
716*x^15*c^7*b*a^6 + 1/14*x^14*b^14 - 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*
a^2 - 4290*x^14*c^3*b^8*a^3 + 15015*x^14*c^4*b^6*a^4 - 18018*x^14*c^5*b^4*a
^5 + 6006*x^14*c^6*b^2*a^6 - 1716/7*x^14*c^7*a^7 - x^13*b^13*a + 78*x^13*c*
b^11*a^2 - 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 - 18018*x^13*c^4*b
^5*a^5 + 12012*x^13*c^5*b^3*a^6 - 1716*x^13*c^6*b*a^7 + 13/2*x^12*b^12*a^2
- 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 - 12012*x^12*c^3*b^6*a^5 +
15015*x^12*c^4*b^4*a^6 - 5148*x^12*c^5*b^2*a^7 + 429/2*x^12*c^6*a^8 - 26*x^
11*b^11*a^3 + 715*x^11*c*b^9*a^4 - 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b
^5*a^6 - 8580*x^11*c^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4
- 1287*x^10*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 - 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 - 143*x^10*c^5*a^9 - 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*
a^6 - 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 - 715*x^9*c^4*b*a^9 + 429
/2*x^8*b^8*a^6 - 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 - 1430*x^8*c^3
*b^2*a^9 + 143/2*x^8*c^4*a^10 - 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 - 1
430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 - 715*x^6*c*b^
4*a^9 + 429*x^6*c^2*b^2*a^10 - 26*x^6*c^3*a^11 - 143*x^5*b^5*a^9 + 286*x^5*
c*b^3*a^10 - 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 - 78*x^4*c*b^2*a^11 + 1
3/2*x^4*c^2*a^12 - 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*x^2*b^2*a^12 -
x^2*c*a^13 - x*b*a^13
```

giac [B] time = 0.47, size = 218, normalized size = 12.11

$$\frac{1}{14}\left(cx^2+bx\right)^{14}-\left(cx^2+bx\right)^{13}a+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2-26\left(cx^2+bx\right)^{11}a^3+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4-143\left(cx^2+bx\right)^9a^5-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14 - (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 - 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 - 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 - 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 - 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 - 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 - (c*x^2 + b*x)*a^13
```

maple [B] time = 0.00, size = 47685, normalized size = 2649.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^13,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.44, size = 16, normalized size = 0.89

$$\frac{1}{14}\left(cx^2+bx-a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x - a)^14
```

mupad [B] time = 1.38, size = 1208, normalized size = 67.11

$$x^{12}\left(\frac{429\,a^8\,c^6}{2}-5148\,a^7\,b^2\,c^5+15015\,a^6\,b^4\,c^4-12012\,a^5\,b^6\,c^3+\frac{6435\,a^4\,b^8\,c^2}{2}-286\,a^3\,b^{10}\,c+\frac{13\,a^2\,b^{12}}{2}\right)+x^{16}\left(\frac{4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^13,x)
```

```
[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 - 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148*a^5*b^2*c^7) - x^13*(a*b^13 - 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 - 12012*a^6*b^3*c^5) + x^15*(b^13*c - 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*
b^2*c^2) - x^22*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) - x^18*(143*a^5*c^9 - (1
43*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 - (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 - (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 -
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 - 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 - 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 - 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 - x^2*(a^13*c - (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 - 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/2 + b
*c^13*x^27 - (c^12*x^26*(2*a*c - 13*b^2))/2 - a^13*b*x - (143*a^7*b*x^7*(12
*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 - 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) - 13*a^3*b*x
^11*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 - 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 -
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c) - 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 - 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 - 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c - 2*b^2) - 13*b*c^11*x^25*(a*c - 2*b^2)
```

sympy [B] time = 0.36, size = 1326, normalized size = 73.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

```
[Out] -a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(-a*c**13 + 13*b**2*c**
12/2) + x**25*(-13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 - 78
*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 - 286*a*b**3*c**
10 + 143*b**5*c**9) + x**22*(-26*a**3*c**11 + 429*a**2*b**2*c**10 - 715*a*b
**4*c**9 + 429*b**6*c**8/2) + x**21*(-286*a**3*b*c**10 + 1430*a**2*b**3*c**
9 - 1287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 - 1430*a
**3*b**2*c**9 + 6435*a**2*b**4*c**8/2 - 1716*a*b**6*c**7 + 429*b**8*c**6/2)
 + x**19*(715*a**4*b*c**9 - 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 - 171
6*a*b**7*c**6 + 143*b**9*c**5) + x**18*(-143*a**5*c**9 + 6435*a**4*b**2*c**
8/2 - 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 - 1287*a*b**8*c**5 + 143*b*
```

```
*10*c**4/2) + x**17*(-1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 - 12012*a**3*b
**5*c**6 + 5148*a**2*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(
429*a**6*c**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 - 12012*a**3*b
**6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**12*c**2/2) + x*
*15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 - 8580*
a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b**11*c**2 + b**13*c) + x**14*(
-1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 - 18018*a**5*b**4*c**5 + 15015*a**4
*b**6*c**4 - 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 - 13*a*b**12*c + b**
14/14) + x**13*(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 - 18018*a**5*b**5*
c**4 + 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 78*a**2*b**11*c - a*b**1
3) + x**12*(429*a**8*c**6/2 - 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 -
12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 - 286*a**3*b**10*c + 13*a**2*b
**12/2) + x**11*(1287*a**8*b*c**5 - 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c
**3 - 5148*a**5*b**7*c**2 + 715*a**4*b**9*c - 26*a**3*b**11) + x**10*(-143*
a**9*c**5 + 6435*a**8*b**2*c**4/2 - 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c*
*2 - 1287*a**5*b**8*c + 143*a**4*b**10/2) + x**9*(-715*a**9*b*c**4 + 4290*a
**8*b**3*c**3 - 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c - 143*a**5*b**9) + x
**8*(143*a**10*c**4/2 - 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 - 1716*
a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 - 1430*a**9*b**3*c*
*2 + 1287*a**8*b**5*c - 1716*a**7*b**7/7) + x**6*(-26*a**11*c**3 + 429*a**1
0*b**2*c**2 - 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(-78*a**11*b*c**2 +
 286*a**10*b**3*c - 143*a**9*b**5) + x**4*(13*a**12*c**2/2 - 78*a**11*b**2*
c + 143*a**10*b**4/2) + x**3*(13*a**12*b*c - 26*a**11*b**3) + x**2*(-a**13*
c + 13*a**12*b**2/2)
```

3.98 $\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=20

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

```
[Out] 1/28*(-c*x^4-b*x^2+a)^14
```

Rubi [A] time = 0.32, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a - b*x^2 - c*x^4)^14/28
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^2\right)\\ &= \frac{1}{28}\left(a-bx^2-cx^4\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.17, size = 233, normalized size = 11.65

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(-14a^{13}+91a^{12}x^2\left(b+cx^2\right)-364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3-2002a^9x^8\left(b+cx^2\right)^4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x^4*(b + c*
x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^
8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x
^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 - 364*a^3
*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x^2)^1
2 + x^26*(b + c*x^2)^13))/28
```

fricas [B] time = 0.50, size = 1454, normalized size = 72.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 - 1/2*x^52*c^13*a + 1
3*x^50*c^11*b^3 - 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*b^4 - 39*x^48*c^11*b
^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^5 - 143*x^46*c^10*b^3*a + 39*x
^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6 - 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^1
0*b^2*a^2 - 13*x^44*c^11*a^3 + 858/7*x^42*c^7*b^7 - 1287/2*x^42*c^8*b^5*a +
 715*x^42*c^9*b^3*a^2 - 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 - 858*x^40
*c^7*b^6*a + 6435/4*x^40*c^8*b^4*a^2 - 715*x^40*c^9*b^2*a^3 + 143/4*x^40*c^
10*a^4 + 143/2*x^38*c^5*b^9 - 858*x^38*c^6*b^7*a + 2574*x^38*c^7*b^5*a^2 -
2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143/4*x^36*c^4*b^10 - 1287/2
*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2 - 4290*x^36*c^7*b^4*a^3 + 6435/4*x^
36*c^8*b^2*a^4 - 143/2*x^36*c^9*a^5 + 13*x^34*c^3*b^11 - 715/2*x^34*c^4*b^9
*a + 2574*x^34*c^5*b^7*a^2 - 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4
- 1287/2*x^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 - 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 - 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*a^4 - 2574
*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^13 - 39*x^30*c^2*b^11
*a + 715*x^30*c^3*b^9*a^2 - 4290*x^30*c^4*b^7*a^3 + 9009*x^30*c^5*b^5*a^4 -
 6006*x^30*c^6*b^3*a^5 + 858*x^30*c^7*b*a^6 + 1/28*x^28*b^14 - 13/2*x^28*c*
b^12*a + 429/2*x^28*c^2*b^10*a^2 - 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4
*b^6*a^4 - 9009*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 - 858/7*x^28*c^7*a
^7 - 1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 - 715*x^26*c^2*b^9*a^3 + 4290*x^2
6*c^3*b^7*a^4 - 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^6 - 858*x^26*c^
6*b*a^7 + 13/4*x^24*b^12*a^2 - 143*x^24*c*b^10*a^3 + 6435/4*x^24*c^2*b^8*a^
4 - 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24*c^4*b^4*a^6 - 2574*x^24*c^5*b^2*a^
7 + 429/4*x^24*c^6*a^8 - 13*x^22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 - 2574*x^2
2*c^2*b^7*a^5 + 6006*x^22*c^3*b^5*a^6 - 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22
*c^5*b*a^8 + 143/4*x^20*b^10*a^4 - 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 - 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 - 143/2*x^20*c^5*a^
9 - 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 - 2574*x^18*c^2*b^5*a^7 + 2145*
x^18*c^3*b^3*a^8 - 715/2*x^18*c^4*b*a^9 + 429/4*x^16*b^8*a^6 - 858*x^16*c*b
^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 - 715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^
10 - 858/7*x^14*b^7*a^7 + 1287/2*x^14*c*b^5*a^8 - 715*x^14*c^2*b^3*a^9 + 14
3*x^14*c^3*b*a^10 + 429/4*x^12*b^6*a^8 - 715/2*x^12*c*b^4*a^9 + 429/2*x^12*
c^2*b^2*a^10 - 13*x^12*c^3*a^11 - 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*a^10
- 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 - 39*x^8*c*b^2*a^11 + 13/4*x^8*c^
2*a^12 - 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/4*x^4*b^2*a^12 - 1/2*x^4*
c*a^13 - 1/2*x^2*b*a^13
```

giac [B] time = 0.57, size = 246, normalized size = 12.30

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}-\frac{1}{2}\left(cx^4+bx^2\right)^{13}a+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2-13\left(cx^4+bx^2\right)^{11}a^3+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4-\frac{143}{2}\left(cx^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 - 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 - 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 - 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 - 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 - 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 - 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 - 1/2*(c*x^4 + b*x^2)*a^13
```

maple [B] time = 0.00, size = 47688, normalized size = 2384.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.52, size = 1242, normalized size = 62.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 143/2*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^
13*b*x^2 - 143/14*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b^3 - a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

mupad [B] time = 3.25, size = 1214, normalized size = 60.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 - 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 - 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 - 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 - 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
- 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 - 2574*a^5*b^2*c^7) - x^26*((a*b
^13)/2 - 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 - 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 - 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 - 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 - 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 - 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 - 13*a^11*c^3 - (715*a^9*b^4*c)/2
+ (429*a^10*b^2*c^2)/2) - x^44*(13*a^3*c^11 - (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 - (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 - (143*a^9*c^5)/2 -
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 - 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
^4)/4) - x^36*((143*a^5*c^9)/2 - (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 - 30
```

```
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 -
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 - 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 - 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 - (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 - 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 - 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 - 858*a*b^6*c^7
+ (6435*a^2*b^4*c^8)/4 - 715*a^3*b^2*c^9) + (c^14*x^56)/28 - x^4*((a^13*c)/
2 - (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 - (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 - (c^12*x^52*(2*a*c - 13*b^2))/4 - (143*a^7*b*x^14*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 - (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 - (13*a^3*b*x^2
2*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/2 - (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c - 2*b^2))/2 - (13*b*c^11*x^50*(a*c - 2*b^2
))/2
```

sympy [B] time = 0.36, size = 1384, normalized size = 69.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)
```

```
[Out] -a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(-a*c**13/2 + 13
*b**2*c**12/4) + x**50*(-13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c
**12/4 - 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 - 143
*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(-13*a**3*c**11 + 429*a**2*b**2*c*
*10/2 - 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(-143*a**3*b*c**10 + 7
15*a**2*b**3*c**9 - 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4
*c**10/4 - 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 - 858*a*b**6*c**7 + 4
29*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 - 2145*a**3*b**3*c**8 + 2574*a**
2*b**5*c**7 - 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(-143*a**5*c**9/2
+ 6435*a**4*b**2*c**8/4 - 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 - 1287*
a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(-1287*a**5*b*c**8/2 + 4290*a**4*
b**3*c**7 - 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 - 715*a*b**9*c**4/2 +
 13*b**11*c**3) + x**32*(429*a**6*c**8/4 - 2574*a**5*b**2*c**7 + 15015*a**4
*b**4*c**6/2 - 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 - 143*a*b**10*c*
*3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 - 6006*a**5*b**3*c**6 + 9009
*a**4*b**5*c**5 - 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 - 39*a*b**11*c**
2 + b**13*c/2) + x**28*(-858*a**7*c**7/7 + 3003*a**6*b**2*c**6 - 9009*a**5*
b**4*c**5 + 15015*a**4*b**6*c**4/2 - 2145*a**3*b**8*c**3 + 429*a**2*b**10*c
**2/2 - 13*a*b**12*c/2 + b**14/28) + x**26*(-858*a**7*b*c**6 + 6006*a**6*b*
*3*c**5 - 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 - 715*a**3*b**9*c**2 +
39*a**2*b**11*c - a*b**13/2) + x**24*(429*a**8*c**6/4 - 2574*a**7*b**2*c**5
 + 15015*a**6*b**4*c**4/2 - 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 - 1
43*a**3*b**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 - 4290*a**7*
b**3*c**4 + 6006*a**6*b**5*c**3 - 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 -
 13*a**3*b**11) + x**20*(-143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 - 4290*a*
*7*b**4*c**3 + 3003*a**6*b**6*c**2 - 1287*a**5*b**8*c/2 + 143*a**4*b**10/4)
 + x**18*(-715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 - 2574*a**7*b**5*c**2 +
858*a**6*b**7*c - 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 - 715*a**9*b**
2*c**3 + 6435*a**8*b**4*c**2/4 - 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14
*(143*a**10*b*c**3 - 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 - 858*a**7*b**
7/7) + x**12*(-13*a**11*c**3 + 429*a**10*b**2*c**2/2 - 715*a**9*b**4*c/2 +
429*a**8*b**6/4) + x**10*(-39*a**11*b*c**2 + 143*a**10*b**3*c - 143*a**9*b*
```

```
*5/2) + x**8*(13*a**12*c**2/4 - 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*
(13*a**12*b*c/2 - 13*a**11*b**3) + x**4*(-a**13*c/2 + 13*a**12*b**2/4)
```

3.99 $\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^{13} \, dx$

Optimal. Leaf size=20

$$\frac{1}{42}\left(a-bx^3-cx^6\right)^{14}$$

```
[Out] 1/42*(-c*x^6-b*x^3+a)^14
```

Rubi [A] time = 0.31, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1468, 629}

$$\frac{1}{42}\left(a-bx^3-cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a - b*x^3 - c*x^6)^14/42
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^{13} \, dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13} \, dx, x, x^3\right)\\ &= \frac{1}{42}\left(a-bx^3-cx^6\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.17, size = 233, normalized size = 11.65

$$\frac{1}{42}x^3\left(b+cx^3\right)\left(-14a^{13}+91a^{12}x^3\left(b+cx^3\right)-364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3-2002a^9x^{12}\left(b+cx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x^6*(b + c*
x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a
^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*
x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 - 364*a^
3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*x^3)^
12 + x^39*(b + c*x^3)^13))/42
```

fricas [B] time = 0.54, size = 1454, normalized size = 72.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 - 1/3*x^78*c^13*a + 2
6/3*x^75*c^11*b^3 - 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 - 26*x^72*c^11
*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 - 286/3*x^69*c^10*b^3*a +
26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 - 715/3*x^66*c^9*b^4*a + 143*x^66*c
^10*b^2*a^2 - 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 - 429*x^63*c^8*b^5*a
+ 1430/3*x^63*c^9*b^3*a^2 - 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 - 57
2*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 - 1430/3*x^60*c^9*b^2*a^3 + 143/
6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 - 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b
^5*a^2 - 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10
 - 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 - 2860*x^54*c^7*b^4*a^3 + 214
5/2*x^54*c^8*b^2*a^4 - 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 - 715/3*x^51
*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 - 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*
b^3*a^4 - 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 - 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 - 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 -
1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 - 26*x^45*c^2*
b^11*a + 1430/3*x^45*c^3*b^9*a^2 - 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^
5*a^4 - 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 - 13/3*
x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 - 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c
^4*b^6*a^4 - 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 - 572/7*x^42*c^7
*a^7 - 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 - 1430/3*x^39*c^2*b^9*a^3 + 286
0*x^39*c^3*b^7*a^4 - 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 - 572*x^
39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 - 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2
*b^8*a^4 - 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 - 1716*x^36*c^5*b^
2*a^7 + 143/2*x^36*c^6*a^8 - 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 - 17
16*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 - 2860*x^33*c^4*b^3*a^7 + 429*x
^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 - 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 - 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 - 143/3*x^30*c^5*a^
9 - 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 - 1716*x^27*c^2*b^5*a^7 + 1430*
x^27*c^3*b^3*a^8 - 715/3*x^27*c^4*b*a^9 + 143/2*x^24*b^8*a^6 - 572*x^24*c*b
^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 - 1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4
*a^10 - 572/7*x^21*b^7*a^7 + 429*x^21*c*b^5*a^8 - 1430/3*x^21*c^2*b^3*a^9 +
 286/3*x^21*c^3*b*a^10 + 143/2*x^18*b^6*a^8 - 715/3*x^18*c*b^4*a^9 + 143*x^
18*c^2*b^2*a^10 - 26/3*x^18*c^3*a^11 - 143/3*x^15*b^5*a^9 + 286/3*x^15*c*b^
3*a^10 - 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 - 26*x^12*c*b^2*a^11 + 13
/6*x^12*c^2*a^12 - 26/3*x^9*b^3*a^11 + 13/3*x^9*c*b*a^12 + 13/6*x^6*b^2*a^1
2 - 1/3*x^6*c*a^13 - 1/3*x^3*b*a^13
```

giac [B] time = 0.68, size = 246, normalized size = 12.30

$$\frac{1}{42}\left(cx^6+bx^3\right)^{14}-\frac{1}{3}\left(cx^6+bx^3\right)^{13}a+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2-\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4-\frac{143}{3}\left(cx^6+bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 - 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 - 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 - 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 - 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 - 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
(c*x^6 + b*x^3)^4*a^10 - 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 - 1/3*(c*x^6 + b*x^3)*a^13
```

maple [B] time = 0.00, size = 47688, normalized size = 2384.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.50, size = 1242, normalized size = 62.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 143/3*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21
*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*b^3 - a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

mupad [B] time = 1.28, size = 1214, normalized size = 60.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 - (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 - 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 - 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 - (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 - 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 - 1716*a^5*b^2*c^7) - x^39*((a*b^1
3)/3 - 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 - 2860*a^4*b^7*
c^3 + 6006*a^5*b^5*c^4 - 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 - 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 - 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 - 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 - (26*a^11*c^3)/3 - (715*a
^9*b^4*c)/3 + 143*a^10*b^2*c^2) - x^66*((26*a^3*c^11)/3 - (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 - 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 - (143*a^9*
c^5)/3 - 429*a^5*b^8*c + 2002*a^6*b^6*c^2 - 2860*a^7*b^4*c^3 + (2145*a^8*b^
2*c^4)/2) - x^54*((143*a^5*c^9)/3 - (143*b^10*c^4)/6 + 429*a*b^8*c^5 - 2002
```

```
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 - (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 - (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 - 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 - 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 - (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 - 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 - (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 - 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 - (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 - x^6*((a^13*c)/3
 - (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 - (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 - (c^12*x^78*(2*a*c - 13*b^2))/6 - (143*a^7*b*x^21*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 - (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 - (13*a^3*b*x^3
3*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/3 - (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c - 2*b^2))/3 - (13*b*c^11*x^75*(a*c - 2*b^2
))/3
```

sympy [B] time = 0.36, size = 1394, normalized size = 69.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)
```

```
[Out] -a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(-a*c**13/3 + 13
*b**2*c**12/6) + x**75*(-13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2
*c**12/6 - 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 - 2
86*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(-26*a**3*c**11/3 + 143*a**2*b
**2*c**10 - 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(-286*a**3*b*c**10
/3 + 1430*a**2*b**3*c**9/3 - 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(14
3*a**4*c**10/6 - 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 - 572*a*b**6
*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 - 1430*a**3*b**3*c**8 +
 1716*a**2*b**5*c**7 - 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(-143*a**
5*c**9/3 + 2145*a**4*b**2*c**8/2 - 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**
6 - 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(-429*a**5*b*c**8 + 2860*a*
*4*b**3*c**7 - 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 - 715*a*b**9*c**4/
3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 - 1716*a**5*b**2*c**7 + 5005*
a**4*b**4*c**6 - 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 - 286*a*b**10*
c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 - 4004*a**5*b**3*c**6 +
6006*a**4*b**5*c**5 - 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 - 26*a*b*
*11*c**2 + b**13*c/3) + x**42*(-572*a**7*c**7/7 + 2002*a**6*b**2*c**6 - 600
6*a**5*b**4*c**5 + 5005*a**4*b**6*c**4 - 1430*a**3*b**8*c**3 + 143*a**2*b**
10*c**2 - 13*a*b**12*c/3 + b**14/42) + x**39*(-572*a**7*b*c**6 + 4004*a**6*
b**3*c**5 - 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 - 1430*a**3*b**9*c**2
/3 + 26*a**2*b**11*c - a*b**13/3) + x**36*(143*a**8*c**6/2 - 1716*a**7*b**2
*c**5 + 5005*a**6*b**4*c**4 - 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 -
 286*a**3*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 - 2860*a**7
*b**3*c**4 + 4004*a**6*b**5*c**3 - 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3
- 26*a**3*b**11/3) + x**30*(-143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 - 2860
*a**7*b**4*c**3 + 2002*a**6*b**6*c**2 - 429*a**5*b**8*c + 143*a**4*b**10/6)
 + x**27*(-715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 - 1716*a**7*b**5*c**2 +
572*a**6*b**7*c - 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 - 1430*a**9*b*
*2*c**3/3 + 2145*a**8*b**4*c**2/2 - 572*a**7*b**6*c + 143*a**6*b**8/2) + x*
*21*(286*a**10*b*c**3/3 - 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c - 572*a**
7*b**7/7) + x**18*(-26*a**11*c**3/3 + 143*a**10*b**2*c**2 - 715*a**9*b**4*c
/3 + 143*a**8*b**6/2) + x**15*(-26*a**11*b*c**2 + 286*a**10*b**3*c/3 - 143*
a**9*b**5/3) + x**12*(13*a**12*c**2/6 - 26*a**11*b**2*c + 143*a**10*b**4/6)
```

```
 + x**9*(13*a**12*b*c/3 - 26*a**11*b**3/3) + x**6*(-a**13*c/3 + 13*a**12*b*
*2/6)
```

3.100 $\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

```
[Out] 1/14*(a-b*x^n-c*x^(2*n))^14/n
```

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx, x, x^n\right)}{n}\\ &= \frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.96

$$\frac{\left(x^n\left(b+cx^n\right)-a\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (-a + x^n*(b + c*x^n))^14/(14*n)
```

fricas [B] time = 0.76, size = 1299, normalized size = 51.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas
")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*b^2*c^12 -
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 - 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 -
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 -
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 - 18*a*b
^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 - 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^(1
4*n) - 14*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a^3*b^11 - 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 - 2*a^9*c^5)*x^(10*n) - 2002*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*
b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 - 24*a^7*
b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) - 286*(12*a^7*b
^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^(6*n) - 182*(11*a^9*b^5
 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 - 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) - 182*(2*a^11*b^3 - a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
- 2*a^13*c)*x^(2*n))/n
```

giac [B] time = 1.09, size = 1693, normalized size = 67.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) - 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) - 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) - 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) - 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) - 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) - 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) - 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) - 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) - 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) - 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) - 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) - 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) - 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) - 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) - 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) - 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) - 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) - 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) - 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) - 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) - 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) - 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) - 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
```

```
) - 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
- 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) - 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) - 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) - 3432*a^7*c^7*x^(14*n) - 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) - 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) - 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) - 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) - 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) - 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) - 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) - 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) - 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) - 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) - 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) - 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) - 2002*a^9*c
^5*x^(10*n) - 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) - 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) - 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) - 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) - 200
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) - 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) - 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) - 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) - 364*a^11*c^3*x^(6*n) - 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) -
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) - 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) - 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) - 14*a^13*c*x^(2*n) - 14*a^13*b*x^n)/n
```

maple [B] time = 0.06, size = 2046, normalized size = 81.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 26*b^11*c^3/n*(x^n)^17-1716/7/n*(x^n)^14*a^7*c^7-1716/7*b^7*a^7/n*(x^n)^7+1
43*b^9*c^5/n*(x^n)^19+26*b^3*c^11/n*(x^n)^25-a*b^13/n*(x^n)^13-143*a^5*b^9/
n*(x^n)^9+1716/7*b^7*c^7/n*(x^n)^21+143*b^5*c^9/n*(x^n)^23+143/2*a^10/n*(x^
n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8-143*b^5*a^9/n*(x^n)^5-26*b^11*a^3/n*(x^n)^
11+b^13*c/n*(x^n)^15+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4-26*a^
11/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6-143*a^9/n*(x^n)^10*c^5+143/2*a^4/n
*(x^n)^10*b^10+429/2*c^8/n*(x^n)^22*b^6-c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)
^26*b^2+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^12-143*c^9/n*(x^n)^1
8*a^5+143/2*c^4/n*(x^n)^18*b^10+143/2*c^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)
^20*b^8-26*c^11/n*(x^n)^22*a^3+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12
*b^12-26*a^11*b^3/n*(x^n)^3+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*
b^4-a^13/n*(x^n)^2*c+13/2*a^12/n*(x^n)^2*b^2-a^13*b/n*x^n+b*c^13/n*(x^n)^27
-1287*b^5*c^8/n*(x^n)^21*a+78*b*c^11/n*(x^n)^23*a^2-286*b^3*c^10/n*(x^n)^23
*a+286*b*a^10/n*(x^n)^7*c^3-1430*b^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)
^7*c+715*b*c^9/n*(x^n)^19*a^4-4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x
^n)^19*a^2-1716*b^7*c^6/n*(x^n)^19*a-5148*a^7/n*(x^n)^12*b^2*c^5+15015*a^6/
n*(x^n)^12*b^4*c^4-12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n)^12*b^8*c
^2+1/14*c^14/n*(x^n)^28-715*a^9/n*(x^n)^6*b^4*c+1/14/n*(x^n)^14*b^14+6435/2
*a^8/n*(x^n)^10*b^2*c^4-8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6
*c^2-1287*a^5/n*(x^n)^10*b^8*c-1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n
)^8*b^4*c^2-1716*a^7/n*(x^n)^8*b^6*c-78*b*a^11/n*(x^n)^5*c^2+286*b^3*a^10/n
*(x^n)^5*c+1287*b*a^8/n*(x^n)^11*c^5-1716*a^7*b/n*(x^n)^13*c^6+12012*a^6*b^
3/n*(x^n)^13*c^5-18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^13*c^3-1
430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c-715*a^9*b/n*(x^n)^9*c^4
+4290*a^8*b^3/n*(x^n)^9*c^3-5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)
^9*c-286*b*c^10/n*(x^n)^21*a^3+1430*b^3*c^9/n*(x^n)^21*a^2-8580*b^3*a^7/n*(
x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3-5148*b^7*a^5/n*(x^n)^11*c^2+715*b^
9*a^4/n*(x^n)^11*c+1716*b*c^7/n*(x^n)^15*a^6-12012*b^3*c^6/n*(x^n)^15*a^5+1
8018*b^5*c^5/n*(x^n)^15*a^4-8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n
```

```
)^15*a^2-78*b^11*c^2/n*(x^n)^15*a-13*b*c^12/n*(x^n)^25*a-1430*c^9/n*(x^n)^2
0*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4-1716*c^7/n*(x^n)^20*a*b^6+429*c^10/
n*(x^n)^22*a^2*b^2-715*c^9/n*(x^n)^22*a*b^4+13*a^12*b/n*(x^n)^3*c-78*c^11/n
*(x^n)^24*a*b^2-78*a^11/n*(x^n)^4*b^2*c+429*a^10/n*(x^n)^6*b^2*c^2-1287*b*c
^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4-12012*b^5*c^6/n*(x^n)^17*a^3+
5148*b^7*c^5/n*(x^n)^17*a^2-715*b^9*c^4/n*(x^n)^17*a+6006/n*(x^n)^14*a^6*b^
2*c^6-18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4-4290/n*(x^n
)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b^10*c^2-13/n*(x^n)^14*a*b^12*c-286*a^3
/n*(x^n)^12*b^10*c-5148*c^7/n*(x^n)^16*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4
-12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*(x^n)^16*a^2*b^8-286*c^3/n*(x^n)
^16*a*b^10+6435/2*c^8/n*(x^n)^18*a^4*b^2-8580*c^7/n*(x^n)^18*a^3*b^4+6006*c
^6/n*(x^n)^18*a^2*b^6-1287*c^5/n*(x^n)^18*a*b^8
```

maxima [B] time = 0.83, size = 2045, normalized size = 81.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima
")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n - a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n - 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n - 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n - 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n - 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n - 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n - 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n - 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n - 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n - 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n - 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 - 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n - 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n - 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n - 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n - 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n - 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n -
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n - 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n - 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n - 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
16*n)/n + b^13*c*x^(15*n)/n - 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n - 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n - 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n - 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n - 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n - 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n - 1716/7*a^7*c^7*x^(14*n)/n - a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n - 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n - 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n - 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n - 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n - 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n - 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n - 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n - 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n - 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n - 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n - 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n - 143*a^9*c^5*x^(10*n)/n - 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n - 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n - 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n - 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n - 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n - 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n - 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
```

```
8*b^6*x^(6*n)/n - 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n - 26
*a^11*c^3*x^(6*n)/n - 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n - 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n - 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n - 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n - a^13*c*x^(2*n)/n - a^13*b*x^n/n
```

mupad [B] time = 5.78, size = 1401, normalized size = 56.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^13,x)
```

```
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 - 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148
*a^5*b^2*c^7))/n - (x^(12*n + 1)*(a*b^13 - 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
5))/n + (x^(14*n + 1)*(b^13*c - 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n - (x^(21*n + 1)*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n - (x^
(17*n + 1)*(143*a^5*c^9 - (143*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
- (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 - 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 - 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
- 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
- 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) - (a^12*x^(n + 1)*(a*c - (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n - (c^12
*x^(25*n + 1)*(a*c - (13*b^2)/2))/n - (a^13*b*x)/n - (143*a^7*b*x^(6*n + 1)
*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) - (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 -
30*a^3*b^2*c^3 - 12*a*b^6*c))/n - (13*a^3*b*x^(10*n + 1)*(2*b^10 - 99*a^5*c
^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n - (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 - 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 - 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c - 2*b^2))/n - (13*b*c^11*x^(24*
n + 1)*(a*c - 2*b^2))/n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

3.101 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x)^14
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

fricas [B] time = 0.48, size = 154, normalized size = 10.27

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

giac [A] time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

maple [B] time = 0.00, size = 155, normalized size = 10.33

$$\frac{1}{14}c^{14}x^{28}+b\,c^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

mupad [B] time = 2.09, size = 154, normalized size = 10.27

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

sympy [B] time = 0.13, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

3.102 $\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx &= \int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)\\ &= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

fricas [B] time = 0.77, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14}+\frac{1}{2}x^{54}c^{13}b+\frac{13}{4}x^{52}c^{12}b^2+13x^{50}c^{11}b^3+\frac{143}{4}x^{48}c^{10}b^4+\frac{143}{2}x^{46}c^9b^5+\frac{429}{4}x^{44}c^8b^6+\frac{858}{7}x^{42}c^7b^7+\frac{429}{4}x^{40}c^6b^8+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

giac [A] time = 0.38, size = 15, normalized size = 0.94

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}b^2c^{12}x^{52}+13b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

maxima [B] time = 0.43, size = 156, normalized size = 9.75

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

mupad [B] time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

sympy [B] time = 0.13, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

3.103 $\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1584, 446, 74}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^{13} \, dx &= \int x^{41} \left(b+cx^3\right)^{13} \left(b+2cx^3\right) \, dx \\
&= \frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx) \, dx, x, x^3\right) \\
&= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

fricas [B] time = 0.70, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14}+\frac{1}{3}x^{81}c^{13}b+\frac{13}{6}x^{78}c^{12}b^2+\frac{26}{3}x^{75}c^{11}b^3+\frac{143}{6}x^{72}c^{10}b^4+\frac{143}{3}x^{69}c^9b^5+\frac{143}{2}x^{66}c^8b^6+\frac{572}{7}x^{63}c^7b^7+\frac{143}{2}x^{60}c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

giac [A] time = 0.51, size = 15, normalized size = 0.94

$$\frac{1}{42}\left(cx^6+bx^3\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}b\,c^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

maxima [B] time = 0.44, size = 156, normalized size = 9.75

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

mupad [B] time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

sympy [B] time = 0.13, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^{2}x^{48}}{6}+\frac{26b^{11}c^{3}x^{51}}{3}+\frac{143b^{10}c^{4}x^{54}}{6}+\frac{143b^{9}c^{5}x^{57}}{3}+\frac{143b^{8}c^{6}x^{60}}{2}+\frac{572b^{7}c^{7}x^{63}}{7}+\frac{143b^{6}c^{8}x^{66}}{2}+\frac{14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

3.104 $\int x^{-1+n} (b + 2cx^n) \left(bx^n + cx^{2n}\right)^{13} \, dx$

Optimal. Leaf size=21

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

```
[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n} (b + 2cx^n) \left(bx^n + cx^{2n}\right)^{13} \, dx &= \int x^{-1+14n} (b + cx^n)^{13} (b + 2cx^n) \, dx\\ &= \frac{\text{Subst}\left(\int x^{13}(b + cx)^{13}(b + 2cx) \, dx, x, x^n\right)}{n}\\ &= \frac{x^{14n} (b + cx^n)^{14}}{14n}\end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

fricas [B] time = 0.68, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} + 3432}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

giac [B] time = 0.43, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} + 3432}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

maple [B] time = 0.04, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n}+\frac{b^{13}c\,x^{15n}}{n}+\frac{13b^{12}c^2x^{16n}}{2n}+\frac{26b^{11}c^3x^{17n}}{n}+\frac{143b^{10}c^4x^{18n}}{2n}+\frac{143b^9c^5x^{19n}}{n}+\frac{429b^8c^6x^{20n}}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+\frac{429b^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*b^3*c^11
/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*b^5*c^9/n*(x^n)^23+429/2*c^8/n*(x
^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^5/n
*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)
^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

maxima [B] time = 0.48, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^2c^{12}x^{26\,n}}{2\,n}+\frac{26\,b^3c^{11}x^{25\,n}}{n}+\frac{143\,b^4c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^5c^9x^{23\,n}}{n}+\frac{429\,b^6c^8x^{22\,n}}{2\,n}+\frac{1716\,b^7c^7x^{21\,n}}{7\,n}+\frac{4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

mupad [B] time = 2.63, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{b+2cx}{a+bx+cx^2} \, dx$

Optimal. Leaf size=11

$$\log \left(a+bx+cx^2\right)$$

[Out] ln(c*x^2+b*x+a)

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {628}

$$\log \left(a+bx+cx^2\right)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(a + b*x + c*x^2),x]

[Out] Log[a + b*x + c*x^2]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{a+bx+cx^2} \, dx = \log \left(a+bx+cx^2\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.91

$$\log(a+x(b+cx))$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]

[Out] Log[a + x*(b + c*x)]

fricas [A] time = 0.75, size = 11, normalized size = 1.00

$$\log \left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] log(c*x^2 + b*x + a)

giac [A] time = 0.35, size = 12, normalized size = 1.09

$$\log \left(\left|cx^2+bx+a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")

[Out] log(abs(c*x^2 + b*x + a))

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\ln \left(c\,x^{2}+b x +a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a),x)
```

```
[Out] ln(c*x^2+b*x+a)
```

maxima [A] time = 0.45, size = 11, normalized size = 1.00

$$\log\left(cx^{2} + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x + a)
```

mupad [B] time = 1.96, size = 11, normalized size = 1.00

$$\ln\left(c\,x^2+b\,x+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2),x)
```

```
[Out] log(a + b*x + c*x^2)
```

sympy [A] time = 0.16, size = 10, normalized size = 0.91

$$\log{\left(a + bx + cx^{2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a),x)
```

```
[Out] log(a + b*x + c*x**2)
```

3.106 $\int \frac{x(b+2cx^2)}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

```
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1247, 628}

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(a+bx^2+cx^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

fricas [A] time = 0.71, size = 15, normalized size = 0.88

$$\frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

giac [A]   time = 1.73, size = 16, normalized size = 0.94

$$\frac{1}{2} \log \left(\left|cx^4 + bx^2 + a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2 + a))
```

maple [A]   time = 0.00, size = 16, normalized size = 0.94

$$\frac{\ln \left(c\,x^4 + b\,x^2 + a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x)
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

maxima [A]   time = 0.43, size = 15, normalized size = 0.88

$$\frac{1}{2} \log \left(cx^4 + bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

mupad [B]   time = 1.96, size = 15, normalized size = 0.88

$$\frac{\ln \left(c\,x^4 + b\,x^2 + a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x)
[Out] log(a + b*x^2 + c*x^4)/2
```

sympy [A]   time = 0.28, size = 14, normalized size = 0.82

$$\frac{\log \left(a + bx^2 + cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
[Out] log(a + b*x**2 + c*x**4)/2
```

3.107 $\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=17

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

```
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 628}

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\log\left(a+bx^3+cx^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

fricas [A] time = 0.63, size = 15, normalized size = 0.88

$$\frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

giac [A] time = 1.09, size = 16, normalized size = 0.94

$$\frac{1}{3}\log\left(\left|cx^6+bx^3+a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3 + a))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{\ln\left(c\,x^6+b\,x^3+a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x)
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

maxima [A] time = 0.44, size = 15, normalized size = 0.88

$$\frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

mupad [B] time = 0.05, size = 15, normalized size = 0.88

$$\frac{\ln\left(c\,x^6+b\,x^3+a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x)
[Out] log(a + b*x^3 + c*x^6)/3
```

sympy [A] time = 0.41, size = 14, normalized size = 0.82

$$\frac{\log\left(a+bx^3+cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)
[Out] log(a + b*x**3 + c*x**6)/3
```

3.108 $\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

```
[Out] ln(a+b*x^n+c*x^(2*n))/n
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1468, 628\}$

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\log\left(a+bx^n+cx^{2n}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.10, size = 19, normalized size = 1.00

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

fricas [A] time = 0.70, size = 19, normalized size = 1.00

$$\frac{\log\left(cx^{2n}+bx^n+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

giac [A] time = 0.46, size = 19, normalized size = 1.00

$$\frac{\log\left(cx^{2n} + bx^{n} + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

maple [A] time = 0.02, size = 24, normalized size = 1.26

$$\frac{\ln\left(b\,\mathrm{e}^{n\ln(x)} + c\,\mathrm{e}^{2n\ln(x)} + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)+a),x)
[Out] 1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
```

maxima [A] time = 0.60, size = 23, normalized size = 1.21

$$\frac{\log\left(\frac{cx^{2n}+bx^{n}+a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log((c*x^(2*n) + b*x^n + a)/c)/n
```

mupad [B] time = 2.32, size = 121, normalized size = 6.37

$$-\frac{2\,b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)-\ln\left(a+b\,x^n+c\,x^{2\,n}\right)\,\sqrt{4\,a\,c-b^2}}{n\,\sqrt{4\,a\,c-b^2}}-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2-4\,a\,c}}\right)}{n\,\sqrt{b^2-4\,a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x)
[Out] - (2*b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^n)/(4*a*c - b^2)^(1/2)) - log(a
+ b*x^n + c*x^(2*n))*(4*a*c - b^2)^(1/2))/(n*(4*a*c - b^2)^(1/2)) - (2*b*at
anh((b + 2*c*x^n)/(b^2 - 4*a*c)^(1/2)))/(n*(b^2 - 4*a*c)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.109 $\int \frac{b+2cx}{(a+bx+cx^2)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{7\left(a+bx+cx^2\right)^7}$$

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {629}

$$-\frac{1}{7\left(a+bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(a + b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8} dx = -\frac{1}{7\left(a+bx+cx^2\right)^7}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$-\frac{1}{7(a+x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(a + x*(b + c*x))^7
```

fricas [B] time = 0.64, size = 350, normalized size = 21.88

$$-\frac{}{7\left(c^7x^{14}+7\,bc^6x^{13}+7\left(3\,b^2c^5+ac^6\right)x^{12}+7\left(5\,b^3c^4+6\,abc^5\right)x^{11}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{10}+7\left(3\,b^5c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5*b^3*c^4 +
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2
```

```
+ 140*a^3*b*c^3)*x^7 + a^7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 + 7*(5*a^3*b^
4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 + 7*(3*a^
5*b^2 + a^6*c)*x^2)
```

giac [A] time = 0.40, size = 14, normalized size = 0.88

$$-\frac{1}{7\left(cx^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{7\left(c\,x^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a)^8,x)
```

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

maxima [A] time = 0.44, size = 14, normalized size = 0.88

$$-\frac{1}{7\left(cx^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

mupad [B] time = 3.62, size = 358, normalized size = 22.38

$$-\frac{}{7\left(x^5\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\left(140\,a^3\,b\,c^3+210\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*(x^5*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 +
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 + 42*a*b^5*c) + x^3*(35*a^4*b^3 + 42*a^5*b*c) + x^11*(35*b^3*c^4 + 42*a*b
*c^5) + x^4*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^6*(7*a*b^6 + 35*a^4*c^3 + 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c + 21*a^5*b^2) + x^12*(7*a*c^6 + 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

sympy [B] time = 4.79, size = 359, normalized size = 22.44

$$-\frac{}{7a^7+49a^6bx+49bc^6x^{13}+7c^7x^{14}+x^{12}\left(49ac^6+147b^2c^5\right)+x^{11}\left(294abc^5+245b^3c^4\right)+x^{10}\left(147a^2c^5+735\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)
```

```
[Out] -1/(7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(49*a*c**
6 + 147*b**2*c**5) + x**11*(294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a**2
*c**5 + 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 + 980*a*b*
*3*c**3 + 147*b**5*c**2) + x**8*(245*a**3*c**4 + 1470*a**2*b**2*c**3 + 735*
a*b**4*c**2 + 49*b**6*c) + x**7*(980*a**3*b*c**3 + 1470*a**2*b**3*c**2 + 29
4*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 + 1470*a**3*b**2*c**2 + 735*a**2
*b**4*c + 49*a*b**6) + x**5*(735*a**4*b*c**2 + 980*a**3*b**3*c + 147*a**2*b
**5) + x**4*(147*a**5*c**2 + 735*a**4*b**2*c + 245*a**3*b**4) + x**3*(294*a
**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c + 147*a**5*b**2))
```

3.110 $\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

```
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 629}

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{\left(a+bx^2+cx^4\right)^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx, x, x^2\right)\\ &= -\frac{1}{14\left(a+bx^2+cx^4\right)^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(a + b*x^2 + c*x^4)^7
```

fricas [B] time = 0.73, size = 352, normalized size = 19.56

$$-\frac{}{14\left(c^7x^{28}+7\,bc^6x^{26}+7\left(3\,b^2c^5+ac^6\right)x^{24}+7\left(5\,b^3c^4+6\,abc^5\right)x^{22}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{20}+7\left(3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

giac [A] time = 6.78, size = 16, normalized size = 0.89

$$-\frac{1}{14\left(cx^4+bx^2+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 + a)^7
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{14\left(c\,x^4+b\,x^2+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x)
```

```
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

maxima [B] time = 0.95, size = 352, normalized size = 19.56

$$-\frac{}{14\left(c^7x^{28}+7\,bc^6x^{26}+7\left(3\,b^2c^5+ac^6\right)x^{24}+7\left(5\,b^3c^4+6\,abc^5\right)x^{22}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{20}+7\left(3\,b^5c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

mupad [B] time = 12.16, size = 360, normalized size = 20.00

$$-\frac{}{14\left(x^{10}\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{18}\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{14}\left(140\,a^3\,b\,c^3+210\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^6*(35*a^4*b^3 + 42*a^5*b*c) + x^22*(35*b^3*c^4 + 42
*a*b*c^5) + x^8*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^12*(7*a*b^6 + 35*a^4*c^3 + 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c + 21*a^5*b^2) + x^24*(7*a*c^6 + 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

sympy [B] time = 7.66, size = 360, normalized size = 20.00

$$-\frac{}{14a^7 + 98a^6bx^2 + 98bc^6x^{26} + 14c^7x^{28} + x^{24}\left(98ac^6 + 294b^2c^5\right) + x^{22}\left(588abc^5 + 490b^3c^4\right) + x^{20}\left(294a^2c^5 + 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)
```

```
[Out] -1/(14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(98*
a*c**6 + 294*b**2*c**5) + x**22*(588*a*b*c**5 + 490*b**3*c**4) + x**20*(294
*a**2*c**5 + 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4 +
1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(490*a**3*c**4 + 2940*a**2*b**2*c
**3 + 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(1960*a**3*b*c**3 + 2940*a**2*b
**3*c**2 + 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 + 2940*a**3*b**2*
c**2 + 1470*a**2*b**4*c + 98*a*b**6) + x**10*(1470*a**4*b*c**2 + 1960*a**3*
b**3*c + 294*a**2*b**5) + x**8*(294*a**5*c**2 + 1470*a**4*b**2*c + 490*a**3
*b**4) + x**6*(588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c + 294*a**5*b
**2))
```

3.111 $\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

```
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 629}

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(b+2cx^3\right)}{\left(a+bx^3+cx^6\right)^8}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx, x, x^3\right)\\
&= -\frac{1}{21\left(a+bx^3+cx^6\right)^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(a + b*x^3 + c*x^6)^7
```

fricas [B] time = 0.65, size = 352, normalized size = 19.56

$$-\frac{}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{36}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{33}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{30}+7\left(3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

giac [A] time = 22.37, size = 16, normalized size = 0.89

$$-\frac{1}{21\left(cx^{6}+bx^{3}+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 + a)^7
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{21\left(c\,x^{6}+b\,x^{3}+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

maxima [B] time = 0.95, size = 352, normalized size = 19.56

$$-\frac{}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{36}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{33}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{30}+7\left(3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

mupad [B] time = 18.21, size = 360, normalized size = 20.00

$$-\frac{}{21\left(x^{15}\left(105\,a^{4}\,b\,c^{2}+140\,a^{3}\,b^{3}\,c+21\,a^{2}\,b^{5}\right)+x^{27}\left(105\,a^{2}\,b\,c^{4}+140\,a\,b^{3}\,c^{3}+21\,b^{5}\,c^{2}\right)+x^{21}\left(140\,a^{3}\,b\,c^{3}+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^9*(35*a^4*b^3 + 42*a^5*b*c) + x^33*(35*b^3*c^4 + 42
*a*b*c^5) + x^12*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^18*(7*a*b^6 + 35*a^4*c^3 + 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c + 21*a^5*b^2) + x^36*(7*a*c^6 +
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

sympy [B] time = 11.76, size = 360, normalized size = 20.00

$$-\frac{}{21a^7 + 147a^6bx^3 + 147bc^6x^{39} + 21c^7x^{42} + x^{36}\left(147ac^6 + 441b^2c^5\right) + x^{33}\left(882abc^5 + 735b^3c^4\right) + x^{30}\left(441a^2c^5 + 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)
```

```
[Out] -1/(21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(1
47*a*c**6 + 441*b**2*c**5) + x**33*(882*a*b*c**5 + 735*b**3*c**4) + x**30*(
441*a**2*c**5 + 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c**4
 + 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(735*a**3*c**4 + 4410*a**2*b**
2*c**3 + 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(2940*a**3*b*c**3 + 4410*a*
*2*b**3*c**2 + 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 + 4410*a**3*b
**2*c**2 + 2205*a**2*b**4*c + 147*a*b**6) + x**15*(2205*a**4*b*c**2 + 2940*
a**3*b**3*c + 441*a**2*b**5) + x**12*(441*a**5*c**2 + 2205*a**4*b**2*c + 73
5*a**3*b**4) + x**9*(882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*c + 441
*a**5*b**2))
```

3.112 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(a+bx^n+cx^{2n}\right)^8}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

```
[Out] -1/7/n/(a+b*x^n+c*x^(2*n))^7
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1468, 629}

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{\left(a+bx^n+cx^{2n}\right)^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx, x, x^n\right)}{n}\\ &= -\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.96

$$-\frac{1}{7n\left(a+x^n\left(b+cx^n\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*(a + x^n*(b + c*x^n))^7)
```

fricas [B] time = 0.75, size = 394, normalized size = 17.13

$$-\frac{}{7\left(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n+a^7n+7\left(3\,b^2c^5+ac^6\right)nx^{12\,n}+7\left(5\,b^3c^4+6\,abc^5\right)nx^{11\,n}+7\left(5\,b^4c^3+15\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 + 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3
*c^4)*n*x^(8*n) + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*n*x^
(7*n) + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4 + 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n*x^(3*n) + 7*(3
*a^5*b^2 + a^6*c)*n*x^(2*n))
```

giac [A] time = 0.63, size = 21, normalized size = 0.91

$$-\frac{1}{7\left(cx^{2\,n}+bx^n+a\right)^7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n + a)^7*n)
```

maple [A] time = 0.06, size = 22, normalized size = 0.96

$$-\frac{1}{7\left(b\,x^n+c\,x^{2n}+a\right)^7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)+a)^8,x)
```

```
[Out] -1/7/n/(a+b*x^n+c*(x^n)^2)^7
```

maxima [B] time = 2.39, size = 416, normalized size = 18.09

$$-\frac{}{7\left(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n+a^7n+7\left(3\,b^2c^5n+ac^6n\right)x^{12\,n}+7\left(5\,b^3c^4n+6\,abc^5n\right)x^{11\,n}+7\left(5\,b^4c^3n+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5*n + a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n + 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n + 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n + 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n + 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n + 5*a^3*c^4*n)*x^(8*n) + (b^7*n + 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n + 140*a^3*b*c^3*n)*x^(7*n) + 7*(a*b^6*n + 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n + 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n + 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) + 7*(5*a^3*b^4*n + 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n + 6*a^5*b*c*n)*x^(3*n) + 7*(3*a^5*b^2*n + a^6*c*n)*x^(2*n))
```

mupad [B] time = 23.01, size = 496, normalized size = 21.57

$$-\frac{}{7\,a^7\,n+7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,a^6\,b\,n\,x^n+49\,a\,b^6\,n\,x^{6\,n}+49\,a^6\,c\,n\,x^{2\,n}+49\,a\,c^6\,n\,x^{12\,n}+49\,b^6\,c\,n\,x^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*a^7*n + 7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n + 49*a*b^
6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) + 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n)
 + 49*b*c^6*n*x^(13*n) + 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) + 24
5*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) + 147*a^5*c^2*n*x^(4*n) + 245*a
^4*c^3*n*x^(6*n) + 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5
*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*
c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) + 980*a^3*b^3*c*n*x^(5*n) + 735*a^
4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) + 980*a^3*b*c^3*n*x^(7*n) + 735
*a*b^4*c^2*n*x^(8*n) + 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) +
735*a*b^2*c^4*n*x^(10*n) + 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*
x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) + 294*a^5*b*c*n*x^(3*n) + 294*a*b^5*c*
n*x^(7*n) + 294*a*b*c^5*n*x^(11*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

3.113 $\int \frac{b+2cx}{-a+bx+cx^2}\,dx$

Optimal. Leaf size=13

$$\log\left(a-bx-cx^2\right)$$

```
[Out] ln(-c*x^2-b*x+a)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {628}

$$\log\left(a-bx-cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[a - b*x - c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx=\log\left(a-bx-cx^2\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.92

$$\log(x(b+cx)-a)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[-a + x*(b + c*x)]
```

fricas [A] time = 0.49, size = 13, normalized size = 1.00

$$\log\left(cx^2+bx-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x - a)
```

giac [A] time = 0.39, size = 14, normalized size = 1.08

$$\log\left(\left|cx^2+bx-a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="giac")
```

```
[Out] log(abs(c*x^2 + b*x - a))
```

maple [**A**] time = 0.00, size = 14, normalized size = 1.08

$$\ln\left(c\,x^{2}+bx-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a),x)
[Out] ln(c*x^2+b*x-a)
```

maxima [**A**] time = 0.44, size = 13, normalized size = 1.00

$$\log\left(cx^{2}+bx-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="maxima")
[Out] log(c*x^2 + b*x - a)
```

mupad [**B**] time = 0.05, size = 13, normalized size = 1.00

$$\ln\left(c\,x^{2}+b\,x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2),x)
[Out] log(b*x - a + c*x^2)
```

sympy [**A**] time = 0.15, size = 10, normalized size = 0.77

$$\log\left(-a+bx+cx^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a),x)
[Out] log(-a + b*x + c*x**2)
```

3.114 $\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

```
[Out] 1/2*ln(-c*x^4-b*x^2+a)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 628}

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a - b*x^2 - c*x^4]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{-a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(a-bx^2-cx^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\log\left(-a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[-a + b*x^2 + c*x^4]/2
```

fricas [A] time = 0.56, size = 17, normalized size = 0.89

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="fricas")
```

[Out] 1/2*log(c*x^4 + b*x^2 - a)

giac [A] time = 1.62, size = 18, normalized size = 0.95

$$\frac{1}{2}\,\log\left(\left|cx^4 + bx^2 - a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="giac")

[Out] 1/2*log(abs(c*x^4 + b*x^2 - a))

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\ln\left(c\,x^4 + b\,x^2 - a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x)

[Out] 1/2*ln(c*x^4+b*x^2-a)

maxima [A] time = 0.44, size = 17, normalized size = 0.89

$$\frac{1}{2}\,\log\left(cx^4 + bx^2 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="maxima")

[Out] 1/2*log(c*x^4 + b*x^2 - a)

mupad [B] time = 0.05, size = 17, normalized size = 0.89

$$\frac{\ln\left(c\,x^4 + b\,x^2 - a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4),x)

[Out] log(b*x^2 - a + c*x^4)/2

sympy [A] time = 0.28, size = 14, normalized size = 0.74

$$\frac{\log\left(-a + bx^2 + cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)

[Out] log(-a + b*x**2 + c*x**4)/2

3.115 $\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

```
[Out] 1/3*ln(-c*x^6-b*x^3+a)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1468, 628}

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{-a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\log\left(a-bx^3-cx^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{3}\log\left(-a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[-a + b*x^3 + c*x^6]/3
```

fricas [A] time = 0.62, size = 17, normalized size = 0.89

$$\frac{1}{3}\log\left(cx^6+bx^3-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="fricas")
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

giac [A] time = 1.04, size = 18, normalized size = 0.95

$$\frac{1}{3}\log\left(\left|cx^6 + bx^3 - a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3 - a))
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\ln\left(c\,x^6 + b\,x^3 - a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x)
[Out] 1/3*ln(c*x^6+b*x^3-a)
```

maxima [A] time = 0.43, size = 17, normalized size = 0.89

$$\frac{1}{3}\log\left(cx^6 + bx^3 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="maxima")
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.89

$$\frac{\ln\left(c\,x^6 + b\,x^3 - a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6),x)
[Out] log(b*x^3 - a + c*x^6)/3
```

sympy [A] time = 0.39, size = 14, normalized size = 0.74

$$\frac{\log\left(-a + bx^3 + cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
[Out] log(-a + b*x**3 + c*x**6)/3
```

3.116 $\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=21

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

```
[Out] ln(a-b*x^n-c*x^(2*n))/n
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 628}

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{-a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\log\left(a-bx^n-cx^{2n}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

fricas [A] time = 0.68, size = 21, normalized size = 1.00

$$\frac{\log\left(cx^{2n}+bx^n-a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

giac [A] time = 0.37, size = 21, normalized size = 1.00

$$\frac{\log\left(cx^{2n} + bx^n - a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

maple [A] time = 0.02, size = 26, normalized size = 1.24

$$\frac{\ln\left(-b\,\mathrm{e}^{n\ln(x)} - c\,\mathrm{e}^{2n\ln(x)} + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x)
[Out] 1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)
```

maxima [A] time = 0.60, size = 25, normalized size = 1.19

$$\frac{\log\left(\frac{cx^{2n}+bx^n-a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log((c*x^(2*n) + b*x^n - a)/c)/n
```

mupad [B] time = 2.68, size = 199, normalized size = 9.48

$$\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,\left(b+2\,c\,x^n\right)\right)\,\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)+\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,\left(b+2\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n)),x)
[Out] log((2*c*x^n)/n - (1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*
c*x^n))*(1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) + log((2*c*x^n)/n
 - (1/n - (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*c*x^n))*(1/n -
(b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) - (2*b*atanh((b + 2*c*x^n)/(4*a*
c + b^2)^(1/2)))/(n*(4*a*c + b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.117 $\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8} dx$

Optimal. Leaf size=18

$$\frac{1}{7\left(a-bx-cx^2\right)^7}$$

```
[Out] 1/7/(-c*x^2-b*x+a)^7
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {629}

$$\frac{1}{7\left(a-bx-cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - b*x - c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8} dx = \frac{1}{7\left(a-bx-cx^2\right)^7}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.89

$$\frac{1}{7(a-x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - x*(b + c*x))^7)
```

fricas [B] time = 0.67, size = 354, normalized size = 19.67

$$-\frac{}{7\left(c^7x^{14}+7\,bc^6x^{13}+7\left(3\,b^2c^5-ac^6\right)x^{12}+7\left(5\,b^3c^4-6\,abc^5\right)x^{11}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{10}+7\left(3\,b^5c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 - a*c^6)*x^12 + 7*(5*b^3*c^4 -
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 - 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2
```

```
- 140*a^3*b*c^3)*x^7 - a^7 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 - 7*(5*a^3*b^
4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^3 - 7*(3*a^
5*b^2 - a^6*c)*x^2)
```

giac [A] time = 0.43, size = 16, normalized size = 0.89

$$-\frac{1}{7\left(cx^2+bx-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{7\left(c\,x^2+bx-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a)^8,x)
```

```
[Out] -1/7/(c*x^2+b*x-a)^7
```

maxima [A] time = 0.43, size = 16, normalized size = 0.89

$$-\frac{1}{7\left(cx^2+bx-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

mupad [B] time = 5.22, size = 358, normalized size = 19.89

$$-\frac{}{7\left(x^5\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\left(-140\,a^3\,b\,c^3+210\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2)^8,x)
```

```
[Out] -1/(7*(x^5*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 -
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 - 42*a*b^5*c) + x^3*(35*a^4*b^3 - 42*a^5*b*c) + x^11*(35*b^3*c^4 - 42*a*b
*c^5) - x^4*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^6*(7*a*b^6 - 35*a^4*c^3 - 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c - 21*a^5*b^2) - x^12*(7*a*c^6 - 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

sympy [B] time = 5.06, size = 359, normalized size = 19.94

$$-\frac{}{-7a^7+49a^6bx+49bc^6x^{13}+7c^7x^{14}+x^{12}\left(-49ac^6+147b^2c^5\right)+x^{11}\left(-294abc^5+245b^3c^4\right)+x^{10}\left(147a^2c^5-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)
```

```
[Out] -1/(-7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(-49*a*c
**6 + 147*b**2*c**5) + x**11*(-294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a
**2*c**5 - 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 - 980*a
*b**3*c**3 + 147*b**5*c**2) + x**8*(-245*a**3*c**4 + 1470*a**2*b**2*c**3 -
735*a*b**4*c**2 + 49*b**6*c) + x**7*(-980*a**3*b*c**3 + 1470*a**2*b**3*c**2
 - 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 - 1470*a**3*b**2*c**2 + 735
*a**2*b**4*c - 49*a*b**6) + x**5*(735*a**4*b*c**2 - 980*a**3*b**3*c + 147*a
**2*b**5) + x**4*(-147*a**5*c**2 + 735*a**4*b**2*c - 245*a**3*b**4) + x**3*
(-294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c - 147*a**5*b**2))
```

3.118 $\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{14\left(a-bx^2-cx^4\right)^7}$$

```
[Out] 1/14/(-c*x^4-b*x^2+a)^7
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{1}{14\left(a-bx^2-cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] 1/(14*(a - b*x^2 - c*x^4)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{\left(-a+bx^2+cx^4\right)^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx, x, x^2\right)\\ &= \frac{1}{14\left(a-bx^2-cx^4\right)^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{14\left(-a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(-a + b*x^2 + c*x^4)^7
```

fricas [B] time = 0.76, size = 356, normalized size = 17.80

$$-\frac{}{14\left(c^7x^{28}+7bc^6x^{26}+7\left(3b^2c^5-ac^6\right)x^{24}+7\left(5b^3c^4-6abc^5\right)x^{22}+7\left(5b^4c^3-15ab^2c^4+3a^2c^5\right)x^{20}+7\left(3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

giac [A] time = 7.19, size = 18, normalized size = 0.90

$$-\frac{1}{14\left(cx^4+bx^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 - a)^7
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{1}{14\left(c\,x^4+b\,x^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x)
```

```
[Out] -1/14/(c*x^4+b*x^2-a)^7
```

maxima [B] time = 0.97, size = 356, normalized size = 17.80

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+7\left(3\,b^2c^5-ac^6\right)x^{24}+7\left(5\,b^3c^4-6\,abc^5\right)x^{22}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{20}+7\left(3\,b^5c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

mupad [B] time = 11.04, size = 360, normalized size = 18.00

$$-\frac{1}{14\left(x^{10}\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{18}\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{14}\left(-140\,a^3\,b\,c^3+210\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^6*(35*a^4*b^3 - 42*a^5*b*c) + x^22*(35*b^3*c^4 - 42
*a*b*c^5) - x^8*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^12*(7*a*b^6 - 35*a^4*c^3 - 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c - 21*a^5*b^2) - x^24*(7*a*c^6 - 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

sympy [B] time = 8.09, size = 360, normalized size = 18.00

$$-\frac{}{-14a^7 + 98a^6bx^2 + 98bc^6x^{26} + 14c^7x^{28} + x^{24}\left(-98ac^6 + 294b^2c^5\right) + x^{22}\left(-588abc^5 + 490b^3c^4\right) + x^{20}\left(294a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)
```

```
[Out] -1/(-14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(-9
8*a*c**6 + 294*b**2*c**5) + x**22*(-588*a*b*c**5 + 490*b**3*c**4) + x**20*(
294*a**2*c**5 - 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4
 - 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(-490*a**3*c**4 + 2940*a**2*b*
*2*c**3 - 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(-1960*a**3*b*c**3 + 2940*a
**2*b**3*c**2 - 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 - 2940*a**3*
b**2*c**2 + 1470*a**2*b**4*c - 98*a*b**6) + x**10*(1470*a**4*b*c**2 - 1960*
a**3*b**3*c + 294*a**2*b**5) + x**8*(-294*a**5*c**2 + 1470*a**4*b**2*c - 49
0*a**3*b**4) + x**6*(-588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c - 294
*a**5*b**2))
```

3.119 $\int \frac{x^2\left(b+2cx^3\right)}{\left(-a+bx^3+cx^6\right)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

```
[Out] 1/21/(-c*x^6-b*x^3+a)^7
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{\left(-a+bx^3+cx^6\right)^8}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx,x,x^3\right)\\ &= \frac{1}{21\left(a-bx^3-cx^6\right)^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{21\left(-a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(-a + b*x^3 + c*x^6)^7
```

fricas [B] time = 0.53, size = 356, normalized size = 17.80

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+7\left(3\,b^{2}c^{5}-ac^{6}\right)x^{36}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)x^{33}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{30}+7\left(3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

giac [A] time = 22.35, size = 18, normalized size = 0.90

$$-\frac{1}{21\left(cx^{6}+bx^{3}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 - a)^7
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{1}{21\left(c\,x^{6}+b\,x^{3}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3-a)^7
```

maxima [B] time = 0.94, size = 356, normalized size = 17.80

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+7\left(3\,b^{2}c^{5}-ac^{6}\right)x^{36}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)x^{33}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{30}+7\left(3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

mupad [B] time = 16.60, size = 360, normalized size = 18.00

$$-\frac{1}{21\,\left(x^{15}\,\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{27}\,\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{21}\,\left(-140\,a^3\,b\,c^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^9*(35*a^4*b^3 - 42*a^5*b*c) + x^33*(35*b^3*c^4 - 42
*a*b*c^5) - x^12*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^18*(7*a*b^6 - 35*a^4*c^3 - 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c - 21*a^5*b^2) - x^36*(7*a*c^6 -
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

sympy [B] time = 11.79, size = 360, normalized size = 18.00

$$-\frac{}{-21a^7 + 147a^6bx^3 + 147bc^6x^{39} + 21c^7x^{42} + x^{36}\left(-147ac^6 + 441b^2c^5\right) + x^{33}\left(-882abc^5 + 735b^3c^4\right) + x^{30}\left(441a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

```
[Out] -1/(-21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(
-147*a*c**6 + 441*b**2*c**5) + x**33*(-882*a*b*c**5 + 735*b**3*c**4) + x**3
0*(441*a**2*c**5 - 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c
**4 - 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(-735*a**3*c**4 + 4410*a**2
*b**2*c**3 - 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(-2940*a**3*b*c**3 + 44
10*a**2*b**3*c**2 - 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 - 4410*a
**3*b**2*c**2 + 2205*a**2*b**4*c - 147*a*b**6) + x**15*(2205*a**4*b*c**2 -
2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(-441*a**5*c**2 + 2205*a**4*b**2*
c - 735*a**3*b**4) + x**9*(-882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*
c - 441*a**5*b**2))
```

3.120 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(-a+bx^n+cx^{2n}\right)^8}\,dx$

Optimal. Leaf size=25

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

```
[Out] 1/7/n/(a-b*x^n-c*x^(2*n))^7
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{\left(-a+bx^n+cx^{2n}\right)^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx,x,x^n\right)}{n}\\ &= \frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}\end{aligned}$$

Mathematica [A] time = 0.07, size = 23, normalized size = 0.92

$$\frac{1}{7n\left(a-x^n\left(b+cx^n\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - x^n*(b + c*x^n))^7)
```

fricas [B] time = 0.70, size = 397, normalized size = 15.88

$$-\frac{}{7\left(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+7\,a^{6}bnx^{n}-a^{7}n+7\left(3\,b^{2}c^{5}-ac^{6}\right)nx^{12\,n}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)nx^{11\,n}+7\left(5\,b^{4}c^{3}-15\,ab\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 - 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3
*c^4)*n*x^(8*n) + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*n*x^
(7*n) - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) - 7*(5*a^3*b^4 - 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b*c)*n*x^(3*n) - 7*(3
*a^5*b^2 - a^6*c)*n*x^(2*n))
```

giac [A] time = 0.79, size = 23, normalized size = 0.92

$$-\frac{1}{7\left(cx^{2\,n}+bx^{n}-a\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n - a)^7*n)
```

maple [A] time = 0.07, size = 24, normalized size = 0.96

$$\frac{1}{7\left(-b\,x^{n}-c\,x^{2n}+a\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x)
```

```
[Out] 1/7/n/(-c*(x^n)^2-b*x^n+a)^7
```

maxima [B] time = 2.42, size = 419, normalized size = 16.76

$$-\frac{}{7\left(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+7\,a^{6}bnx^{n}-a^{7}n+7\left(3\,b^{2}c^{5}n-ac^{6}n\right)x^{12\,n}+7\left(5\,b^{3}c^{4}n-6\,abc^{5}n\right)x^{11\,n}+7\left(5\,b^{4}c^{3}n-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5*n - a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n - 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n - 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n - 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n - 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n - 5*a^3*c^4*n)*x^(8*n) + (b^7*n - 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n - 140*a^3*b*c^3*n)*x^(7*n) - 7*(a*b^6*n - 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n - 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n - 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) - 7*(5*a^3*b^4*n - 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n - 6*a^5*b*c*n)*x^(3*n) - 7*(3*a^5*b^2*n - a^6*c*n)*x^(2*n))
```

mupad [B] time = 22.40, size = 496, normalized size = 19.84

$$-\frac{}{7 b^7 n x^{7 n} - 7 a^7 n + 7 c^7 n x^{14 n} + 49 a^6 b n x^n - 49 a b^6 n x^{6 n} + 49 a^6 c n x^{2 n} - 49 a c^6 n x^{12 n} + 49 b^6 c n x^{8 n} +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) - 7*a^7*n + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n - 49*a*b^
6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) - 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n)
 + 49*b*c^6*n*x^(13*n) - 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) - 24
5*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) - 147*a^5*c^2*n*x^(4*n) + 245*a
^4*c^3*n*x^(6*n) - 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5
*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*
c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) - 980*a^3*b^3*c*n*x^(5*n) + 735*a^
4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) - 980*a^3*b*c^3*n*x^(7*n) - 735
*a*b^4*c^2*n*x^(8*n) - 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) -
735*a*b^2*c^4*n*x^(10*n) - 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*
x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) - 294*a^5*b*c*n*x^(3*n) - 294*a*b^5*c*
n*x^(7*n) - 294*a*b*c^5*n*x^(11*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

3.121 $\int \frac{b+2cx}{bx+cx^2} dx$

Optimal. Leaf size=10

$$\log \left(bx+cx^2\right)$$

```
[Out] ln(c*x^2+b*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {628}

$$\log \left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{bx+cx^2} dx = \log \left(bx+cx^2\right)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.90

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

fricas [A] time = 0.81, size = 10, normalized size = 1.00

$$\log \left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

giac [A] time = 0.46, size = 11, normalized size = 1.10

$$\log \left(\left|cx^2+bx\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] log(abs(c*x^2 + b*x))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\ln\left(\left(cx+b\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x)
[Out] ln(x*(c*x+b))
```

maxima [A] time = 0.44, size = 10, normalized size = 1.00

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
[Out] log(c*x^2 + b*x)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\ln\left(x\ \left(b+c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
[Out] log(x*(b + c*x))
```

sympy [A] time = 0.12, size = 8, normalized size = 0.80

$$\log\left(bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
[Out] log(b*x + c*x**2)
```

3.122 $\int \frac{x\left(b+2cx^2\right)}{bx^2+cx^4} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2}\log\left(bx^2+cx^4\right)$$

```
[Out] 1/2*ln(c*x^4+b*x^2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(b+2cx^2\right)}{bx^2+cx^4} \, dx &= \int \frac{b+2cx^2}{x\left(b+cx^2\right)} \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right) \, dx, x, x^2\right)\\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
[Out] Log[x] + Log[b + c*x^2]/2
```

fricas [A] time = 0.84, size = 13, normalized size = 0.81

$$\frac{1}{2}\log\left(cx^2 + b\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

giac [A] time = 0.49, size = 15, normalized size = 0.94

$$\frac{1}{2}\log\left(\left|cx^4 + bx^2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2))
```

maple [A] time = 0.01, size = 14, normalized size = 0.88

$$\ln(x) + \frac{\ln\left(c\,x^2 + b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

maxima [A] time = 0.43, size = 17, normalized size = 1.06

$$\frac{1}{2}\log\left(cx^2 + b\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.81

$$\frac{\ln\left(c\,x^2 + b\right)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x)
[Out] log(b + c*x^2)/2 + log(x)
```

sympy [A] time = 0.19, size = 12, normalized size = 0.75

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)
[Out] log(x) + log(b/c + x**2)/2
```

3.123 $\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx$

Optimal. Leaf size=16

$$\frac{1}{3}\log\left(bx^3+cx^6\right)$$

```
[Out] 1/3*ln(c*x^6+b*x^3)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1584, 446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(b+2cx^3\right)}{bx^3+cx^6}\,dx &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^3\right)\\
&= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
[Out] Log[x] + Log[b + c*x^3]/3
```

fricas [A] time = 0.84, size = 13, normalized size = 0.81

$$\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="fricas")
[Out] 1/3*log(c*x^3 + b) + log(x)
```

giac [A] time = 0.34, size = 15, normalized size = 0.94

$$\frac{1}{3}\log\left(\left|cx^6+bx^3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3))
```

maple [A] time = 0.01, size = 14, normalized size = 0.88

$$\ln(x)+\frac{\ln\left(c\,x^3+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x)
[Out] ln(x)+1/3*ln(c*x^3+b)
```

maxima [A] time = 0.43, size = 17, normalized size = 1.06

$$\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="maxima")
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

mupad [B] time = 1.99, size = 13, normalized size = 0.81

$$\frac{\ln\left(c\,x^3+b\right)}{3}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x)
[Out] log(b + c*x^3)/3 + log(x)
```

sympy [A] time = 0.20, size = 12, normalized size = 0.75

$$\log{\left(x \right)}+\frac{\log{\left(\frac{b}{c}+x^{3} \right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)
[Out] log(x) + log(b/c + x**3)/3
```

3.124 $\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

```
[Out] ln(x)+ln(b+c*x^n)/n
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx &= \int \frac{b+2cx^n}{x(b+cx^n)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^n\right)}{n}\\
&= \log(x)+\frac{\log(b+cx^n)}{n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
[Out] Log[x] + Log[b + c*x^n]/n
```

fricas [A] time = 0.87, size = 17, normalized size = 1.13

$$\frac{n\log(x)+\log\left(cx^n+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (n*log(x) + log(c*x^n + b))/n
```

giac [A] time = 0.37, size = 17, normalized size = 1.13

$$\frac{\log\left(|cx^n+b|\right)}{n}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(abs(c*x^n + b))/n + log(abs(x))
```

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\ln(x)+\frac{\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x)
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

maxima [B] time = 0.44, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

mupad [B] time = 2.23, size = 28, normalized size = 1.87

$$\frac{2\left(\ln\left(b+c\,x^n\right)-\mathrm{atanh}\left(\frac{2\,c\,x^n}{b}+1\right)\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x)
[Out] (2*(log(b + c*x^n) - atanh((2*c*x^n)/b + 1)))/n
```

sympy [A] time = 31.23, size = 48, normalized size = 3.20

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \frac{n^2\log(x)}{n^2-n} - \frac{n\log(x)}{n^2-n} & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (n**2*log(x)/(n**2 - n) - n*log(x)/(n**2 - n), Eq(c, 0)), (log(x) + log
(b/c + x**n)/n, True))
```

3.125 $\int \frac{b+2cx}{\left(bx+cx^2\right)^8} dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

```
[Out] -1/7/(c*x^2+b*x)^7
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} dx = -\frac{1}{7\left(bx+cx^2\right)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

fricas [B] time = 0.82, size = 81, normalized size = 5.40

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

giac [A] time = 0.32, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

maple [B] time = 0.02, size = 177, normalized size = 11.80

$$\frac{c^7}{7\left(cx+b\right)^7 b^7}+\frac{c^7}{\left(cx+b\right)^6 b^8}+\frac{4c^7}{\left(cx+b\right)^5 b^9}+\frac{12c^7}{\left(cx+b\right)^4 b^{10}}+\frac{30c^7}{\left(cx+b\right)^3 b^{11}}+\frac{66c^7}{\left(cx+b\right)^2 b^{12}}+\frac{132c^7}{\left(cx+b\right) b^{13}}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] -1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4
-4/b^9*c^2/x^5+1/b^8*c/x^6+132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^
11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^
6+1/7*c^7/b^7/(c*x+b)^7
```

maxima [A] time = 0.42, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

mupad [B] time = 4.30, size = 12, normalized size = 0.80

$$-\frac{1}{7\,x^7\,\left(b+c\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

sympy [B] time = 0.88, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

3.126 $\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} \, dx &= \int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} \, dx, x, x^2\right) \\
&= -\frac{1}{14x^{14}(b+cx^2)^7}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

fricas [B] time = 0.88, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

giac [A] time = 0.45, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^{4}+bx^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{2}+b\right)^{7}c}-\frac{b^{5}}{\left(c\,x^{2}+b\right)^{6}c}-\frac{4b^{4}}{\left(c\,x^{2}+b\right)^{5}c}-\frac{12b^{3}}{\left(c\,x^{2}+b\right)^{4}c}-\frac{30b^{2}}{\left(c\,x^{2}+b\right)^{3}c}-\frac{66b}{\left(c\,x^{2}+b\right)^{2}c}-\frac{132}{\left(c\,x^{2}+b\right)c}\right)c^{8}}{2b^{13}}-\frac{66c^{6}}{b^{13}x^{2}}+\frac{33c^{5}}{b^{12}x^{4}}-\frac{15c^{4}}{b^{11}x^{6}}+\frac{6c^{3}}{b^{10}x^{8}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x)
```

```
[Out] -1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x
^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12-1/2*c^8/b^13*(-12*b^3/c/(c*x^2+b)^4-30*b^2
/c/(c*x^2+b)^3-132/c/(c*x^2+b)-b^5/c/(c*x^2+b)^6-4*b^4/c/(c*x^2+b)^5-66*b/c
/(c*x^2+b)^2-1/7*b^6/c/(c*x^2+b)^7)
```

maxima [B] time = 0.52, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

mupad [B] time = 2.33, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\left(c\,x^{2}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

sympy [B] time = 1.38, size = 87, normalized size = 5.44

$$-\frac{1}{14b^7x^{14} + 98b^6cx^{16} + 294b^5c^2x^{18} + 490b^4c^3x^{20} + 490b^3c^4x^{22} + 294b^2c^5x^{24} + 98bc^6x^{26} + 14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

3.127 $\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{\left(bx^3+cx^6\right)^8}\,dx &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx\\ &= \frac{1}{3}\,\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)\\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}\end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

fricas [B] time = 0.81, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

giac [A] time = 0.61, size = 15, normalized size = 0.94

$$-\frac{1}{21\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

maple [B] time = 0.01, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{3}+b\right)^{7}c}-\frac{b^{5}}{\left(c\,x^{3}+b\right)^{6}c}-\frac{4b^{4}}{\left(c\,x^{3}+b\right)^{5}c}-\frac{12b^{3}}{\left(c\,x^{3}+b\right)^{4}c}-\frac{30b^{2}}{\left(c\,x^{3}+b\right)^{3}c}-\frac{66b}{\left(c\,x^{3}+b\right)^{2}c}-\frac{132}{\left(c\,x^{3}+b\right)c}\right)c^{8}}{3b^{13}}-\frac{44c^{6}}{b^{13}x^{3}}+\frac{22c^{5}}{b^{12}x^{6}}-\frac{10c^{4}}{b^{11}x^{9}}+\frac{4c}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x)
```

```
[Out] -1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^3/x
^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18-1/3*c^8/b^13*(-12*b^3/c/(c*x^3+b)^4-30*
b^2/c/(c*x^3+b)^3-132/c/(c*x^3+b)-b^5/c/(c*x^3+b)^6-4*b^4/c/(c*x^3+b)^5-66*
b/c/(c*x^3+b)^2-1/7*b^6/c/(c*x^3+b)^7)
```

maxima [B] time = 0.51, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

mupad [B] time = 5.07, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,\left(c\,x^{3}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

sympy [**B**] time = 1.90, size = 87, normalized size = 5.44

$$-\frac{1}{21b^7x^{21} + 147b^6cx^{24} + 441b^5c^2x^{27} + 735b^4c^3x^{30} + 735b^3c^4x^{33} + 441b^2c^5x^{36} + 147bc^6x^{39} + 21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

3.128 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(bx^n+cx^{2n}\right)^8} \, dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}\left(b+2cx^n\right)}{\left(bx^n+cx^{2n}\right)^8} \, dx &= \int \frac{x^{-1-7n}\left(b+2cx^n\right)}{\left(b+cx^n\right)^8} \, dx \\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} \, dx, x, x^n\right)}{n} \\
&= -\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

fricas [B] time = 0.74, size = 105, normalized size = 5.00

$$-\frac{1}{7\left(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+21\,b^2c^5nx^{12\,n}+35\,b^3c^4nx^{11\,n}+35\,b^4c^3nx^{10\,n}+21\,b^5c^2nx^{9\,n}+7\,b^6cnx^{8\,n}+b^7nx^{7\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

giac [A] time = 0.44, size = 20, normalized size = 0.95

$$-\frac{1}{7\left(cx^{2\,n}+bx^n\right)^7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n)^7*n)
```

maple [B] time = 0.05, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^7n}+\frac{c\,x^{-6n}}{b^8n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{66c^5x^{-2n}}{b^{12}n}+\frac{\left(9009b^5c\,x^n+20020b^4c^2x^{2n}+24024b^3c^3x^{3n}+16380\right.}{7\left(c\,x^n+b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

maxima [B] time = 0.66, size = 612, normalized size = 29.14

$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}+401}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5nx^{12\,n}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
```

```
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

mupad [**B**] time = 2.36, size = 107, normalized size = 5.10

$$-\frac{1}{7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,b^6\,c\,n\,x^{8\,n}+49\,b\,c^6\,n\,x^{13\,n}+147\,b^5\,c^2\,n\,x^{9\,n}+245\,b^4\,c^3\,n\,x^{10\,n}+245\,b^3\,c^4\,n\,x^{11\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*b^6*c*n*x^(8*n) + 49*b*c^6*n*x^
(13*n) + 147*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(
11*n) + 147*b^2*c^5*n*x^(12*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

3.129 $\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (c*x^2+b*x+a)^(1+p)/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {629}

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx=\frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{(a+x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (a + x*(b + c*x))^(1 + p)/(1 + p)
```

fricas [A] time = 0.97, size = 28, normalized size = 1.40

$$\frac{\left(cx^2+bx+a\right)\left(cx^2+bx+a\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)
```

giac [A] time = 0.41, size = 20, normalized size = 1.00

$$\frac{\left(cx^2+bx+a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="giac")
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{\left(c\,x^{2}+bx+a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^p,x)
[Out] (c*x^2+b*x+a)^(p+1)/(p+1)
```

maxima [A] time = 0.42, size = 20, normalized size = 1.00

$$\frac{\left(cx^{2}+bx+a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

mupad [B] time = 2.04, size = 39, normalized size = 1.95

$$\left(\frac{a}{p+1}+\frac{b\,x}{p+1}+\frac{c\,x^{2}}{p+1}\right)\,\left(c\,x^{2}+b\,x+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^p,x)
[Out] (a/(p + 1) + (b*x)/(p + 1) + (c*x^2)/(p + 1))*(a + b*x + c*x^2)^p
```

sympy [B] time = 57.11, size = 104, normalized size = 5.20

$$\begin{cases} \frac{a\left(a+bx+cx^{2}\right)^{p}}{p+1}+\frac{bx\left(a+bx+cx^{2}\right)^{p}}{p+1}+\frac{cx^{2}\left(a+bx+cx^{2}\right)^{p}}{p+1} & \text{for}\ p\neq -1\\ \log\left(\frac{b}{2c}+x-\frac{\sqrt{-4ac+b^{2}}}{2c}\right)+\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^{2}}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)
[Out] Piecewise((a*(a + b*x + c*x**2)**p/(p + 1) + b*x*(a + b*x + c*x**2)**p/(p +
 1) + c*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x -
sqrt(-4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)),
True))
```

3.130 $\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 629}

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)\\ &= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

fricas [A] time = 0.76, size = 33, normalized size = 1.32

$$\frac{\left(cx^4+bx^2+a\right)\left(cx^4+bx^2+a\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

giac [A] time = 0.46, size = 23, normalized size = 0.92

$$\frac{\left(cx^4 + bx^2 + a\right)^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
[Out] 1/2*(c*x^4 + b*x^2 + a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{\left(c\,x^4 + b\,x^2 + a\right)^{p+1}}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x)
[Out] 1/2*(c*x^4+b*x^2+a)^(p+1)/(p+1)
```

maxima [A] time = 0.60, size = 33, normalized size = 1.32

$$\frac{\left(cx^4 + bx^2 + a\right)\left(cx^4 + bx^2 + a\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

mupad [B] time = 2.09, size = 49, normalized size = 1.96

$$\left(c\,x^4 + b\,x^2 + a\right)^p \left(\frac{a}{2\,p+2} + \frac{b\,x^2}{2\,p+2} + \frac{c\,x^4}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x)
[Out] (a + b*x^2 + c*x^4)^p*(a/(2*p + 2) + (b*x^2)/(2*p + 2) + (c*x^4)/(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)
[Out] Timed out
```

3.131 $\int x^2 \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[Out] 1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 629}

$$\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^p \, dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx,x,x^3\right)\\ &= \frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

fricas [A] time = 0.79, size = 33, normalized size = 1.32

$$\frac{\left(cx^6+bx^3+a\right)\left(cx^6+bx^3+a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

giac [A] time = 0.32, size = 23, normalized size = 0.92

$$\frac{\left(cx^6+bx^3+a\right)^{p+1}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3 + a)^(p + 1)/(p + 1)
```

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{\left(c x^6+b x^3+a\right)^{p+1}}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x)
[Out] 1/3*(c*x^6+b*x^3+a)^(p+1)/(p+1)
```

maxima [A] time = 0.58, size = 33, normalized size = 1.32

$$\frac{\left(cx^6+bx^3+a\right)\left(cx^6+bx^3+a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

mupad [B] time = 2.12, size = 49, normalized size = 1.96

$$\left(c x^6+b x^3+a\right)^p \left(\frac{a}{3p+3}+\frac{b x^3}{3p+3}+\frac{c x^6}{3p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x)
[Out] (a + b*x^3 + c*x^6)^p*(a/(3*p + 3) + (b*x^3)/(3*p + 3) + (c*x^6)/(3*p + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

3.132 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (a+b*x^n+c*x^(2*n))^(1+p)/n/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1468, 629}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{(a+x^n(b+cx^n))^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

fricas [A] time = 0.88, size = 38, normalized size = 1.41

$$\frac{\left(cx^{2\,n}+bx^n+a\right)\left(cx^{2\,n}+bx^n+a\right)^p}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*p + n)
```

giac [A] time = 0.84, size = 27, normalized size = 1.00

$$\frac{\left(cx^{2n} + bx^n + a\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n + a)^(p + 1)/(n*(p + 1))
```

maple [A] time = 0.06, size = 40, normalized size = 1.48

$$\frac{\left(b\,x^n + c\,x^{2n} + a\right)\left(b\,x^n + c\,x^{2n} + a\right)^p}{\left(p+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n)+a)^p,x)
```

```
[Out] (a+b*x^n+c*(x^n)^2)/(p+1)/n*(a+b*x^n+c*(x^n)^2)^p
```

maxima [A] time = 0.72, size = 39, normalized size = 1.44

$$\frac{\left(cx^{2n} + bx^n + a\right)\left(cx^{2n} + bx^n + a\right)^p}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*(p + 1))
```

mupad [B] time = 2.57, size = 56, normalized size = 2.07

$$\left(a + b\,x^n + c\,x^{2\,n}\right)^p\,\left(\frac{a}{n\,\left(p+1\right)} + \frac{b\,x^n}{n\,\left(p+1\right)} + \frac{c\,x^{2\,n}}{n\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] (a + b*x^n + c*x^(2*n))^p*(a/(n*(p + 1)) + (b*x^n)/(n*(p + 1)) + (c*x^(2*n)
)/(n*(p + 1)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.133 $\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=22

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (c*x^2+b*x-a)^(1+p)/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{(x(b+cx)-a)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + x*(b + c*x))^(1 + p)/(1 + p)
```

fricas [A] time = 0.86, size = 32, normalized size = 1.45

$$\frac{\left(cx^2+bx-a\right)\left(cx^2+bx-a\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)
```

giac [A] time = 0.40, size = 22, normalized size = 1.00

$$\frac{\left(cx^2+bx-a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{\left(c\,x^{2}+bx-a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^p,x)
```

```
[Out] (c*x^2+b*x-a)^(p+1)/(p+1)
```

maxima [A] time = 0.42, size = 22, normalized size = 1.00

$$\frac{\left(cx^{2}+bx-a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

mupad [B] time = 2.05, size = 42, normalized size = 1.91

$$\left(\frac{b\,x}{p+1}-\frac{a}{p+1}+\frac{c\,x^{2}}{p+1}\right)\,\left(c\,x^{2}+b\,x-a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^p,x)
```

```
[Out] ((b*x)/(p + 1) - a/(p + 1) + (c*x^2)/(p + 1))*(b*x - a + c*x^2)^p
```

sympy [B] time = 56.66, size = 104, normalized size = 4.73

$$\begin{cases} -\frac{a\left(-a+bx+cx^{2}\right)^{p}}{p+1}+\frac{bx\left(-a+bx+cx^{2}\right)^{p}}{p+1}+\frac{cx^{2}\left(-a+bx+cx^{2}\right)^{p}}{p+1} & \text{for}\ p\neq -1 \\ \log\left(\frac{b}{2c}+x-\frac{\sqrt{4ac+b^{2}}}{2c}\right)+\log\left(\frac{b}{2c}+x+\frac{\sqrt{4ac+b^{2}}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

```
[Out] Piecewise((-a*(-a + b*x + c*x**2)**p/(p + 1) + b*x*(-a + b*x + c*x**2)**p/(
p + 1) + c*x**2*(-a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) +
x - sqrt(4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(4*a*c + b**2)/(2*c))
, True))
```

3.134 $\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^2\right)\\ &= \frac{\left(-a+bx^2+cx^4\right)^{1+p}}{2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

fricas [A] time = 0.91, size = 37, normalized size = 1.37

$$\frac{\left(cx^4+bx^2-a\right)\left(cx^4+bx^2-a\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

giac [A] time = 0.45, size = 25, normalized size = 0.93

$$\frac{\left(cx^4+bx^2-a\right)^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4 + b*x^2 - a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{\left(c\,x^4+b\,x^2-a\right)^{p+1}}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x)
```

```
[Out] 1/2*(c*x^4+b*x^2-a)^(p+1)/(p+1)
```

maxima [A] time = 0.60, size = 37, normalized size = 1.37

$$\frac{\left(cx^4+bx^2-a\right)\left(cx^4+bx^2-a\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

mupad [B] time = 2.05, size = 52, normalized size = 1.93

$$\left(c\,x^4+b\,x^2-a\right)^p\,\left(\frac{b\,x^2}{2\,p+2}-\frac{a}{2\,p+2}+\frac{c\,x^4}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^p,x)
```

```
[Out] (b*x^2 - a + c*x^4)^p*((b*x^2)/(2*p + 2) - a/(2*p + 2) + (c*x^4)/(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
```

```
[Out] Timed out
```

3.135 $\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

[Out] 1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]

[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^p \, dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx) \left(-a+bx+cx^2\right)^p \, dx, x, x^3\right)\\ &= \frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]

[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

fricas [A] time = 0.60, size = 37, normalized size = 1.37

$$\frac{\left(cx^6+bx^3-a\right)\left(cx^6+bx^3-a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

giac [A] time = 0.39, size = 25, normalized size = 0.93

$$\frac{\left(cx^6 + bx^3 - a\right)^{p+1}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3 - a)^(p + 1)/(p + 1)
```

maple [A] time = 0.01, size = 26, normalized size = 0.96

$$\frac{\left(c\,x^6 + b\,x^3 - a\right)^{p+1}}{3p + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x)
[Out] 1/3*(c*x^6+b*x^3-a)^(p+1)/(p+1)
```

maxima [A] time = 0.59, size = 37, normalized size = 1.37

$$\frac{\left(cx^6 + bx^3 - a\right)\left(cx^6 + bx^3 - a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

mupad [B] time = 2.08, size = 52, normalized size = 1.93

$$\left(c\,x^6 + b\,x^3 - a\right)^p\,\left(\frac{b\,x^3}{3\,p+3} - \frac{a}{3\,p+3} + \frac{c\,x^6}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^p,x)
[Out] (b*x^3 - a + c*x^6)^p*((b*x^3)/(3*p + 3) - a/(3*p + 3) + (c*x^6)/(3*p + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)
[Out] Timed out
```

3.136 $\int x^{-1+n}\left(b+2 c x^n\right)\left(-a+b x^n+c x^{2 n}\right)^p dx$

Optimal. Leaf size=29

$$\frac{\left(-a+b x^n+c x^{2 n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (-a+b*x^n+c*x^(2*n))^(1+p)/n/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{\left(-a+b x^n+c x^{2 n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\left(b+2 c x^n\right)\left(-a+b x^n+c x^{2 n}\right)^p dx &= \frac{\text{Subst}\left(\int (b+2 c x)\left(-a+b x+c x^2\right)^p dx, x, x^n\right)}{n}\\ &= \frac{\left(-a+b x^n+c x^{2 n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.97

$$\frac{\left(x^n\left(b+c x^n\right)-a\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

fricas [A] time = 0.93, size = 42, normalized size = 1.45

$$\frac{\left(c x^{2 n}+b x^n-a\right)\left(c x^{2 n}+b x^n-a\right)^p}{n p+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)
```

giac [A] time = 0.82, size = 29, normalized size = 1.00

$$\frac{\left(cx^{2\,n} + bx^n - a\right)^{p+1}}{n\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))
```

maple [A] time = 0.06, size = 45, normalized size = 1.55

$$-\frac{\left(-b\,x^{n} - c\,x^{2n} + a\right)\left(b\,x^{n} + c\,x^{2n} - a\right)^{p}}{\left(p + 1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] -(-c*(x^n)^2-b*x^n+a)/(p+1)/n*(-a+b*x^n+c*(x^n)^2)^p
```

maxima [A] time = 0.71, size = 43, normalized size = 1.48

$$\frac{\left(cx^{2\,n} + bx^n - a\right)\left(cx^{2\,n} + bx^n - a\right)^{p}}{n\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*(p + 1))
```

mupad [B] time = 2.54, size = 59, normalized size = 2.03

$$\left(\frac{b\,x^n}{n\,\left(p+1\right)}-\frac{a}{n\,\left(p+1\right)}+\frac{c\,x^{2\,n}}{n\,\left(p+1\right)}\right)\,{\left(b\,x^n-a+c\,x^{2\,n}\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^p,x)
```

```
[Out] ((b*x^n)/(n*(p + 1)) - a/(n*(p + 1)) + (c*x^(2*n))/(n*(p + 1)))*(b*x^n - a
+ c*x^(2*n))^p
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.137 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

fricas [A] time = 0.81, size = 26, normalized size = 1.37

$$\frac{\left(cx^2+bx\right)\left(cx^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

giac [A] time = 0.41, size = 19, normalized size = 1.00

$$\frac{\left(cx^2+bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$\frac{(cx+b)\,x\left(c\,x^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)
```

```
[Out] (c*x+b)*x/(p+1)*(c*x^2+b*x)^p
```

maxima [A] time = 0.43, size = 19, normalized size = 1.00

$$\frac{\left(cx^2+bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

mupad [B] time = 2.03, size = 23, normalized size = 1.21

$$\frac{x\left(c\,x^2+b\,x\right)^p\,\left(b+c\,x\right)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

```
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

sympy [A] time = 0.66, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx\left(bx+cx^2\right)^p}{p+1}+\frac{cx^2\left(bx+cx^2\right)^p}{p+1} & \text{for } p\neq -1 \\ \log\left(x\right)+\log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.138 $\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=24

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] 1/2*(c*x^4+b*x^2)^(1+p)/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1588}

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{\left(bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 4.04

$$\frac{x^2\left(x^2\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^2*(x^2*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((
c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/
b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

fricas [A] time = 0.76, size = 31, normalized size = 1.29

$$\frac{\left(cx^4+bx^2\right)\left(cx^4+bx^2\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)
```

giac [A] time = 0.40, size = 22, normalized size = 0.92

$$\frac{\left(cx^4+bx^2\right)^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="giac")
[Out] 1/2*(c*x^4 + b*x^2)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 1.29

$$\frac{\left(c\,x^2+b\right)x^2\left(c\,x^4+b\,x^2\right)^p}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x)
[Out] 1/2*(c*x^2+b)*x^2/(p+1)*(c*x^4+b*x^2)^p
```

maxima [A] time = 0.59, size = 35, normalized size = 1.46

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

mupad [B] time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^2\,\left(c\,x^2+b\right)\,\left(c\,x^4+b\,x^2\right)^p}{2\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x)
[Out] (x^2*(b + c*x^2)*(b*x^2 + c*x^4)^p)/(2*(p + 1))
```

sympy [B] time = 17.12, size = 85, normalized size = 3.54

$$\begin{cases} \frac{bx^2\left(bx^2+cx^4\right)^p}{2p+2}+\frac{cx^4\left(bx^2+cx^4\right)^p}{2p+2} & \text{for}\ p\neq -1 \\ \log\left(x\right)+\frac{\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{c}}+x\right)}{2}+\frac{\log\left(i\sqrt{b}\,\sqrt{\frac{1}{c}}+x\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)
[Out] Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2 + c*x**4)
**p/(2*p + 2), Ne(p, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/c) + x)/2 + log(
I*sqrt(b)*sqrt(1/c) + x)/2, True))
```

3.139 $\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=24

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

[Out] 1/3*(c*x^6+b*x^3)^(1+p)/(1+p)

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1588}

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]

[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx = \frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 4.04

$$\frac{x^3 \left(x^3 \left(b+cx^3\right)\right)^p \left(\frac{cx^3}{b}+1\right)^{-p} \left(2c(p+1)x^3 \, {}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2) \, {}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]

[Out] (x^3*(x^3*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)

fricas [A] time = 0.73, size = 31, normalized size = 1.29

$$\frac{\left(cx^6+bx^3\right)\left(cx^6+bx^3\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="fricas")

```
[Out] 1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)
```

giac [A] time = 0.67, size = 22, normalized size = 0.92

$$\frac{\left(cx^6+bx^3\right)^{p+1}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 1.29

$$\frac{\left(c\,x^3+b\right)x^3\left(c\,x^6+b\,x^3\right)^p}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x)
[Out] 1/3*(c*x^3+b)*x^3/(p+1)*(c*x^6+b*x^3)^p
```

maxima [A] time = 0.59, size = 35, normalized size = 1.46

$$\frac{\left(cx^6+bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

mupad [B] time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^3\,\left(c\,x^3+b\right)\,\left(c\,x^6+b\,x^3\right)^p}{3\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x)
[Out] (x^3*(b + c*x^3)*(b*x^3 + c*x^6)^p)/(3*(p + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
[Out] Timed out
```

3.140 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=26

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (b*x^n+c*x^(2*n))^(1+p)/n/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2034, 629}

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx&=\frac{\text{Subst}\left(\int (b+2cx)\left(bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\&=\frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [C] time = 0.13, size = 111, normalized size = 4.27

$$\frac{x^{-np}\left(x^n(b+cx^n)\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3\right.\right.}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((x^n*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p,
2 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 +
p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^(n*p)*(1 + (c*x^n)/b)^p)
```

fricas [**A**] time = 0.64, size = 36, normalized size = 1.38

$$\frac{\left(cx^{2n} + bx^n\right)\left(cx^{2n} + bx^n\right)^p}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] (c*x^(2*n) + b*x^n)*(c*x^(2*n) + b*x^n)^p/(n*p + n)
```

giac [**A**] time = 0.83, size = 26, normalized size = 1.00

$$\frac{\left(cx^{2n} + bx^n\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] (c*x^(2*n) + b*x^n)^(p + 1)/(n*(p + 1))
```

maple [**C**] time = 0.11, size = 155, normalized size = 5.96

$$\frac{(c\,x^n + b)\,x^n \mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(i\left(c\,x^n+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^n+b\right)x^n\right)+i\pi\,\mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(i\left(c\,x^n+b\right)x^n\right)^2+i\pi\,\mathrm{csgn}\left(i\left(c\,x^n+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^n+b\right)x^n\right)^2-i\pi\mathrm{csgn}\left(i\left(c\,x^n+b\right)x^n\right)^3+2\ln(}{2}}}{\left(p+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x)
[Out] x^n*(c*x^n+b)/(p+1)/n*exp(1/2*p*(-I*Pi*csgn(I*x^n*(c*x^n+b))^3+I*Pi*csgn(I*
x^n*(c*x^n+b))^2*csgn(I*x^n)+I*Pi*csgn(I*x^n*(c*x^n+b))^2*csgn(I*(c*x^n+b))
-I*Pi*csgn(I*x^n*(c*x^n+b))*csgn(I*x^n)*csgn(I*(c*x^n+b))+2*ln(x^n)+2*ln(c*
x^n+b)))
```

maxima [**A**] time = 0.77, size = 40, normalized size = 1.54

$$\frac{\left(cx^{2n} + bx^n\right)e^{\left(p\log\left(cx^n+b\right)+p\log\left(x^n\right)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] (c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))
```

mupad [**B**] time = 2.13, size = 34, normalized size = 1.31

$$\frac{x^n\ (b + c\,x^n)\ \left(b\,x^n + c\,x^{2n}\right)^p}{n\ \left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x)
[Out] (x^n*(b + c*x^n)*(b*x^n + c*x^(2*n))^p)/(n*(p + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.141 $\int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=196

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(f*
x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1560, 364}

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^
2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 +
m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m\left(d+ex^n\right)}{a+bx^n+cx^{2n}}\,dx &= \int \left(\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\right)dx\\ &= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx+\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{b-\sqrt{b^2-4ac}+}\\ &= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1\right.}{\left(b+\sqrt{b^2-4a\right.}}\end{aligned}$$

Mathematica [A] time = 0.31, size = 158, normalized size = 0.81

$$\frac{x(fx)^m\left(\left(d\sqrt{b^2-4ac}-2ae+bd\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(d\sqrt{b^2-4ac}+2ae-bd\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};\right.\right.}{2a(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^
2 - 4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)\left(fx\right)^m}{cx^{2n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex^n+d)\left(fx\right)^m}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{(e\,x^n+d)\left(fx\right)^m}{b\,x^n+c\,x^{2n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a),x)
```

```
[Out] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex^n+d)\left(fx\right)^m}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(f\,x\right)^m\,\left(d+e\,x^n\right)}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.142 $\int \frac{(fx)^m(d+ex^n)}{\left(a+bx^n+cx^{2n}\right)^2}\,dx$

Optimal. Leaf size=374

$$-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(\cdots\right.}{\cdots}$$

```
[Out] (f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n/(a+
b*x^n+c*x^(2*n))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(-4*a*c*d*(1+m-2*n)+b^2*d*(1+
m-n)-2*a*b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b-(-4*a*c+b^2
)^(1/2))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(4*a*c*d*(1+m-2*n)-b^2*d*(1+m-n)+2*a*
b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 1.38, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1558, 1560, 364}

$$-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4
*a*c)*f*n*(a + b*x^n + c*x^(2*n))) - (c*((b*d - 2*a*e)*(1 + m - n) - (4*a*c
*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*n) - (c
*((b*d - 2*a*e)*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) +
2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b +
Sqrt[b^2 - 4*a*c])*f*(1 + m)*n)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1558

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)}{\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \frac{(fx)^m\left(-abe(1+m)-2acd(1+m-2n)+b^2d(1+m\cdots\right.}{a+bx^n+cx^{2n}}}{a\left(b^2-4ac\right)\cdots}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int\left(\frac{\left(c(bd-2ae)(1+m-n)+\frac{c\left(b^2d-4acd+b^2dm-4acdm\cdots\right)}{\sqrt{b^2-4\cdots}}\right)}{b-\sqrt{b^2-4ac}+2cx^n\cdots}\cdots\right)}{\cdots}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\left(c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-\cdots}{\cdots}\right.\right.}{a\left(b^2\cdots\right.}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-2\cdots}{\cdots}\right.}{a\left(b^2-4\cdots\right.}$$

Mathematica [B] time = 6.52, size = 5363, normalized size = 14.34

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
[Out] Result too large to show
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(ex^n+d)\left(fx\right)^m}{c^2x^{4n}+b^2x^{2n}+2\,abx^n+a^2+2\,(bcx^n+ac)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
[Out] integral((e*x^n + d)*(f*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 +
 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)\left(fx\right)^m}{\left(cx^{2n}+bx^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^{n}+d)\left(f x\right)^{m}}{\left(b\,x^{n}+c\,x^{2n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^2,x)
```

```
[Out] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^{2}df^{m}-\left(2\,cdf^{m}+bef^{m}\right)a\right)xx^{m}+\left(bcdf^{m}-2\,acef^{m}\right)xe^{\left(m\log(x)+n\log(x)\right)}}{a^{2}b^{2}n-4\,a^{3}cn+\left(ab^{2}cn-4\,a^{2}c^{2}n\right)x^{2\,n}+\left(ab^{3}n-4\,a^{2}bcn\right)x^{n}}-\int \frac{\left(b^{2}df^{m}(m-n+1)-\left(2\,cdf^{m}(m-2\right.\right.}{a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b^2*d*f^m - (2*c*d*f^m + b*e*f^m)*a)*x*x^m + (b*c*d*f^m - 2*a*c*e*f^m)*x*
e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)
*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(((b^2*d*f^m*(m - n + 1)
 - (2*c*d*f^m*(m - 2*n + 1) + b*e*f^m*(m + 1))*a)*x^m + (b*c*d*f^m*(m - n +
 1) - 2*a*c*e*f^m*(m - n + 1))*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*
c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x\right)^{m}\,\left(d+e\,x^{n}\right)}{\left(a+b\,x^{n}+c\,x^{2\,n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

3.143 $\int \frac{(fx)^m(d+ex^n)}{\left(a+bx^n+cx^{2n}\right)^3}\,dx$

Optimal. Leaf size=816

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)+\frac{-d\left(m^2+(2-3n)m\right.}{}\right.}{2a}$$

```
[Out] 1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n
/(a+b*x^n+c*x^(2*n))^2+1/2*(f*x)^(1+m)*((-2*a*c+b^2)*(a*b*e*(1+m)+2*a*c*d*(
1+m-4*n)-b^2*d*(1+m-2*n))+a*b*c*(-2*a*e+b*d)*(1+m-3*n)+c*(a*b^2*e*(1+m)+2*a
*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*x^n)/a^2/(-4*a*c+b^
2)^2/f/n^2/(a+b*x^n+c*x^(2*n))-1/2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1
+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(2+2*m-
7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*(1+m-n)+(a*b^3*e*(1+m)*(1+m-n)-4*
a^2*b*c*e*(1+m^2+m*(2-n)-n-3*n^2)-b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)+6*a*b^2
*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)-8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+8*n^2))/(
-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/f/(1+m)/n^2/(b-(-4*a*c+b^2)^(1/2))-1/
2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)
^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1
+m-2*n))*(1+m-n)+(-a*b^3*e*(1+m)*(1+m-n)+4*a^2*b*c*e*(1+m^2+m*(2-n)-n-3*n^2
)+b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)-6*a*b^2*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)
+8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+8*n^2))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b
^2)^2/f/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 4.55, antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1558, 1560, 364}

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)+\frac{-d\left(m^2+(2-3n)m\right.}{}\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 -
 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + ((f*x)^(1 + m)*((b^2 - 2*a*c)*(a*b
*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1 + m - 2*n)) + a*b*c*(b*d - 2*
a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2
*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*f*n^
2*(a + b*x^n + c*x^(2*n))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*
n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b^3*e*
(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n)
 - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b
^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 -
4*a*c])*f*(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*b^3*e*(1 +
 m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1
+ m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4
*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 -
 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*
c])*f*(1 + m)*n^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1558

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)}{\left(a+bx^n+cx^{2n}\right)^3}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{(fx)^m\left(-abe(1+m)-2acd(1+m-4n)+b^2d(1+m-2n\right.}{\left(a+bx^n+cx^{2n}\right)^2}}{2a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd\right.\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd\right.\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd\right.\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd\right.\right.}{}$$

Mathematica [B] time = 7.54, size = 20515, normalized size = 25.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)(fx)^m}{c^3x^{6n}+b^3x^{3n}+3ab^2x^{2n}+3a^2bx^n+a^3+3(bc^2x^n+ac^2)x^{4n}+3(b^2cx^{2n}+2abcx^n+a^2c)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) +
 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a
*b*c*x^n + a^2*c)*x^(2*n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^n+d)(fx)^m}{(b\,x^n+c\,x^{2n}+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^3,x)
```

```
[Out] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*b^4*d*f^m*(m - 3*n + 1) + 2*(b*c*e*f^m*(2*m - 5*n + 2) + 2*c^2*d*f
^m*(m - 6*n + 1))*a^3 - (b^2*c*d*f^m*(5*m - 21*n + 5) + b^3*e*f^m*(m - n +
1))*a^2)*x*x^m + (b^3*c^2*d*f^m*(m - 2*n + 1) + 4*a^2*c^3*e*f^m*(m - 3*n +
1) - (2*b*c^3*d*f^m*(2*m - 7*n + 2) + b^2*c^2*e*f^m*(m + 1))*a)*x*e^(m*log(
x) + 3*n*log(x)) + (2*b^4*c*d*f^m*(m - 2*n + 1) + 2*(b*c^2*e*f^m*(4*m - 9*n
 + 4) + 2*c^3*d*f^m*(m - 4*n + 1))*a^2 - (b^2*c^2*d*f^m*(9*m - 29*n + 9) +
2*b^3*c*e*f^m*(m + 1))*a)*x*e^(m*log(x) + 2*n*log(x)) + (b^5*d*f^m*(m - 2*n
 + 1) + 4*a^3*c^2*e*f^m*(m - 5*n + 1) + (b^2*c*e*f^m*(3*m - 4*n + 3) + 2*b*
c^2*d*f^m*n)*a^2 - (4*b^3*c*d*f^m*(m - 3*n + 1) + b^4*e*f^m*(m + 1))*a)*x*e
^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (
a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*
c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^
3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 +
16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n +
 1)*b^4*d*f^m + 2*(2*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*c^2*d*f^m + (2
```

```
*m^2 - m*(5*n - 4) - 5*n + 2)*b*c*e*f^m)*a^2 - ((5*m^2 - m*(21*n - 10) + 16
*n^2 - 21*n + 5)*b^2*c*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^3*e*f^m)*a)*x^m
+ ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d*f^m + 4*(m^2 - 2*m*(2*n -
1) + 3*n^2 - 4*n + 1)*a^2*c^2*e*f^m - (2*(2*m^2 - m*(9*n - 4) + 7*n^2 - 9*n
 + 2)*b*c^2*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^2*c*e*f^m)*a)*e^(m*log(x) +
 n*log(x)))/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^
2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*
c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x\right)^m\,\left(d+e\,x^n\right)}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

3.144 $\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx$

Optimal. Leaf size=47

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

```
[Out] -3*ln(c^(2/3)-c^(1/3)*d^(1/3)*x^(1/3)+d^(2/3)*x^(2/3))/c^(1/3)/d^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1594, 1468, 628}

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx &= \int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{\left(c\sqrt[3]{d}-c^{2/3}d^{2/3}\sqrt[3]{x}+\sqrt[3]{c}dx^{2/3}\right)x^{2/3}}\,dx\\
&= 3\,\text{Subst}\left(\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{c\sqrt[3]{d}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^2}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3
)*x + c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

fricas [A] time = 0.89, size = 33, normalized size = 0.70

$$-\frac{3\log\left(dx^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+c^{\frac{2}{3}}d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="fricas")
```

```
[Out] -3*log(d*x^(2/3) - c^(1/3)*d^(2/3)*x^(1/3) + c^(2/3)*d^(1/3))/(c^(1/3)*d^(2
/3))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[%%%{%%{[%%%{1,[1]%%%},0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%},0]:[1
,0,0,%%%{-1,[1]%%%}]%%},[2]%%%}+%%%{%%%{%%{[%%%{-1,[1]%%%},0,0]:[1,0,0,%%%{
-1,[1]%%%}]%%},[1]%%%},[1]%%%}+%%%{%%{[%%%{%%%{1,[2]%%%},[0]%%%},0,0]:[1,0,
0,%%%{-1,[1]%%%}]%%},[0]%%%} / %%%{%%%{%%{[-1,0]:[1,0,0,%%%{-1,[1]%%%}]%%},
[2]%%%},[2]%%%}+%%%{%%{[%%%{%%{[1,0,0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%},0,
0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%}+%%%{%%{[%%%{%%%{-1,[1]%%%},[1]%%%},0]:
[1,0,0,%%%{-1,[1]%%%}]%%},[0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$-\frac{3\ln\left(-c^{\frac{1}{3}}d\,x^{\frac{2}{3}}+c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}-c\,d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3
)*d*x^(4/3)),x)
```

```
[Out] -3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*x^(2/3)*d-c*d^(1/3))
```

maxima [A] time = 0.45, size = 34, normalized size = 0.72

$$-\frac{3\log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}}-c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="maxima")
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d^
(2/3))
```

mupad [B] time = 2.46, size = 31, normalized size = 0.66

$$-\frac{3\,\ln\left(x^{2/3}+\frac{c^{2/3}}{d^{2/3}}-\frac{c^{1/3}\,x^{1/3}}{d^{1/3}}\right)}{c^{1/3}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x)
```

```
[Out] -(3*log(x^(2/3) + c^(2/3)/d^(2/3) - (c^(1/3)*x^(1/3))/d^(1/3)))/(c^(1/3)*d^
(2/3))
```

sympy [C] time = 6.32, size = 126, normalized size = 2.68

$$-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}-\frac{\sqrt{3}\,i\sqrt{c^{\frac{4}{3}}}\,\sqrt{d^{\frac{4}{3}}}}{2\sqrt[3]{c}\,d}\right)}{\sqrt[3]{c}\,d^{\frac{2}{3}}}-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}+\frac{\sqrt{3}\,i\sqrt{c^{\frac{4}{3}}}\,\sqrt{d^{\frac{4}{3}}}}{2\sqrt[3]{c}\,d}\right)}{\sqrt[3]{c}\,d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**(
2/3)*x+c**(1/3)*d*x**(4/3)),x)
```

```
[Out] -3*log(-c**(1/3)/(2*d**(1/3)) + x**(1/3) - sqrt(3)*I*sqrt(c**(4/3))*sqrt(d*
*(4/3))/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3)) - 3*log(-c**(1/3)/(2*d**(1/3))
+ x**(1/3) + sqrt(3)*I*sqrt(c**(4/3))*sqrt(d**(4/3))/(2*c**(1/3)*d))/(c**(1
/3)*d**(2/3))
```

3.145 $\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$

Optimal. Leaf size=245

$$\frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m}{}\right.}{f(m+1)\sqrt{b^2-4ac}}$$

```
[Out] 2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m+n)/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(b-(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m+n)
/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(-4*a*
c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.54, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1556, 511, 510}

$$\frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m}{}\right.}{f(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b -
Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e
*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 +
 m)*(1 + (e*x^n)/d)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{(fx)^m}{b+\sqrt{b^2-}}}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(fx)^{1+m}\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{1+m}{n};1,-q;\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{2c(fx)^{1+}}{}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)^q\left(fx\right)^m}{cx^{2\,n}+bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q\left(fx\right)^m}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m (e\,x^n+d)^q}{b\,x^n+c\,x^{2n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
[Out] int((f*x)^m*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q\left(fx\right)^m}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x\right)^m\,{\left(d+e\,x^n\right)}^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.146 $\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=210

$$-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\right.}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] -2/3*c*x^3*(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^3*
(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^
n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.50, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1556, 511, 510}

$$-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\right.}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2 (d + ex^n)^q}{a + bx^n + cx^{2n}} dx = \frac{(2c) \int \frac{x^2(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{x^2(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2 - 4ac}}$$

$$= \frac{\left(2c (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{x^2\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2 - 4ac}} - \frac{\left(2c (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{x^2\left(1+\frac{ex^n}{d}\right)}{b+\sqrt{b^2-4ac}+ \cdots}}{\sqrt{b^2 - 4ac}}$$

$$= -\frac{2cx^3 (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(\frac{3}{n};1,-q;\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2 - 4ac - b\sqrt{b^2 - 4ac}\right)} - \frac{2cx^3 (d + ex^n)^q \left(1 + \frac{ex^n}{d} \cdots\right.}{3\left(b^2 \cdots\right.}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^2 (e x^n + d)^q}{b x^n + c x^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
[Out] int(x^2*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(d+e\,x^n\right)}^q}{a+b\,x^n+c\,x^{2\,n}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.147 $\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=206

$$-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2c}{b+\sqrt{b}}\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -c*x^2*(d+e*x^n)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*(d+e*x^n
)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/((1
+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.38, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1556, 511, 510}

$$-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2c}{b+\sqrt{b}}\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e
*x^n)/d)^q)) - (c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(1 + (e*x^n)/d)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{x(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x\left(1+\right.}{b+\sqrt{b^2-4}}}{\sqrt{b^2-4ac}}$$

$$= -\frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2(d+ex^n)^q\left(1+\frac{ex}{d}\right.}{b^2}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x(e\,x^n+d)^q}{b\,x^n+c\,x^{2n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
[Out] int(x*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.148 $\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=194

$$-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -2*c*x*(d+e*x^n)^q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),
-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(d+e*x^n)^
q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/((1+e*x
^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.30, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1428, 430, 429}

$$-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+}}{\sqrt{b^2-4ac}}$$

$$= -\frac{2cx\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right.}{b^2}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)^q}{cx^{2n}+bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^n+d)^q}{b\,x^n+c\,x^{2n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
[Out] int((e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q}{cx^{2n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x)
[Out] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.149 $\int \frac{(d+ex^n)^q}{x\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=263

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2\right.}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+\right.\right.}$$

```
[Out] -(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^n/d)/a/d/n/(1+q)+c*(d+e*x^n
)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/
2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+c
*(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))
```

Rubi [A] time = 0.73, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1474, 960, 65, 830, 68}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2\right.}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*n*(1
 + q)) - ((d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n
)/d])/(a*d*n*(1 + q))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax} + \frac{(-b-cx)(d+ex)^q}{a\left(a+bx+cx^2\right)}\right) dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx, x, x^n\right)}{an}$$

$$= -\frac{(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)} + \frac{\text{Subst}\left(\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-c+\cdots\right.}{b+\sqrt{\cdots}}\right.\right.}{an}$$

$$= -\frac{(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+\cdots}\right.}{an}$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\cdots}{a\cdots}$$

Mathematica [A]   time = 0.67, size = 218, normalized size = 0.83

$$\frac{(d+ex^n)^{q+1}\left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,q+1;q+2;\frac{ex^n}{d}+\cdots\right)}{d}\right.}{an(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] ((d + e*x^n)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^n))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^n)/d]/d))/(a*n*(1 + q))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)^q}{cxx^{2\,n}+bxx^n+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x*x^(2*n) + b*x*x^n + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q}{\left(cx^{2\,n}+bx^n+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^n+d)^q}{\left(b\,x^n+c\,x^{2n}+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^n+d)^q/x/(b*x^n+c*x^(2*n)+a),x)
[Out] int((e*x^n+d)^q/x/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q}{\left(cx^{2\,n}+bx^n+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^q}{x\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.150 $\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})} dx$

Optimal. Leaf size=212

$$\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\right.}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] 2*c*(d+e*x^n)^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2))
,-e*x^n/d)/x/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*(d+e*x^n)
^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/x/
((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.49, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1556, 511, 510}

$$\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\right.}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x*(1
 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt
[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{}{x^2(}}{\sqrt{b^2-4ac}}$$

$$= \frac{2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c\,(d+ex^n)^q\left(1+\right.}{(}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^q}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
[Out] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)^q}{cx^2x^{2n}+bx^2x^n+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q}{\left(cx^{2n}+bx^n+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^n+d)^q}{\left(b\,x^n+c\,x^{2n}+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^n+d)^q/x^2/(b*x^n+c*x^(2*n)+a),x)
[Out] int((e*x^n+d)^q/x^2/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x^n)^q}{x^2\,(a + b\,x^n + c\,x^{2\,n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.151 $\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=210

$$\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] c*(d+e*x^n)^q*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-
e*x^n/d)/x^2/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*(d+e*x^n)^q
*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/x^2/
((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.48, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1556, 511, 510}

$$\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2*(1 +
(e*x^n)/d)^q) + (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4
*a*c])*x^2*(1 + (e*x^n)/d)^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int}{\sqrt{b^2-4a}}$$

$$= \frac{c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c\,(d+ex^n)^q\left(1+\right.}{}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
[Out] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)^q}{cx^3x^{2n}+bx^3x^n+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n+d)^q}{\left(cx^{2n}+bx^n+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^n+d)^q}{\left(b\,x^n+c\,x^{2n}+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^n+d)^q/x^3/(b*x^n+c*x^(2*n)+a),x)
[Out] int((e*x^n+d)^q/x^3/(b*x^n+c*x^(2*n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{\left(cx^{2n} + bx^n + a\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x^n)^q}{x^3\,\left(a + b\,x^n + c\,x^{2\,n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.152 $\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=498

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c
*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+2*d
*e*x^(1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+2*n)
/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)
/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))
^p)+e^2*x^(1+2*n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+2*n)/n,-p,-p,
(1+m+3*n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)/(1+m+2*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b
^2)^(1/2)))^p)
```

Rubi [A] time = 0.61, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1560, 1385, 510, 20}

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1
 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x
^(2*n))^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 +
(2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m +
 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \int \left(d^2(fx)^m \left(a+bx^n+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p + e^2x^{2n}\right.$$

$$= d^2 \int (fx)^m \left(a+bx^n+cx^{2n}\right)^p dx + (2de) \int x^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p$$

$$= \left(2dex^{-m}(fx)^m\right) \int x^{m+n} \left(a+bx^n+cx^{2n}\right)^p dx + \left(e^2x^{-m}(fx)^m\right) \int x^{m+2n}$$

$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\right.}{f(1+m)}$$

$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\right.}{f(1+m)}$$

Mathematica [A] time = 1.14, size = 391, normalized size = 0.79

$$\frac{x(fx)^m \left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p} (a+x^n(b+cx^n))^p \left(d^2\left(m^2+m(3n+2)+2n^2+3n+1\right)F_1\left(\frac{m+1}{n};\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n
))*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^n*(2*d*(1 + m + 2*
n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m + n)*x^n*Appell
F1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + m)*(1 + m + n)*(1 + m +
2*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^{2n}+2\,dex^n+d^2\right)\left(cx^{2n}+bx^n+a\right)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,
 x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming x near 0Simplificati
on assuming f near 0Simplification assuming x near 0Simplification assuming
 f near 0Unable to divide, perhaps due to rounding error%%%{128,[1,0,5,3,0,
5,4,1,6,1]%%%}+%%%{512,[1,0,5,3,0,5,3,1,6,1]%%%}+%%%{768,[1,0,5,3,0,5,2,1,6
,1]%%%}+%%%{512,[1,0,5,3,0,5,1,1,6,1]%%%}+%%%{128,[1,0,5,3,0,5,0,1,6,1]%%%}
+%%%{-96,[1,0,5,3,0,4,4,3,5,1]%%%}+%%%{-384,[1,0,5,3,0,4,3,3,5,1]%%%}+%%%{-
576,[1,0,5,3,0,4,2,3,5,1]%%%}+%%%{-384,[1,0,5,3,0,4,1,3,5,1]%%%}+%%%{-96,[1
,0,5,3,0,4,0,3,5,1]%%%}+%%%{24,[1,0,5,3,0,3,4,5,4,1]%%%}+%%%{96,[1,0,5,3,0,
3,3,5,4,1]%%%}+%%%{144,[1,0,5,3,0,3,2,5,4,1]%%%}+%%%{96,[1,0,5,3,0,3,1,5,4,
1]%%%}+%%%{24,[1,0,5,3,0,3,0,5,4,1]%%%}+%%%{-2,[1,0,5,3,0,2,4,7,3,1]%%%}+%%
%{-8,[1,0,5,3,0,2,3,7,3,1]%%%}+%%%{-12,[1,0,5,3,0,2,2,7,3,1]%%%}+%%%{-8,[1,
0,5,3,0,2,1,7,3,1]%%%}+%%%{-2,[1,0,5,3,0,2,0,7,3,1]%%%}+%%%{64,[1,0,5,2,1,5
,3,1,6,1]%%%}+%%%{192,[1,0,5,2,1,5,2,1,6,1]%%%}+%%%{192,[1,0,5,2,1,5,1,1,6,
1]%%%}+%%%{64,[1,0,5,2,1,5,0,1,6,1]%%%}+%%%{-48,[1,0,5,2,1,4,3,3,5,1]%%%}+%
%%{-144,[1,0,5,2,1,4,2,3,5,1]%%%}+%%%{-144,[1,0,5,2,1,4,1,3,5,1]%%%}+%%%{-4
8,[1,0,5,2,1,4,0,3,5,1]%%%}+%%%{12,[1,0,5,2,1,3,3,5,4,1]%%%}+%%%{36,[1,0,5,
2,1,3,2,5,4,1]%%%}+%%%{36,[1,0,5,2,1,3,1,5,4,1]%%%}+%%%{12,[1,0,5,2,1,3,0,5
,4,1]%%%}+%%%{-1,[1,0,5,2,1,2,3,7,3,1]%%%}+%%%{-3,[1,0,5,2,1,2,2,7,3,1]%%%}
+%%%{-3,[1,0,5,2,1,2,1,7,3,1]%%%}+%%%{-1,[1,0,5,2,1,2,0,7,3,1]%%%}+%%%{64,[
1,0,5,2,0,5,3,1,6,1]%%%}+%%%{192,[1,0,5,2,0,5,2,1,6,1]%%%}+%%%{192,[1,0,5,2
,0,5,1,1,6,1]%%%}+%%%{64,[1,0,5,2,0,5,0,1,6,1]%%%}+%%%{-48,[1,0,5,2,0,4,3,3
,5,1]%%%}+%%%{-144,[1,0,5,2,0,4,2,3,5,1]%%%}+%%%{-144,[1,0,5,2,0,4,1,3,5,1]
%%%}+%%%{-48,[1,0,5,2,0,4,0,3,5,1]%%%}+%%%{12,[1,0,5,2,0,3,3,5,4,1]%%%}+%%%
{36,[1,0,5,2,0,3,2,5,4,1]%%%}+%%%{36,[1,0,5,2,0,3,1,5,4,1]%%%}+%%%{12,[1,0,
5,2,0,3,0,5,4,1]%%%}+%%%{-1,[1,0,5,2,0,2,3,7,3,1]%%%}+%%%{-3,[1,0,5,2,0,2,2
,7,3,1]%%%}+%%%{-3,[1,0,5,2,0,2,1,7,3,1]%%%}+%%%{-1,[1,0,5,2,0,2,0,7,3,1]%%
%}+%%%{128,[0,0,5,2,1,5,3,0,7,1]%%%}+%%%{384,[0,0,5,2,1,5,2,0,7,1]%%%}+%%%{
384,[0,0,5,2,1,5,1,0,7,1]%%%}+%%%{128,[0,0,5,2,1,5,0,0,7,1]%%%}+%%%{-96,[0,
0,5,2,1,4,3,2,6,1]%%%}+%%%{-288,[0,0,5,2,1,4,2,2,6,1]%%%}+%%%{-288,[0,0,5,2
,1,4,1,2,6,1]%%%}+%%%{-96,[0,0,5,2,1,4,0,2,6,1]%%%}+%%%{24,[0,0,5,2,1,3,3,4
,5,1]%%%}+%%%{72,[0,0,5,2,1,3,2,4,5,1]%%%}+%%%{72,[0,0,5,2,1,3,1,4,5,1]%%%}
+%%%{24,[0,0,5,2,1,3,0,4,5,1]%%%}+%%%{-2,[0,0,5,2,1,2,3,6,4,1]%%%}+%%%{-6,[
0,0,5,2,1,2,2,6,4,1]%%%}+%%%{-6,[0,0,5,2,1,2,1,6,4,1]%%%}+%%%{-2,[0,0,5,2,1
,2,0,6,4,1]%%%}+%%%{128,[0,0,5,2,0,5,3,0,7,1]%%%}+%%%{384,[0,0,5,2,0,5,2,0,
7,1]%%%}+%%%{384,[0,0,5,2,0,5,1,0,7,1]%%%}+%%%{128,[0,0,5,2,0,5,0,0,7,1]%%%
}+%%%{-96,[0,0,5,2,0,4,3,2,6,1]%%%}+%%%{-288,[0,0,5,2,0,4,2,2,6,1]%%%}+%%%{
-288,[0,0,5,2,0,4,1,2,6,1]%%%}+%%%{-96,[0,0,5,2,0,4,0,2,6,1]%%%}+%%%{24,[0,
0,5,2,0,3,3,4,5,1]%%%}+%%%{72,[0,0,5,2,0,3,2,4,5,1]%%%}+%%%{72,[0,0,5,2,0,3
,1,4,5,1]%%%}+%%%{24,[0,0,5,2,0,3,0,4,5,1]%%%}+%%%{-2,[0,0,5,2,0,2,3,6,4,1]
%%%}+%%%{-6,[0,0,5,2,0,2,2,6,4,1]%%%}+%%%{-6,[0,0,5,2,0,2,1,6,4,1]%%%}+%%%{
-2,[0,0,5,2,0,2,0,6,4,1]%%%} / %%%{64,[0,0,5,3,0,5,3,0,6,0]%%%}+%%%{192,[0,
0,5,3,0,5,2,0,6,0]%%%}+%%%{192,[0,0,5,3,0,5,1,0,6,0]%%%}+%%%{64,[0,0,5,3,0,
5,0,0,6,0]%%%}+%%%{-48,[0,0,5,3,0,4,3,2,5,0]%%%}+%%%{-144,[0,0,5,3,0,4,2,2,
5,0]%%%}+%%%{-144,[0,0,5,3,0,4,1,2,5,0]%%%}+%%%{-48,[0,0,5,3,0,4,0,2,5,0]%%
%}+%%%{12,[0,0,5,3,0,3,3,4,4,0]%%%}+%%%{36,[0,0,5,3,0,3,2,4,4,0]%%%}+%%%{36
,[0,0,5,3,0,3,1,4,4,0]%%%}+%%%{12,[0,0,5,3,0,3,0,4,4,0]%%%}+%%%{-1,[0,0,5,3
,0,2,3,6,3,0]%%%}+%%%{-3,[0,0,5,3,0,2,2,6,3,0]%%%}+%%%{-3,[0,0,5,3,0,2,1,6,
3,0]%%%}+%%%{-1,[0,0,5,3,0,2,0,6,3,0]%%%} Error: Bad Argument Value
```

maple [**F**] time = 0.07, size = 0, normalized size = 0.00

$$\int (e\,x^{n}+d)^{2} \left(f x\right)^{m} \left(b\,x^{n}+c\,x^{2n}+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)^2*(b*x^n+c*x^(2*n)+a)^p,x)
[Out] int((f*x)^m*(e*x^n+d)^2*(b*x^n+c*x^(2*n)+a)^p,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^{n}+d)^{2}\left(cx^{2\,n}+bx^{n}+a\right)^{p} \left(fx\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f\,x\right)^{m} (d+e\,x^{n})^{2} \left(a+b\,x^{n}+c\,x^{2\,n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

3.153 $\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{f(m+1)}$$

```
[Out] d*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x
^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e*x^(
1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+2*n)/n,-2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)/((1+2
*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.37, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1560, 1385, 510, 20}

$$\frac{d(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
 m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n)
)^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int (fx)^m (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p dx = \int \left(d(fx)^m\left(a+bx^n+cx^{2n}\right)^p + ex^n(fx)^m\left(a+bx^n+cx^{2n}\right)^p\right) dx$$
$$= d\int (fx)^m\left(a+bx^n+cx^{2n}\right)^p dx + e\int x^n(fx)^m\left(a+bx^n+cx^{2n}\right)^p dx$$
$$= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+bx^n+cx^{2n}\right)^p dx + \left(d\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\right.$$
$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+}{n}\right.}{f(1+m)}$$
$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+}{n}\right.}{f(1+m)}$$

Mathematica [A] time = 0.60, size = 273, normalized size = 0.85

$$\frac{x(fx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p}(a+x^n(b+cx^n))^p\left(d(m+n+1)F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4}}\right.\right.}{(m+1)(m+n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d*(1 + m + n)*AppellF1[(1 + m)/n, -p, -
p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + e*(1 + m)*x^n*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
]))/((1 + m)*(1 + m + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left((ex^n+d)\left(cx^{2n}+bx^n+a\right)^p(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (e\,x^{n}+d)\left(f x\right)^{m}\left(b\,x^{n}+c\,x^{2n}+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)*(b*x^n+c*x^(2*n)+a)^p,x)
[Out] int((f*x)^m*(e*x^n+d)*(b*x^n+c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^{n}+d)\left(cx^{2\,n}+bx^{n}+a\right)^{p}\left(fx\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f\,x\right)^{m}\,\left(d+e\,x^{n}\right)\,\left(a+b\,x^{n}+c\,x^{2\,n}\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

3.154 $\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=158

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

```
[Out] (f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 510}

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (fx)^m \left(1+\right.$$

$$= \frac{(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\right.}{f(1+m)}$$

Mathematica [A] time = 0.30, size = 181, normalized size = 1.15

$$\frac{x(fx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+x^n\left(b+cx^n\right)\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \left(b\,x^n + c\,x^{2n} + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p,x)
```

```
[Out] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(f\,x\right)}^m\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.155 $\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$

Optimal. Leaf size=34

$$\operatorname{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx=\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Mathematica [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(cx^{2n}+bx^n+a\right)^p\left(fx\right)^m}{ex^n+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^p\left(fx\right)^m}{ex^n+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(b\,x^{n}+c\,x^{2n}+a\right)^{p}}{e\,x^{n}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d),x)
[Out] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2\,n}+bx^{n}+a\right)^{p} \left(fx\right)^{m}}{ex^{n}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(f\,x\right)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p}{d+e\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x)
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Timed out
```

3.156 $\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx=\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

fricas [A] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^{2n}+bx^n+a\right)^p\left(fx\right)^m}{e^2x^{2n}+2\,dex^n+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^p\left(fx\right)^m}{(ex^n+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(f x\right)^{m} \left(b\,x^{n}+c\,x^{2 n}+a\right)^{p}}{\left(e\,x^{n}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d)^2,x)
[Out] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2\,n}+bx^{n}+a\right)^{p} \left(f x\right)^{m}}{\left(ex^{n}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(f\,x\right)^{m}\,\left(a+b\,x^{n}+c\,x^{2\,n}\right)^{p}}{\left(d+e\,x^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x)
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```